Can catastrophic events be another source of environmental traps?

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Abstract

This paper aims to present a new explanation for environmental traps, by the presence of endogenous hazard rate. We show that adaptation and mitigation policies have different effects on the occurrence of environmental traps: the former could cause an environmental trap, whereas the latter could help society avoid such a trap, since it decreases the catastrophic event probability. As a result, we present a new trade-off between adaptation and mitigation policies other than the usual dynamic trade-off highlighted in many studies, which is crucial for developing countries. Contrary to the literature, when the economy is in a trap, the economy at the high environmental quality equilibrium tends to be more conservative for resource exploitation than the low environmental quality equilibrium economy, which implies a heterogeneous reaction against the probability of a catastrophic event occurring.

Keywords : Environmental damage, Catastrophic event, Occurrence hazard, Tipping points, Multiple equilibria, Environmental traps, Adaptation, Mitigation.

JEL Classification : O13, D81,Q2, Q54,

1   Introduction

A social planner should consider ways of avoiding damage as a result of hazardous events that might entail negative consequences. A direct response requires action to reduce the probability of a harmful event taking place. In many cases, mitigation activities are able to reduce the risk of a catastrophic regime shift by improving the environmental quality[1] altough they can not eliminate it completely. In such situations, a possible action could be alleviating the negative consequences of a catastrophic damage. The measures taken to reduce the loss due to the catastrophic event can be considered as adaptation. The management of adaptation and mitigation activities raises an interesting dynamic trade-off that can be described as "adaptation and mitigation dilemma" in the environmental economics literature (Zemel (2015), Tsur and Zemel (2015), Crépin et al. (2012)).

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[1]Since we focus on environmental quality in the paper, mitigation activities are aimed at increasing environmental quality. A possible definition of mitigation activity can be reforestation, which enhances carbon sinks. (IPCC (2007))

To elaborate more on these concepts, we consider concrete examples. Improvements in energy efficiency, activities such as carbon capture and storage and reforestation address the root causes, by decreasing greenhouse gas emissions and reducing the risk of a catastrophic climate event and therefore can be referred to as mitigation. Mitigation activity can also be seen as a tool to avert a catastrophic event (Martin and Pindyck (2015)). Whereas, installing flood defenses and developing irrigation systems aim to reduce the damage inflicted by a possible catastrophic event and hence can be classified as adaptation. In this context, adaptation plays a *proactive* role, which means that it has no concrete effect prior to the catastrophic event (Smit et al. (2000) ; Shalizi and Lecocq (2009)). In this example, the problem is to decide an optimal combination of risk-reducing and damage-reducing measures within a given budget.

In this paper, we study the optimal management of natural resources and its link with adaptation and mitigation policies in a simple growth model under catastrophic event probability depending on the environmental quality level[2]. For example, the arrival rate of catastrophic events such as droughts, crop failures and floods [3] is linked to the exploitation of natural capital. Our model uses a general definition for natural capital which encompasses all natural amenities such as the stock of clean water, soil and air quality, forests, biomass and so on.

The contribution of this paper is threefold: The first contribution of the paper is to analyze the implications of the catastrophic event probability on the occurrence of the multiplicity of equilibria (i.e the environmental trap). We show that one of the reasons behind environmental traps is the catastrophic event probability. The reason why catastrophic events may cause an environmental trap is as follows: when an economy faces a catastrophic event probability, a second trade-off arises between consumption and the catastrophic event probability other than the usual intertemporal trade-off between present and future consumption. An economy with serious environmental quality problems is supposed to be more impatient due to the endogenous hazard rate. Therefore, agents tend to increase their consumption at earlier dates since they face a higher event probability, which again stresses the environmental quality over time. This trade-off between consumption and catastrophic events results in a vicious cycle of "low level of environmental quality and consumption" in the long run that can be defined as an environmental trap.

The possibility of multiple stationary equilibria in growth models with endogenous hazard[4] is already mentioned by Tsur and Zemel (2016). However, one cannot understand the economic intuition behind the multiplicity of equilibria Tsur and Zemel (2016). In this paper, not only do we offer an economic explanation for environmental traps, but we also prove the existence of the multiplicity of equilibria. The mathematical proof of the existence of environmental traps allows us to know within which range of catastrophic damage, the economy finds itself in a multiple equilibria economy.

