Longevity and Environmental Quality in an OLG Model

P-A. Jouvet*, P. Pestieau† and G. Ponthiere‡

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Abstract

Whereas existing OLG models with endogenous longevity neglect the impact of environmental quality on mortality, this paper studies the design of the optimal public intervention in a two-period OLG model where longevity is influenced positively by health expenditures, but negatively by pollution due to production. It is shown that if individuals, when choosing how much to spend on health, do not internalize the impact of their decision on environmental quality (i.e. the space available for each person), the decentralization of the social optimum requires a tax not only on capital income, but also, on health expenditures. The sensitivity of the optimal second-best public intervention is also explored numerically.

1 Introduction

The simultaneous growth of economic activity and longevity observed during the last two centuries in industrialized countries has raised the issue of the relationship between those two phenomena. In the recent years, the study of interactions between economic development and longevity has been particularly enriched by OLG models with endogenous longevity (Chakraborty, 2004; Battacharya and Qiao, 2005; Zhang *et al*, 2006). But those models, by making longevity depend on (private and/or public) health expenditures only, tend to neglect the influence of the natural environment on longevity.

*EconomiX, University Paris X, CORE.
†CREPP, University of Liege, CORE and PSE.
‡CREPP, University of Liege and FRS-FNRS.

However, as this was emphasized by demographers (see Sartor, 2001), the natural environment constitutes a major determinant of longevity. Environmental quality affects longevity through various channels: the climate – i.e. temperature (Kunst *et al*, 1993) and solar radiations (Elwood *et al*, 1974) – the quality of lands (Kjellström, 1986), of waters (Sartor and Rondia, 1983) and of the air (Kinney and Ozkanyak, 1991).

Given that environmental determinants of longevity are significantly influenced by economic activity, introducing the environment in the analysis can contribute to refine the study of the relationship between economic activity and longevity, which is assumed, in existing models, to be merelly 'globally positive'. Although including environmental quality does not question the positive relationship observed, this may, however, be most relevant for public policy analysis, because actual policies may be non-optimal: the observed production and longevity levels may, under actual interactions between production, environment and longevity, differ from what would maximize lifetime welfare.

The goal of this paper is to examine the issue of the optimal public intervention in a two-period OLG economy, in which the length of the second period of life is influenced not only by private health expenditures, but, also, by environmental quality. For that purpose, environmental quality shall enter our model in two distinct ways, each of these involving specific externalities, which can partially offset each others.

First, the production process is assumed to generate polluting emissions, whose negative effects on longevity are not taken into account by producers. The stock of pollution at each point in time, depending on the stock of pollution at the previous period and on current polluting emissions, tends to lower longevity, and, as such, accounts for the – widely documented – negative impact of the pollution of land, waters and the air on longevity.

Second, environmental quality is assumed to influence welfare not only indirectly, through the effects of pollution on longevity, but, also, directly, through the quantity of space available for each person. It is here postulated that individuals, when choosing their health spending, do not internalize the impact of their decisions on the number of persons.[1] However, such a behaviour is not without consequences on environmental quality, because the earth is, in Boulding's (1966) terms, nothing else than a 'spaceship'.[2] Hence,

[1] Actually, the same externalities appear in fertility decision, which involves the well-known tragedy of the commons: with fixed natural resources, free riding agents adopt a too high fertility rate. In the present context, free riding agents tend to live too long.

[2] Boulding's (1966) expression means that the earth is becoming a single spaceship without illimited reservoirs of anything, either for extraction or for pollution, and in which therefore man must find his place in a cyclical ecological system.

individuals, by ignoring that the lengthening of their life causes – under the limiteness of the 'spaceship' – undesirable crowding effects, tend to overspend in health.

What is interesting here is that these two external effects can partially offset each others. Pollution hurts longevity, which can be desirable if longevity decisions do not internalize their effects on environmental quality. The existence of those two partially offsetting externalities raises the question of what public intervention should be: under those externalities, should a government tax or subsidize revenues from production and health expenditures?

This paper aims at casting a new light on the normative study of the growth-longevity relationship, by examining the optimal public intervention in an OLG economy where those two externalities are present. For that purpose, we shall assume that fertility is exogenous and fixed. This constitutes a non negligible restriction of the set of available instruments: undoubtedly, if the issue of 'earth as spaceship' must be tackled, it is preferable to do it through fertility than mortality. However, given that fertility decisions depend on various factors, a government has only a quite imperfect control on it, so that it makes sense, for our purpose, to abstract from fertility, and to concentrate on the interactions between production, environment and longevity under a fixed fertility.

The rest of the paper is organized as follows. Section 2 presents the model. The steady-state is characterized in Section 3. The decentralization of the social optimum is discussed in Section 4. Section 5 analyses the second best policy, whose sensitivity is studied numerically in Section 6. Section 7 concludes.

2 The model

We consider an overlapping generations model with endogenous life longevity and unpriced pollution.

2.1 Firms and pollution

We assume the existence of a neo-classical production sector using a quantity K of capital and L of labor. We assume that capital fully depreciates during the process of production. At each time t, firms produce a good (Y_t) with a well-behaved production function,

$$Y_t = F(K_t, L_t) \tag{1}$$

Within the framework of a competitive equilibrium, each firm, at time t, chooses the quantity of capital and labor which will maximize its profit,

$$\pi_t = F(K_t, L_t) - R_t K_t - w_t L_t \tag{2}$$

At equilibrium, the levels of return from the inputs correspond respectively to their marginal productivity:

$$R_t = F_K(K_t, L_t) \tag{3}$$

$$w_t = F_L(K_t, L_t) \tag{4}$$

where R_t is the interest rate for savings and w_t the wage rate.
At each period, the flow of pollution emission is equal to a proportion η of current production,

$$E_t = \eta F(K_t, L_t) \tag{5}$$

The dynamics of the stock of pollution at time t, P_t, is defined by

$$P_t = (1 - \delta)P_{t-1} + E_t \tag{6}$$

where δ the natural level of pollution absorption, $0 \leq \delta \leq 1$.

2.2 Agents

We suppose that the population is constant, and that at each time t, N identical agents are born. Each agent lives through two life periods. He or she works during the first period of life and is a pensioner for the second period of life.
The first period is of unitary length; the second period lasts a period h, inferior to 1, which can be increased through health spending (like primary prevention in first period), x_t, in the second period and decreased by industrial pollution, P_{t+1} (see Evans and Smith, 2005). The longevity function, $h(x_t, P_{t+1})$, is strictly concave with $h_x > 0$, $h_{xx} < 0$, $h_P < 0$, $h_{PP} > 0$.
Any agent born in period t derives utility from consumption, c_t and the amount of space or land per person, q_t, in her/his first period of life and from consumption, d_{t+1}, and the amount of space or land per person, q_{t+1}, in her/his second period of life.
The preferences of the agents are represented by a utility function, $U(c_t, q_t)$ $+h(x_t, P_{t+1})U(d_{t+1}, q_{t+1})$ where $q_t = \overline{Q}/N(1 + h(x_{t-1}, P_t))$ with $\overline{Q}$ the given total quantity of space. $U(.)$ is supposed to be strictly concave with $U_i > 0$

(for $i = c, d$), $U_q > 0$ and the cross derivative is assumed to be non negative, $U_{iq} \geq 0$.[3]

During the first period of life, each agent supplies one inelastic unit of labor and receives the wage, w_t, which he or she consumes, c_t and saves s_t in the form of capital and spend x_t for health.
Let us introduce a tax ξ on health spending and a tax τ on capital income, along with a lump-sum subsidy a. The budget constraint in the first period of life is:

$$w_t + a = c_t + s_t + (1 + \xi)x_t \tag{7}$$

In the second period of life, the agents receive the return of their saving, $R_{t+1}s_t$ and consume d_{t+1} during $h(x_t, P_{t+1})$. The budget constraint in the second period of life of an agent born in t is therefore:

$$h(x_t, P_{t+1})d_{t+1} = (R_{t+1} - \tau)\, s_t. \tag{8}$$

The problem of each individual is thus to maximize:

$$U(c_t, q_t) + h(x_t, P_{t+1})U(d_{t+1}, q_{t+1})$$

subject to (7) and (8) by choosing x_t and s_t. What is crucial is that, when choosing x_t and s_t, individuals do not perceive the effect of their decisions on the environmental variables: pollution and space. Since the individual does not see the effect of savings on pollution, nor the effect of health spending on total population $N(1 + h(x_t, P_{t+1}))$, one obtains the first order conditions for savings

$$-U_{c_t} + (R_{t+1} - \tau)\, U_{d_{t+1}} = 0, \tag{9}$$

and for health spending

$$-(1 + \xi)U_{c_t} + h_{x_t}U(d_{t+1}, q_{t+1}) - d_{t+1}h_{x_t}U_{d_{t+1}} = 0 \tag{10}$$

3 Equilibrium and steady state

3.1 Intertemporal equilibrium

The intertemporal equilibrium is defined, by a sequence of prices, individual variables and aggregate variables satisfying all the equilibrium conditions.

[3]This assumption converts a complementary effect of space and consumption. An alternative assumption is that space and consumption are substitutable, a negative crossed derivative. For an in-depth discussion of these assumptions we refer the reader to Michel and Rotillon (1995).

Firms maximize their profit (conditions (19) and (20) hold) and consumers their utility (conditions (9) and (10) hold).
The capital stock is equal to savings,

$$K_t = Ns_{t-1} \tag{11}$$

The market of labor clears,

$$L_t = N, \tag{12}$$

as well the market of goods,

$$Y_t = F(K_t, N) = Nc_t + Nx_t + Nh(x_t, P_{t+1})d_t + K_{t+1} \tag{13}$$

In addition, the dynamic equation of pollution holds.

$$P_t = (1 - \delta)P_{t-1} + \eta F(K_t, N) \tag{14}$$

At time 0, consumption of the retirees satisfies:

$$h(x_{-1}, P_1)d_0 = R_0 s_{-1}. \tag{15}$$

Further we take for given the initial capital stock $K_0 = Ns_{-1}$, the pollution stock P_{-1} and health expenditure x_{-1}.