Our study relates also to a substantial literature on resource exploitation under uncertainty. The consideration of catastrophic events for optimal management starts with Cropper (1976), who finds that the depletion of resources is either faster or slower if the available resource stock is uncertain. Clarke and Reed (1994) offer a framework where the stochastic dynamic problem for optimal management is transformed into a deterministic problem that can be solved with the Pontryagin maximum principle. The authors find an ambiguous effect of hazard rate on the optimal pollution emissions when there is a single occurrence

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[2]We use interchangeably the terms environmental quality and natural resource stock.

[3]The depletion of forests in a region increases the probability of floods, since the soil loses its ability to absorb the rainfall.

[4]There is also a large literature on growth models with endogenous discounting pointing out the possibility of multiple equilibria (Das (2003), Drugeon (1996), Schumacher (2009)). However, these papers do not include neither the uncertainty component or the environmental aspects.

event that reduces indefinitely the utility to constant level. A more recent work by Polasky et al. (2011) presents a growth model with exogenous/endogenous collapse risk. They find that the renewable resource stock decreases after the event and the effect of the endogenous risk is ambiguous. A paper by de Zeeuw and Zemel (2012) follows a similar modeling approach to Polasky et al. (2011) and tries to figure out the conditions under which the economy tends to become precautious. They show that the ambiguous effect is due to the characteristics of the post-event value. If there is a doomsday event and economic activity stops, the hazard rate enters as an additional factor to the discount rate, which makes the management policy ambiguous with respect to catastrophic risk. Ren and Polasky (2014) also study optimal management under the risk of potential regime shift and show that it is also possible that the economy adopts an aggressive management policy under a small endogenous event risk, different from many studies which show that the economy becomes either precautious or the overall effect is ambiguous. Apart from the literature on resource exploitation under uncertainty, there are some recent contributions regarding the implications of catastrophic events on the long run behavior of the economy. A paper by van der Ploeg (2014) focuses on how a first-best optimal carbon tax should be adjusted along time when the economy faces a catastrophic event probability. van der Ploeg and de Zeeuw (2017) extend the framework offered by van der Ploeg (2014) and show that a positive saving response can help the economy to dampen the discrete change in consumption at the time of the catastrophic event. A recent study by Akao and Sakamoto (2018) offers a general framework to justify the empirical evidence that shows the positive correlation between disasters and long term economic performances.

The existing literature tries to figure out the reaction of the economy to uncertain events. One of the common arguments in the above-cited papers is that the uncertainty pushes the economy to be more precautious. However, these studies overlook the possibility of multiple equilibria which can imply a heterogeneous reaction to uncertain events. What exactly is the heterogeneity against the uncertainty? An economy can find itself at an equilibrium point with low or high environmental quality. Therefore, an economy at high environmental quality is more precautious than one at low environmental quality level. In other words, an economy at the low environmental quality equilibrium adopts an "aggressive" exploitation policy relative to an economy at the high environmental quality equilibrium.

In this sense, differently from the literature, our paper shows that, given uncertainty, an economy adopts precautionary behaviour, although the level of precaution is not the same when an economy converges to a low or a high environmental quality level.

The second and the main contribution of the paper is to present a new trade-off between adaptation and mitigation other than the usual dynamic trade-off highlighted by numerous studies (Bréchet et al. (2012), Kama and Pommeret (2016), Millner and Dietz (2011)). A recent work by Zemel (2015) studies the time profile of the optimal mix of adaptation and mitigation in a simple growth model with uncertainty. Tsur and Zemel (2015) extend Zemel (2015) by relaxing the linearity assumption on adaptation investments and find optimal interior solutions. However, these studies did not take into account the multiplicity of equilibria and its implications regarding the adaptation and mitigation policies. Indeed, our contribution is to show that adaptation policy increases the possibility of environmental traps. On the other hand, mitigation policy makes more likely that the economy admits a unique equilibrium. Hence, concentrating on environmental traps in an economy with endogenous hazard rate allows us to present a new trade-off between adaptation and mitigation regarding the environmental traps.