3.2 Stationary equilibrium

At the steady state with a given policy, the stock of capital is given by the sum of saving, $K = Ns$ and the economy's resource constraint per young is,

$$f(k) = c + x + h(x, P)d + k \tag{16}$$

where $k = K/N$ is the capital per young and $f(k) = F(k, 1)$. Long-run equilibrium pollution is defined by,

$$P = \frac{\eta}{\delta} N f(k) \tag{17}$$

The amount of space or land per person is defined by,

$$q = \frac{\overline{Q}}{N\left[1 + h(x, P)\right]} \tag{18}$$

Using relation (2), the interest factor (with tax) and the wage rate are,

$$R = f'(k) \tag{19}$$

and

$$w = f(k) - kf'(k) \tag{20}$$

The optimal consumers' decisions are given by,

$$-U_c + (R - \tau)\, U_d = 0 \tag{21}$$

and for health spending

$$-(1+\xi)U_c + h_x U(d,q) - dh_x U_d = 0 \tag{22}$$

4 Social Optimum and optimal policy

We now turn to the analysis of the social optimum and optimal policy. At the long-run equilibrium, we are looking for the maximum possible utility in the economy.

4.1 Social optimum

In a centralized economy, the objective of the central planner is to maximize the welfare of the agents by choosing the level of consumptions (c, d), health spending (x) and the level of capital (k), under constraints of resources, pollution and space per person.

$$\max_{c,d,x,k} U(c,q) + h(x,P)U(d,q)$$

$$s.t.: \begin{cases} f(k) = c + x + h(x,P)d + k \\ P = \frac{\eta}{\delta} N f(k) \\ q = \frac{\overline{Q}}{N[1+h(x,P)]} \end{cases}$$

Denoting by λ the Lagrangian multiplier of resource constraint (16), the Lagrangian $\mathcal{L}$ is defined by

$$\begin{gathered} \mathcal{L}(c,d,x,k) = U\left(c, \frac{\overline{Q}}{N\left[1+h(x,\frac{\eta}{\delta}Nf(k))\right]}\right) \\ +h\left(x,\tfrac{\eta}{\delta}Nf(k)\right)U\left(d, \frac{\overline{Q}}{N\left[1+h(x,\frac{\eta}{\delta}Nf(k))\right]}\right) \\ +\lambda\left[f(k) - c - x - h(x,\tfrac{\eta}{\delta}Nf(k))d - k\right] \end{gathered} \tag{23}$$

One obtains thereby the first order conditions,

$$U_c = \lambda \tag{24}$$

$$U_d = \lambda \tag{25}$$

$$h_x U(d,q) - \frac{N\overline{Q}h_x}{\left(N\left[1+h(x,P)\right]\right)^2}\mu = \lambda(1+h_x d) \tag{26}$$

and

$$\begin{aligned} & h_P\frac{\eta}{\delta}Nf'(k)U(d,q) - \frac{N\overline{Q}h_P\frac{\eta}{\delta}Nf'(k)}{(N\left[1+h(x,P)\right])^2}\mu \\ = \;& \lambda(1+h_P\frac{\eta}{\delta}Nf'(k)d - f'(k)) \end{aligned} \tag{27}$$

where $\mu \equiv U_q(c,q) + h(x,P)U_q(d,q)$ is the marginal lifetime utility of an increase in q.[4]

From (24) and (25) we obtain,

$$U_c = U_d \tag{28}$$

Using (28), (26) and (27), we have

$$U_c\left(c,q\right)\left(1+h_x(x,P)d\right) = h_x U(d,q) - h_x\frac{\overline{Q}}{\left[N\left[1+h(x,P)\right]\right]^2}\mu$$

and

$$f'\left(k\right) = 1 + f'\left(k\right)h_p\frac{\eta}{\delta}N\left[\frac{\overline{Q}}{\left(N(1+h)\right)^2}\frac{\mu}{U_d} - \left(\frac{U(d,q)}{U_d} - d\right)\right] \tag{29}$$

Note that without the environmental variables, these two optimal conditions would reduce to:

$$U_c\left(c,q\right) = h_x\left[U(d,q) - U_d(d,q)d\right]$$

and

$$f'\left(k\right) = 1.$$

The first equation is equivalent to (22) with $\xi = 0$. As to the second, it is the Golden rule (population growth is here 0).

With the environmental variable, we have some externalities. Starting with (29) associated with health spending, the externality comes from the fact that individuals do not internalize in their decisions the effect of increased longevity on the number of inhabitants of a finite earth.

[4] Note that if there is separability between consumption and q, $\frac{N\bar{Q}}{\left(N\left(1+h\right)\right)^2}\mu = v'\left(q\right)q$ with $U(c,q) = u(c) + v(q)$.

The choice of investment (eq. (30)) entails two externalities. The first one is positive. Keeping in mind that investment, production and pollution are closely related, more capital means more pollution and thus less population, which is good for the quality of the environment. The second externality is negative: more pollution means lower longevity and thus lower utility in the second period of life. However, by substituting (29) in (30), we obtain:

$$f'(k) = 1 - f'(k) \frac{h_p}{h_x} \frac{\eta}{\delta} N \tag{30'}$$

This implies that the optimum level of capital accumulation should be lower than that corresponding to the standard golden rule level. In other words, the negative externality more than offsets the positive one.

4.2 Optimal policy

Contrasting the market solution with the first-best optimum, one sees that the social optimum can be decentralized with appropriate choices of "Pigouvian taxes" ξ and τ for the environmental damage:

$$\xi = \frac{h_x \overline{Q}}{N\left[1 + h(x, P)\right]^2} \frac{\mu}{U_c} > 0 \tag{30}$$

and for saving

$$\tau = f'(k) - 1 = \frac{\frac{h_P}{h_x} \frac{\eta}{\delta} N}{1 - \frac{h_P}{h_x} \frac{\eta}{\delta} N} > 0 \tag{31}$$

To these two taxes one should add an intergenerational transfer device that leads to $R - \tau = 1$ from (28). With our environmental externality, we thus have $f'(k) = R > 1$. In other words, the optimal level of capital stock is inferior to that consistent with the standard Golden Rule ($f'(k) = 1$).

5 Second-best policy

We now turn to the second-best setting. We assume that both τ and ξ are available but that we do not have an instrument to achieve the optimal level of capital accumulation. We conduct this second-best analysis in a steady-state framework.

With a zero population growth, we have $s = k$. We can easily show that the resource constraint implies the revenue constraint

$$\begin{aligned} f(k) &= c + x + hd + k \\ &= f'(k) + c + s(1+\xi) + s - a \end{aligned}$$

and thus,

$$a = \tau s + \xi x,$$

where a is a lump-sum transfer given in the first period of life. Note that if, besides a, we had also a transfer in the second period of life, one would get the first-best, with ξ and τ having the values (31) and (32) and the tax transfers leading to $f'(k) - \tau = 1$.

We will express the problem of the social planner using the revenue and not the resource constraint. To keep the notation simple, we assume $N = 1$ without loss of generosity. The Lagrangian expression can now be written as:

$$\begin{aligned} \mathcal{L}(\tau, \xi, a) &= U(c,q) + h\left(x, \frac{\eta}{\delta} f(s)\right) U(d,q) \\ &\quad + \gamma\left[\tau s + \xi x - a\right] \\ &\quad - \mu\left[q - \frac{\bar{Q}}{1 + h\left(x, \frac{\eta}{\delta} f(s)\right)}\right]. \end{aligned}$$

where γ and μ are the multipliers associated with the revenue constraint and with the definition of q.

We maximize $\mathcal{L}$ with respect to τ, ξ and a, our tax parameters and with respect to q, an adjustment variable.

The FOC's are given by:

$$\frac{\partial \mathcal{L}}{\partial a} = U_c + \gamma\left[\tau \frac{\partial s}{\partial a} + \xi \frac{\partial x}{\partial a} - 1\right] - \left[\frac{\mu \bar{Q}}{(1+h)^2} - U(d,q)\right] h_p \frac{\eta}{\delta} f'(s) \frac{\partial s}{\partial a}$$
$$- \frac{\mu \bar{Q}}{(1+h)^2} h_x \frac{\partial x}{\partial a} = 0$$

$$\frac{\partial \mathcal{L}}{\partial \tau} = -U_c \frac{s}{R - \tau} + \gamma\left[\tau \frac{\partial s}{\partial \tau} + \xi \frac{\partial x}{\partial \tau} + s\right] - \left[\frac{\mu \bar{Q}}{(1+h)^2} - U(d,q)\right] h_p \frac{\eta}{\delta} f'(s) \frac{\partial s}{\partial \tau}$$
$$- \frac{\mu \bar{Q}}{(1+h)^2} h_x \frac{\partial x}{\partial \tau} = 0$$

$$\frac{\partial \mathcal{L}}{\partial \xi} = -U_c x + \gamma \left[\tau \frac{\partial s}{\partial \xi} + \xi \frac{\partial x}{\partial \xi} + x\right] - \left[\frac{\mu \bar{Q}}{(1+h)^2} - U(d,q)\right] h_p \frac{\eta}{\delta} f'(s) \frac{\partial s}{\partial \xi} - \frac{\mu \bar{Q}}{(1+h)^2} h_x \frac{\partial x}{\partial \xi} = 0$$

$$\frac{\partial \mathcal{L}}{\partial q} = U_q(c,q) + hU_q(d,q) - \mu = 0 \tag{32}$$

As usual with this type of problem, we express the tax formula in compensated terms. In other words, an increase in either τ or ξ is compensated by an increase in a and we want to know the effect of such a compensated increase on social welfare. We use the superscript c to denote the compensated effects. The multiplier μ in (33) expresses the social value of environmental quality.