What is the role of adaptation and mitigation on the environmental traps mentioned above? Our con-

tribution is to show that adaptation policy can cause multiple equilibria while mitigation can avoid it. This depends largely on the occurrence probability. This is because adaptation capital is shown to decrease the optimal steady state level of environmental quality, since agents worry less about the consequences of a catastrophic event when there is an increasing adaptation capacity. Then, since the catastrophic event probability increases, the trade-off between present consumption and the catastrophic event probability becomes tighter, which is likely to raise multiple equilibria. Contrary to this mechanism, mitigation activity improves the environmental quality and the trade-off between present consumption and catastrophic event probability turns to be weaker. To better understand this result, assume for a moment that mitigation activity can eliminate the catastrophic event risk[5]. Then, the trade-off between present consumption and hazardous event disappears, since the catastrophic event probability disappears. Consequently, the environmental trap is not a possible outcome.

The third contribution is to analyze whether adaptation and mitigation investments are complementary or substitutes. The recent papers by Tsur and Zemel (2015) and Zemel (2015) overlook whether adaptation and mitigation are substitutes or complementary. In this paper, we show that substitutability depends on how the adaptation cost is modeled. When adaptation investment costs are considered as a social disutility, as in Tsur and Zemel (2015) and Zemel (2015), adaptation and mitigation are always complementary. However, when the adaptation cost is taken in the resource constraint[6], adaptation and mitigation can also be substitutes if the marginal benefit from mitigation investments is sufficiently high.

The remainder of the paper is organized as follows: Section 2 presents the benchmark model. Section 3 describes the model with adaptation and mitigation policies and explains in a greater-depth the implications of adaptation and mitigation on the occurrence of environmental traps. Section 4 provides numerical illustrations and Section 5 concludes the paper.

2 Model

Let $S(t)$ represent environmental stock available or environmental quality at time t, e.g, the stock of clean water, soil quality, air quality, forests, biomass. We refer to a broad definition of environmental quality which encompasses all environmental amenities and existing natural capital that have an economic value[7]. Obviously, disamenities such as waste and pollution stemming from consumption decrease environmental quality stock. The stock $S(t)$ evolves in time according to

$$\dot{S}(t) = R(S(t)) - c(t) \tag{1}$$

where the control variable $c(t)$ stands for consumption at time t. With a given initial state $S(0)$, an exploitation policy of environmental stock $c(t)$ generates the state process $S(t)$ according to the equation (1) and provides the utility $u(c(t), S(t))$. Similar to Kama and Schubert (2007), we use a framework where consumption comes directly from environmental services and causes environmental damages.

We make use of the following assumptions.

A.1 The regeneration of environmental quality is characterised by $R(.): \mathbb{R}_+ \to \mathbb{R}_+$, $R(S) > 0$ and $R_{SS}(S) < 0$.

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[5]In our model, a risk of a catastrophic event exists always whatever the environmental quality level is. This is also the justification for a proactive adaptation policy.

[6]This means that adaptation cost is financed by the rents stemming from the natural capital.

[7]We exclude mining and oil industry from our definition of natural capital.

A.2 The utility function $u(.) : \mathbb{R}_+ \to \mathbb{R}_+$ is twice continuously differentiable with the following properties ; $u(c) < 0$, $u_c(c) > 0$, $u_{cc}(c) < 0$, $\forall c$ and $\lim_{c \to 0} u_c(c) = \infty$.

An important issue highlighted by Schumacher (2011) is related to the use of a utility function with a positive domain in growth models with endogenous discount. Schumacher (2011) shows rigorously that the models with endogenous discounting should use a utility function with a positive domain since the utility function shows up in the optimal path of consumption. Indeed, since our model focuses on recurrent events such as droughts and floods, the utility function does not show up in the Keynes-Ramsey rule. Therefore, the use of a utility function with a negative or positive domain does not lead to qualitatively different solutions.