$$\frac{\partial \mathcal{L}^c}{\partial \tau} = \gamma \overbrace{\left[\tau \frac{\partial s^c}{\partial \tau} + \xi \frac{\partial x^c}{\partial \tau}\right]}^{[1]} + \overbrace{\gamma \frac{s}{R}(R - \tau - 1)}^{[2]} - \overbrace{\frac{\mu \bar{Q}}{(1+h)^2} h_x \frac{\partial x^c}{\partial \tau}}^{[3]} - \left[\overbrace{\frac{\mu \bar{Q}}{(1+h)^2}}^{[4]} \overbrace{-U(d,q)}^{[5]}\right] h_p \frac{\eta}{\delta} f'(s) \frac{\partial s^c}{\partial \tau} \tag{33}$$

$$\frac{\partial \mathcal{L}^c}{\partial \xi} = \gamma \overbrace{\left[\tau \frac{\partial s^c}{\partial \xi} + \xi \frac{\partial x^c}{\partial \xi}\right]}^{[1]} - \overbrace{\frac{\mu \bar{Q}}{(1+h)^2} h_x \frac{\partial x^c}{\partial \xi}}^{[3]} - \left[\overbrace{\frac{\mu \bar{Q}}{(1+h)^2}}^{[4]} \overbrace{-U(d,q)}^{[5]}\right] h_p \frac{\eta}{\delta} f'(s) \frac{\partial s^c}{\partial \xi} \tag{34}$$

We distinguish among five terms in those two conditions:

1. the traditional Ramsey formula,
2. the gap between the rate of interest and the population growth rate,
3. the crowding effect induced by longevity-enhancing investment,
4. the decrowding effect induced by pollution,
5. the utility loss arising from shorter lifetime induced by pollution.

To get further insight, we make a quite extreme assumption: the cross-derivatives are negligible. In other words, τ has no influence over x and ξ over s. Then we have:

$$\frac{\partial \mathcal{L}^c}{\partial \tau} = \gamma\tau \overset{[1]}{\frac{\partial s^c}{\partial \tau}} + \gamma \overset{[2]}{\frac{s}{R}(R-1)} - \left[\overset{[4]}{\frac{\mu \bar{Q}}{(1+h)^2}} - \overset{[5]}{U(d,q)} \right] h_p \frac{\eta}{\delta} f'(s) \frac{\partial s^c}{\partial \tau}$$

$$\frac{\partial \mathcal{L}^c}{\partial \xi} = \gamma\zeta \overset{[1]}{\frac{\partial x^c}{\partial \xi}} - \overset{[3]}{\frac{\mu \bar{Q}}{(1+h)^2}} h_x \frac{\partial x^c}{\partial \xi}$$

Starting with τ, we have two positive terms: [2] and [5] and two negative ones [1] and [4]. The terms [2] and [4] are standard. The term [5] reflects the fact that by increasing τ, there is less capital accumulation and thus less pollution, which leads to an increased longevity and thus to an increased utility in period two. The term [4] shows also that a tax on saving has a positive effect on longevity, but longevity has a crowding effect on the fixed environmental quality $\bar{Q}$.

Turning to ξ, we have one negative effects [1] and a positive one [3]. The first one [1] is standard and negative; the second one, positive effect [3] says that by taxing health care, people do not live as long as without such a tax and this has a relief effect on the fixed quality of environment.

Admittedly, the result - i.e. health care ought to be taxed - obtained both in the first-best and in the second-best (under particular conditions) is a bit surprising and, as such, has to be qualified. Subsidizing health care is often recommended on the grounds of other considerations: redistribution, externality, etc. The negative effect underlined in this paper is likely to be dominated by these other considerations.

The two external effects of production and pollution are quite interesting. On prior grounds, one cannot say whether saving ought to be taxed or subsidized. We know that in the first-best it has to be taxed.

6 A numerical application

Let us now consider, in the light of numerical examples, the implications of the present model for public policy. For that purpose, we shall first introduce and calibrate functional forms for preferences, production and longevity. Then, we shall explore the sensitivity of the optimal (second-best) taxes on capital income and health expenditures - $\tau^*$and ξ^*- to various parameters of the model.

6.1 Functional forms

Lifetime welfare is assumed to take the following, additive form:[5]

$$U_t = \frac{\left[(c_t)^{\gamma}(q_t)^{1-\gamma}\right]^{1-\sigma}}{1-\sigma} + \beta h_{t+1}\frac{\left[(d_{t+1})^{\gamma}(q_{t+1})^{1-\gamma}\right]^{1-\sigma}}{1-\sigma} \tag{35}$$

where β is a time preference factor, whereas γ reflects the importance of consumption with respect to environmental quality ($0 \leq \gamma \leq 1$). γ is assumed to be constant across periods, which is a simplification, as old agents may be more or less sensitive to environmental quality than young ones.[6] The parameter σ is the inverse of the intertemporal elasticity of substitution for consumption.

Moreover, we shall assume that production takes a Cobb-Douglas form:

$$Y_t = AK_t^{\alpha}L_t^{1-\alpha} \tag{36}$$

with $0 < \alpha < 1$ and $A > 0$.

Longevity is determined by health expenditures and pollution as follows:[7]

$$h_{t+1} = Bx_t^{\phi}p_t^{\psi} \tag{37}$$

where B denotes the 'natural' longevity level, that is, its level in the hypothetical case where health expenditures and pollution have no influence on longevity ($B > 0$). ϕ is the elasticity of h_{t+1} with respect to health expenditures ($\phi > 0$), whereas ψ is the elasticity of h_{t+1} with respect to the stock of pollution at time t ($\psi < 0$), denoted here in intensive terms (i.e. $p_t = P_t/N$ is the stock of pollution per worker).[8] In order to capture the intergenerational dimension of pollution, longevity is here affected by the stock of pollution faced when being young: individuals, even if they were non-myopic, could choose their longevity only within a range allowed by previous generations, because some of its causes - here p_t - result from past decisions on which they have no control.[9]

[5] Under that functional form, the cross derivatives U_{cq} and U_{dq} are non-negative.

[6] Another non-negligible assumption is that pollution does not enter individual utility directly, but, only indirectly, through its influence on longevity h_{t+1}.

[7] Under $\phi > 0$ and $\psi < 0$, that functional form satisfies the properties mentioned in Section 2: $h_x > 0$, $h_{xx} < 0$, $h_p < 0$ and $h_{pp} > 0$.

[8] Given that each cohort is of constant size, this does not have any influence on the results.

[9] This assumption does not affect the conclusions drawn in previous sections, because these concerned the steady-state, where, by definition, p_{t+1} and p_t are equal.

6.2 Calibration

Regarding preferences, the discount factor β is assumed to be equal to 0.30, which corresponds, given that the length of a full period is 40 years, to a quarterly subjective discount factor of 0.99. In the light of empirical studies showing that the intertemporal elasticity of substitution for consumption is slightly above unity (see Browning *et al*, 1999), the parameter σ is fixed to 0.83, which implies an elasticity of intertemporal substitution of 1.25. Regarding the parameter γ, we shall first consider the benchmark case where the available space does not affect welfare (i.e. $\gamma = 1$), and, in a second stage, use lower values for γ.

As far as production parameters are concerned, α is fixed to 0.36, in conformity with the literature (see de la Croix and Michel, 2002), while the scale parameter A is fixed to 10.

The calibration of pollution parameters δ and η depends on the particular pollution process under study.[10] Given that each period is of length 40 years, it makes sense to assume that δ is relatively high, but its level depends on what pollution consists of. We shall, as a benchmark, assume that δ is equal to 0.9 (i.e. 9/10th of the pollution have vanished after a time lag of 40 years). Regarding η, we shall take 0.10 as a benchmark, and consider also higher values.

Regarding the space available per active person $\bar{Q}/N$, we shall assume that it is equal to about 3000 square-meters (i.e. equal approximately to the available space in countries such as the Netherlands). But, in order to explore how the optimal policy is affected by the 'number problem', we shall also compute (τ^*, ξ^*) under a higher population density (i.e. a lower $\bar{Q}/N$).

Finally, as far as the calibration of the longevity function is concerned, we shall assume that the elasticities of longevity with respect to health spending ϕ and pollution ψ are equal respectively to 0.15 and -0.05. Under those values, assuming that B is equal to 0.30 implies, under k_0 equal to 0.1, and under $\tau = 0$ and $\xi = 0$, an initial longevity equal to about $65 + 0.25(40) = 75$ years.

6.3 Results

Let us now consider the policy to be implemented by a government maximizing steady-state lifetime welfare subject to the budget constraint. The government collects a tax τ on capital income, taxes health expenditures at a rate ξ, and uses the fiscal revenues to fund a transfer a, which is remitted

[10] It should be stressed here that the assumed dynamic expression for pollution may not cover many existing pollution processes, whose dynamics is far more complex.

to individuals during their first period of life. Individuals cannot anticipate the impact of their health spending on a, and take it as given. Given the absence of a second-period transfer, the derived optimal policies are second-best policies.

When choosing their savings and health expenditures, agents do not internalize the impact of those decisions on pollution and on the available space. Moreover, we shall assume, for simplicity, that, when making their decisions, individuals form static expectations for the space available in the next period.[11]

To study the sensitivity of the optimal second-best policy (τ^*, ξ^*), we shall first focus on parameters describing pollution (i.e. η and δ), and, then, on preference parameters (i.e. β, γ and σ). For simplicity, we shall, in a first stage, assume that the available space does not affect welfare (i.e. $\gamma = 1$).

As illustrated by Figure 1, lifetime welfare at the steady-state is a non-monotonic function of the tax on capital income τ.[12] For low taxation levels, a higher τ raises lifetime welfare, whereas the opposite holds when τ is high.

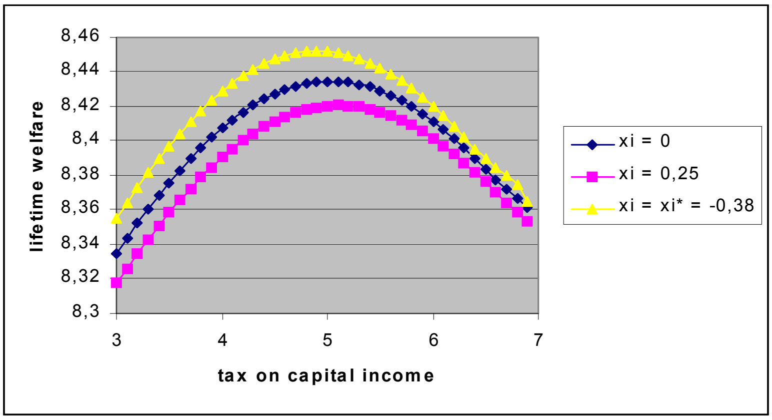

Fig. 1: Steady-state lifetime welfare as a function of τ.