In addition to the fact that the environmental stock $S(t)$ represents a source of consumption for the economy, it also affects the occurrence probability of a catastrophic event. To elaborate this in greater depth, forests, for example influence considerably the environmental conditions in a given area. (Dasgupta and Mäler (1997), chapter 1) and helps to decrease the probability of catastrophic events (see Jie-Sheng et al. (2014), Bradshaw et al. (2007))[8]. When catastrophic events occur, they inflict environmental damages. The consequences of these recurrent catastrophic events are defined by the post-event value $\varphi(S)$ that we will discuss later.

Let T be the event occurrence time and let $F(t) = Pr\{T \le t\}$ and $f(t) = F'(t)$ denote the corresponding probability distribution and density functions, respectively. The environmental stock dependent hazard rate $h(S)$ is related to $F(t)$ and $f(t)$ with respect to

$$h(S(t))\Delta = Pr\{T \in (t, t + \Delta \mid T > t)\} = \frac{f(t)\Delta}{1 - F(t)} \tag{2}$$

where Δ is an infinitesimal time interval. We have $h(S(t)) = -\frac{ln(1-F(t))}{dt}$. The term $h(S(t))\Delta$ specifies the conditional probability that a catastrophic event will occur between $[t, t + \Delta]$, given that the event has not occurred by time t. A formal specification for probability distribution and density functions gives

$$F(t) = 1 - exp\left(-\int_0^t h(S(\tau))d\tau\right) \ \ and \ \ f(t) = h(S(t))[1 - F(t)] \tag{3}$$

Since $S(t)$ is a beneficial state, the hazard rate h is a non-increasing function (a higher environmental quality stock entails a lower occurrence probability, $h_S < 0$).

We consider recurrent events which entail an immediate damage $\bar{\psi}$. Recurrent events are repeated events such as droughts and floods that occur frequently in our current society. In addition, the economy is always under the same risk of more events occurring later on, with value function $V(S)$. The post-value function describing the economy with recurrent events after the occurrence of catastrophic event is defined as

$$\varphi(S) = V(S) - \bar{\psi} \tag{4}$$

Given the uncertain arrival time T, the exploitation policy $c(t)$ yields the following payoff

$$\int_0^T u(c(t))e^{-\rho t}dt + \varphi(S(T))e^{-\rho T} \tag{5}$$

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[8] An interesting real world example could be the reforestation project in Samboja Lestari conducted by Borneo Orangutan Survival Foundation. The project helped to increase rainfall by 25% and to avoid droughts by lowering air temperature by 3 degrees Celcius to 5 degrees Celcius . (Boer (2010), Normile (2009))

where ρ is the social discount rate. Taking expectations of the expression (5) according to distribution of T and considering (3) gives the expected payoff

$$V(S) = \max_{c(t)} \int_0^\infty \left[u(c(t)) + h(S(t)) \left[V\left(S(t) - \bar{\psi}\right) \right] \right] \exp\left(-\int_0^t \left[\rho + h(S(\tau)) \right] d\tau \right) dt \qquad (6)$$

The maximization problem (6) at hand is deterministic, yielding a value that corresponds to the maximum expected value of the uncertainty problem.

The solution of maximising (6) with respect to evolution of environmental stock (1) leads to the Keynes-Ramsey rule (see Appendix (A) for details),

$$\dot{c} = -\frac{u_c(c)}{u_{cc}(c)} \left[R_S(S) - \rho - \frac{\bar{\psi} h_S(S)}{u_c(c)} \right] \qquad (7)$$

When there are recurrent catastrophic events, the economy adopts a precautionary behavior due to the presence of the precaution term $\frac{\bar{\psi} h_S(S)}{u_c(c)}$ in equation (7). This result is similar to de Zeeuw and Zemel (2012), where the authors show that recurrent events induce a precautionary behavior.