Although the relationship between steady-state lifetime welfare and taxation on capital income has the same inverted-U shape for the three values of

[11]Under those assumptions, the optimal saving and health expenditures are equal to: $s^* = \frac{(w-(1+\xi)x^*+a)\left[\beta h_{t+1}\left(\frac{(R-\tau)}{h_{t+1}}\right)^{\gamma(1-\sigma)}\right]^{\frac{1}{\gamma\sigma-\gamma+1}}}{1+\left[\beta h_{t+1}\left(\frac{(R-\tau)}{h_{t+1}}\right)^{\gamma(1-\sigma)}\right]^{\frac{1}{\gamma\sigma-\gamma+1}}}$ and $x^* = \frac{\phi s^*\left[\frac{\gamma\sigma-\gamma+1}{1-\sigma}\right]}{\gamma(1+\xi)}$.

[12]Figure 1 is based on $A = 10, \alpha = 0.36, \beta = 0.3, \gamma = 1, \sigma = 0.83, \delta = 0.9, \eta = 0.1, B = 0.3, \phi = 0.15$ and $\psi = -0.05$. Initial capital is 0.1. $\bar{Q}/N = 3000$.

ξ, the *level* of lifetime welfare is not insensitive to the tax on health spending. Actually, the computation of lifetime welfare under different values of ξ suggests that subsidizing health spending is here socially desirable: the highest inverted-U curve is obtained under ξ equal to -38 %, so that the optimal second-best policy involves τ^* equal to about 4.9, and ξ^* equal to -0.38.

Let us now explore how this optimal second-best policy (τ^*, ξ^*) varies with the various parameters of this model. For that purpose, we shall present steady-state lifetime welfare as a function of τ under different parameters's values, while assuming that the tax on health expenditures takes its optimal level ξ^* for each parametrization.

A first set of parameters concerns the pollution process. As illustrated on Figure 2, which shows steady-state lifetime welfare as a function of τ under low emissions (i.e. $\eta = 0.1$), medium emissions (i.e. $\eta = 0.3$) and large emissions (i.e. $\eta = 0.5$), lifetime welfare is, as expected, lower when emissions are larger, that is, when η is higher.[13] But another important thing to observe is that the optimal second-best policy is not insensitive to η: (τ^*, ξ^*) is equal to $(4.9, -0.38)$ when η equals 0.1, and to respectively $(5, -0.39)$ and $(5.3, -0.41)$ when η equals 0.3 and 0.5. Thus, higher emissions lead here to a higher optimal tax on capital income, and to a higher optimal subsidy on health spending.

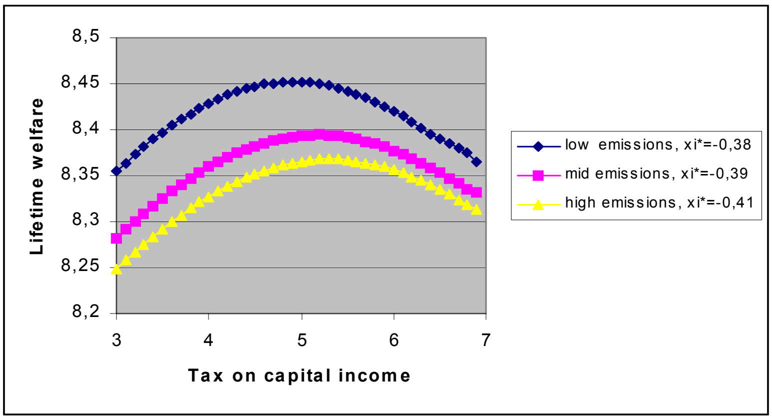

Fig. 2: Relation between U and τ for different η.

One should notice that assuming a lower δ (i.e. a lower natural absorption of pollution) has the same effects as a rise of η: it reduces, *ceteris paribus*,

[13] Figure 2 is based on $A = 10, \alpha = 0.36, \beta = 0.3, \gamma = 1, \sigma = 0.83, \delta = 0.9, B = 0.3, \phi = 0.15$ and $\psi = -0.05$. Initial capital is 0.1. $\bar{Q}/N = 3000$.

steady-state lifetime welfare, and implies a higher optimal tax on capital income, and a higher optimal subsidy on health expenditures.[14]

But the optimal second-best policy is also sensitive to preference parameters β, σ and γ, as we shall now discuss.

The influence of parameters β and σ on the optimal second-best policy is shown on Table 1.[15] Regarding the impact of β, the first three rows of Table 1 illustrate that a lower impatience tends to raise steady-state capital, longevity and welfare. From the point of view of public intervention, a higher β reduces the optimal tax on capital income and the optimal subsidy on health spending.

Table 1: Optimal policy (τ^*, ξ^*) and preference parameters

β	σ	γ	τ^*	ξ^*	k^*	x^*	p^*	h^*	U^*
0.30	0.83	1.00	4.90	-0.38	0.396	0.468	0.796	0.271	8.452
0.40	0.83	1.00	3.50	-0.25	0.628	0.613	0.940	0.280	8.853
0.50	0.83	1.00	2.70	-0.15	0.887	0.764	1.064	0.287	9.217
0.30	0.70	1.00	4.40	-0.59	0.362	0.309	0.771	0.255	5.889
0.30	0.83	1.00	4.90	-0.38	0.396	0.468	0.796	0.271	8.452
0.30	0.95	1.00	4.90	0.16	0.487	1.198	0.857	0.311	23.693

Regarding the influence of σ on the optimal second-best public intervention, Table 1 suggests that a higher σ (i.e. a lower elasticity of intertemporal substitution) tends to reduce the size of the optimal subsity on health expenditures, and may turn it into a tax. However, its impact on τ^* is more ambiguous.