Proposition 1. *(i) A multiple steady state (i.e a environmental trap) is possible if the economy is exposed to endogenous hazard, depending on the environmental stock S. If the catastrophic event probability is exogenous, the multiplicity of equilibria is not a possible outcome.*

(ii) The necessary conditions to have an environmental trap for recurrent events are $\exists S < \bar{S}$[9] such that ;

$$G_S(S) = R_{SS}(S) - \frac{\bar{\psi} h_{SS}(S)}{u_c(R(S))} + \frac{\bar{\psi} h_S(S) u_{cc}(R(S)) R_S(S)}{(u_c(R(S)))^2} > 0 \qquad (8)$$

Proof. See Appendix (B)

In a standard neoclassical growth model, this condition cannot be satisfied, since all terms with endogenous catastrophic event probability vanish and the condition reduces to $R_{SS}(S) > 0$, which is not possible according to A.1.

In a Ramsey-Cass-Koopmans model, the usual inter-temporal trade-off is between the present and future consumption. Moreover, an economy exposed to catastrophic events faces an additional trade-off between present consumption and catastrophic risk. Indeed, this is the reason why the economy could find itself in a trapped equilibrium. More technically, it is possible to understand how the trade-off between present consumption and catastrophic event risk through the precaution term $\frac{\bar{\psi} h_S(S)}{u_c(c)}$ in equation (7). When the marginal utility is at a high level, the weight of the precaution term decreases. This means that agents care less about the hazard rate due to a higher opportunity cost of decreasing present consumption.

Note that without this term, the function $G_S(S)$ becomes linear and the economy always admits a unique equilibrium. The first term relates to convexity of regeneration function which is a negative term. The second and third term in (8) are positive and negative terms respectively.

The economic intuition behind these terms are as follows: the second term stands for the convexity of the hazard rate[10]. The more the hazard rate is sensitive to marginal changes of environmental quality, the more there are chances that $G_S(S) > 0$. The third term relates to the concavity of the utility function. A more concave utility function (i.e, higher risk aversion) decreases $G_S(S)$ if $R_S(S) > 0$. Overall, the multiple

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[9]$\bar{S}$ is the carrying capacity of the natural resource stock.

[10]Assume that the hazard rate has a convex form.

equilibria condition depends on the close relationship between the hazard rate depending on environmental quality and consumption.

One can understand the occurrence of environmental traps due to hazard rate by making a phase diagram analysis. Recall that the consumption rule is $\dot{c}/c = \sigma\left(r - \rho\right)$[11] without catastrophic event probability. Then, the steady-state curve $\dot{c} = 0$ is vertical and implies a unique equilibrium, which is not the case with hazard probability. Considering equation (7), we can observe that steady state curve $\dot{c} = 0$ is non-linear on a phase plane (S, c). Therefore, multiple equilibria is a possible outcome with event risk. To illustrate this explanation, we refer to a phase diagram analysis in the following section.

2.1 Phase Diagram Analysis

Finding directions of arrows on the phase diagram analysis requires some focus, since steady state curve of consumption is a function of environmental quality S. If (c, S) is below (above) the $\dot{S} = 0$, $R\left(S\right) - c > (<)\,0$, which makes $\dot{S} > (<)\,0$. The analysis is not that easy for $\dot{c} = 0$. Therefore, we use the necessary condition (8). Above the $\dot{c} = 0$ line, we have $\dot{c} > (<)\,0$ if $G_S\left(S\right) > (<)\,0$. We define different zones on the phase diagram where the slope of $G\left(S\right)$ changes. The possible phase diagrams of the dynamical system are as follows:
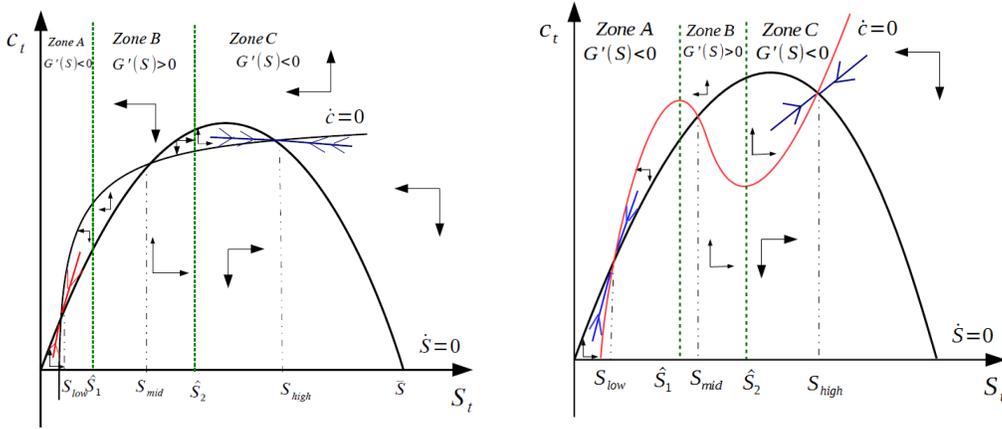

Figure 1: Phase diagram with monotonically and non-monotonically increasing $\dot{c} = 0$ curve

Note that, from multiple steady state condition, between $\left[0; \hat{S}_1\right]$ and $\left[\hat{S}_2; \bar{S}\right]$, we know that $G_S\left(S\right) < 0$ (see Figure (5) in Appendix *(B)*.). Within the zones A and C, the slope of the $\dot{c} = 0$ line is always positive, which means that the curve is always increasing between these intervals (see Appendix *(B)* for details). Conversely, within the zone B, it is not possible to determine exactly whether the slope of $\dot{c} = 0$ line is increasing or decreasing. Consequently, we present two different phase diagrams (see Appendix (C)).

Lemma 1. *The steady-states (S_{low}, c_{low}) and (S_{high}, c_{high}) are saddle path stable. However, (S_{mid}, c_{mid}) could have complex dynamics.*

Proof. See Appendix (D)

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[11]where $r = R_S\left(S\right)$. With usual growth model, one can maximise $\int_0^\infty \frac{c^{1-\sigma}}{1-\sigma}e^{-\rho t}dt$ with respect to $\dot{S} = R\left(S\right) - c$ and find the usual Euler equation.

This means that the economy could converge to either high or low environmental quality equilibrium. Once the economy reaches an equilibrium (low or high), it remains in this state permanently. The economy reaching the low-quality equilibrium is said to be "trapped", where the consumption and environmental quality are lower relative to high-quality equilibrium.

How can a country be trapped in the low-quality equilibrium? It is possible to have a precise answer through the precaution term $\frac{\bar{\psi}h_S(S)}{u_c(c)}$ in equation (7). The economic explanation is as follows: postponing consumption is too costly for survival ($u_c(0) = \infty$) for poor agents and preferences are directed toward the present. In this case, the weight of precaution term decreases and agents are exploiting most of the natural capital at earlier dates and they face a higher event probability. Since the occurrence probability is high, agents tend to be impatient[12] and start to excessively exploit natural resources. Consequently, a vicious cycle occurs due to the trade-off between present consumption (i.e use of environment) and the hazard rate.

With all these elements, one may understand how environmental conditions through catastrophic event probability could cause an environmental trap in a country. Indeed, since exploitation of the environment is one of the major sources of revenue in developing countries[13], countries with a low initial state of environmental quality are likely to suffer from an environmental trap due to the mechanism explained above.

Proposition 2. *When the economy is exposed to a catastrophic event risk and there is a multiplicity of equilibria, a high-equilibrium economy adopts a more conservative exploitation policy than low-equilibrium economy.*

Proof. See Appendix (E)

According to the precaution term in equation (7), when there are recurrent events, the risk of catastrophic event occurring makes always an economy more precautious for resource exploitation (see de Zeeuw and Zemel (2012), Tsur and Zemel (2016), Polasky et al. (2011), Clarke and Reed (1994)). The novelty of this proposition with respect to the literature is that the endogenous event risk leads to different levels of precaution (different steady-states) against catastrophic events when there is a multiplicity of equilibria.
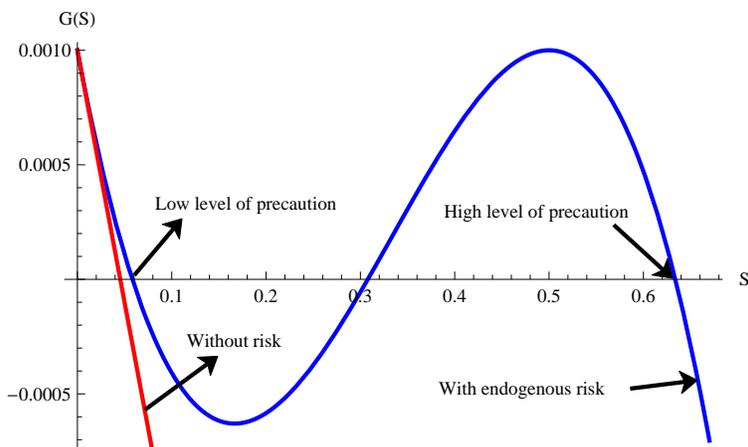