Turning now to the parameter γ, which captures the importance, in welfare terms, of consumption with respect to the available space per person, Table 2 suggests that reducing γ leaves the optimal tax on capital income τ^* unchanged.[16] However, assigning a higher weight to environmental quality tends, *ceteris paribus*, to reduce the subsidy on health expenditures, and turns it into a tax (i.e. $\xi^* > 0$). That result is not surprising: the lower γ

[14] For instance, under the above calibration and $\eta = 0.1$, (τ^*, ξ^*) is equal to $(4.9, -0.38)$ when δ equals 0.9, and to $(5.1, -0.39)$ when δ equals 0.1.

[15] Table 1 is based on $A = 10, \alpha = 0.36, \delta = 0.9, \eta = 0.1, B = 0.3, \phi = 0.15$ and $\psi = -0.05$. Initial capital is 0.1. $\bar{Q}/N = 3000$.

[16] Table 2 is based on $A = 10, \alpha = 0.36, \delta = 0.9, \eta = 0.1, B = 0.3, \phi = 0.15$ and $\psi = -0.05$. Initial capital is 0.1.

is, the larger is the welfare loss due to the non-internalization by agents of the influence of health spending on the available space (through the rise in longevity). Such a larger welfare loss must necessarily lead, *ceteris paribus*, to a higher tax rate ξ^*.

The second part of Table 2 shows the optimal policy (τ^*, ξ^*) when the available space per active person $\bar{Q}/N$ is reduced to a level equal to 1000 square-meters, which corresponds approximately to the space availability in a highly densified country like Bangladesh (where the population density is about 1018 persons per square kilometer).[17]

As shown by the last column of Table 2, postulating a smaller available space $\bar{Q}/N$ tends, *ceteris paribus*, to lower steady-state lifetime welfare, except in the special case where γ equals 1 (i.e. environmental quality does not influence welfare). However, reducing the available space does not, under the postulated functional forms, affect the optimal second-best policy (τ^*, ξ^*), which is the same under $\bar{Q}/N$ equal to 3000 and 1000. Thus, although a smaller space reduces welfare, it does not affect the optimal second-best public intervention.

Table 2: Optimal policy (τ^*, ξ^*) for different γ and $\bar{Q}/N$.

β	σ	γ	τ^*	ξ^*	k^*	x^*	p^*	h^*	U^*
$\bar{Q}/N = 3000$									
0.30	0.83	1.00	4.90	-0.38	0.396	0.468	0.796	0.271	8.452
0.30	0.83	0.75	4.90	-0.19	0.421	0.533	0.814	0.276	10.979
0.30	0.83	0.50	4.90	0.14	0.455	0.644	0.836	0.283	14.246
$\bar{Q}/N = 1000$									
0.30	0.83	1.00	4.90	-0.38	0.396	0.468	0.796	0.271	8.452
0.30	0.83	0.75	4.90	-0.19	0.421	0.533	0.814	0.276	10.478
0.30	0.83	0.50	4.90	0.14	0.455	0.644	0.836	0.283	12.976

7 Conclusions

The goal of this paper was to study the design of the optimal long-run public intervention in an economy where longevity, which is influenced negatively by pollution due to production, tends, by raising population density, to affect environmental quality. For that purpose, we developed a two-period OLG model, where agents do not, when choosing their savings and health expenditures, internalize the impact of their decisions on the natural environment.

[17] Sources: INSEE (2006).

As this was shown with general functional forms for production, preferences and longevity, the first-best public intervention in that economy involves, besides adequate transfers leading to the Golden rule, a positive taxation of capital income, as well as a positive taxation of health expenditures. Whereas the former pigouvian tax allows the internalization of the pollution externality, the latter corrects agents's tendency to overspend in health.

Regarding the optimal public policy under a limited set of instruments (including the two taxes and a first-period transfer), it was shown that the optimal second-best levels of the tax on capital income and on health spending can hardly be signed with certainty, but, rather, depend on various factors, such as the intensity of preferences for environmental quality and the pollution process.

The sensitivity of the optimal second-best policy to those factors was also illustrated by means of numerical examples based on a time-additive CES utility function, a Cobb-Douglas production technology, and a longevity exhibiting constant elasticities with respect to health spending and pollution. As one may expect, pollution processes involving higher emissions tend, *ceteris paribus*, to raise the optimal second-best tax on capital income. Moreover, increasing the importance of environmental quality as a determinant of human welfare tends to turn health care subsidies into a tax.

While this paper does not allow us to draw precise conclusions about what public policy should be in the actual world, this allows us, however, to highlight that public intervention should take into account the multiple relationships between production, longevity and the natural environment, because these determine the corrections to be made by governments. Given the complexity of those relations, it cannot be overemphasized that this paper is only a first step in the study of optimal policy under endogenous longevity and environment.

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