Figure 2: Different reaction against catastrophic events

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[12]The adjusted discount rate is $\rho + h(S)$.

[13]see http://data.worldbank.org/indicator/NY.GDP.FRST.RT.ZS?name_desc=false&view=map for a detailed data on forests rents.

As seen in Figure (2), the hazard rate always pushes the economy to be precautionary with respect to the case without any catastrophic event risk. However, countries characterized by high equilibrium become more conservative about the environment relative to countries characterized by low equilibrium when there is an endogenous hazard rate (see Figure (2)). Environmental Performance Index (EPI hereafter) in 2018[14] supports this result, since many African and Asian countries are listed at the bottom of the rankings.

2.2 An example of multiple equilibria

The multiplicity of steady states is an issue related to functional forms and parameter values. We provide explicit conditions ensuring this multiplicity in a case with functional forms that have been extensively used in the literature. The regeneration of the environment, hazard rate and the utility function are given respectively as follows:

$$R\left(S\right) = g\left(1 - S\right)S \tag{9}$$

$$h\left(S\right) = \left(1 - \bar{h}S^2\right) \tag{10}$$

$$u\left(c\right) = log\left(c\right) \tag{11}$$

where g is the intrinsic growth rate of the environmental quality and $0 < \bar{h} < 1$ shows at which extent the catastrophic event probability depends on the environmental quality level. At the steady-state, the equation (7) can be written as

$$G\left(S\right) = R_S\left(S\right) - \rho - \frac{\bar{\psi}h_S\left(S\right)}{u_c\left(c\right)} = 0 \tag{12}$$

By using the functional specifications, we reformulate the equation (12) combined with (1) at the steady state

$$-2g\bar{\psi}\bar{h}S^3 + 2g\bar{\psi}\bar{h}S^2 - 2gS + \left(g - \rho\right) = 0 \tag{13}$$

Proposition 3. *The sufficient condition to have three positive real roots for the third degree polynomial equation (13) is as follows:*

$$\frac{\left(4g^2 + 36g\left(g - \rho\right) - 27\left(g - \rho\right)^2\right) - \sqrt{\left(4g^2 + 36g\left(g - \rho\right) - 27\left(g - \rho\right)^2\right)^2 - 2^9 g^3\left(g - \rho\right)}}{8\left(g - \rho\right)2g\bar{h}}$$
$$< \bar{\psi} < \frac{\left(4g^2 + 36g\left(g - \rho\right) - 27\left(g - \rho\right)^2\right) + \sqrt{\left(4g^2 + 36g\left(g - \rho\right) - 27\left(g - \rho\right)^2\right)^2 - 2^9 g^3\left(g - \rho\right)}}{8\left(g - \rho\right)2g\bar{h}} \tag{14}$$

Proof. See Appendix (F)

This result suggests that the damage rate due to catastrophic events should not be either too low or too high for the multiplicity of equilibria. The trade-off between present consumption and catastrophic events

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[14]The Environmental Performance Index is a method developed by Yale Center for Environmental Law and Policy, that evaluates environmental policies of countries. see http://epi.yale.edu/country-rankings.

can be understood through this condition as well: When the damage rate is too high, this implies that the economy adopts a precautionary behavior since the marginal benefit of protecting the environment is higher. On the other hand, when the damage rate is too low, the economy tends to care less about the environment and exploits natural resources. Therefore, in these two cases, the trade-off is not very binding. However, when the damage lies in between a certain range of values, the trade-off between catastrophic events and present consumption becomes more binding, since the economy is neither very precautionary nor careless about the environment.

3 Can environmental policy cause/avoid an environmental trap?

In this section, we consider the benchmark model and analyze how adaptation and mitigation policies affect the main results obtained in the model without environmental policy. Since the focus of our study is to figure out the effects of adaptation and mitigation policies on environmental traps, the objective is to understand the effect of these two policies on necessary condition (8) for multiple equilibria. To facilitate understanding of the mechanisms, we prefer to analyze separately the effect of adaptation and mitigation policies on environmental traps.

3.1 An economy with adaptation policy only

As mentioned in the benchmark model, when a catastrophic event occurs, the economy suffers from environmental damage. However, a social planner could reduce the damage ψ via adaptation capital K_A. This way of modeling adaptation is consistent with Zemel (2015) and Tsur and Zemel (2016) but differs from Bréchet et al. (2012) and Kama and Pommeret (2016), where the adaptation capital affects directly the damage function for all time t. In our specification, as mentioned above, adaptation plays a *proactive* role. Hence, tangible benefits of adaptation can be gained only if a catastrophic event occurs. However, this is not to say that investing in the adaptation decision makes no difference. Its contribution is accounted for by the objective function of the social planner. Investing at rate A contributes to adaptation capital K_A, which follows the stock dynamics

$$\dot{K}_A(t) = A(t) - \delta K_A(t) \tag{15}$$

where δ represents the capital depreciation rate and damage function $\psi(K_A)$ decreases when adaptation capital K_A increases. We assume that

A.3 The damage function is characterised by $\psi(.)$: $\mathbb{R}_+ \to \mathbb{R}_+$, $\psi(0) = \bar{\psi}$, $\psi(\infty) = \underline{\psi}$, $\psi(K_A) > 0$, $\psi_{K_A}(K_A) < 0$ and $\psi_{K_A K_A}(K_A) > 0$

When there is no adaptation expenditure, the inflicted damage will be a constant term. Moreover, it is realistic to assume that reduction in damage has a limit, since we cannot get rid of the negative effects of a catastrophic event completely by accumulating adaptation capital. For that reason, we assume that damage function is constrained between an upper $\bar{\psi}$ and a lower bound $\underline{\psi}$.

In an economy with adaptation policy, the expected payoff is

$$V(S, K_A) = \max_{\{c(t), A(t)\}} \int_0^\infty \left[u(c(t)) - Q^1(A(t)) \right.$$
$$\left. + h(S(t)) \left[V(S(t), K_A(t)) \right] - \psi(K_A(t)) \right] \exp\left(-\int_0^t \left[\rho + h(S(\tau)) \right] d\tau \right) dt \quad (16)$$

where $u(c(t))$ is the utility function. $V(S, K_A)$ stands for the value of the maximization problem. $Q^1(A) = \phi_A \frac{A^2}{2}$ is the convex cost function for adaptation that enters in the utility function as a social cost as in Zemel (2015) and Tsur and Zemel (2016). The reason behind the choice of a convex cost function for adaptation activity instead of a linear function is that the accumulation of adaptation capital is linear. Otherwise, a linear cost function for adaptation investments leads to corner solutions (see Zemel (2015) for an example).

The optimal policy is to maximize (16) subject to (I) and (15) (see Appendix G). Optimal dynamics for consumption and adaptation are as follows

$$\dot{c} = -\frac{u_c(c)}{u_{cc}(c)} \left[R_S(S) - \rho - \frac{\psi(K_A) h_S(S)}{u_c(c)} \right] \quad (17)$$

$$\dot{A} = \frac{Q_A^1(A)}{Q_{AA}^1(A)} \left(\rho + \delta + \frac{h(S) \psi_{K_A}(K_A)}{Q_A^1(A)} \right) \quad (18)$$

When an economy invests in adaptation policy, we have additionally equation (18), which shows the optimal dynamics of adaptation investment. In the next part, we seek to figure out the effect of adaptation investments on the long-run equilibrium.

Proposition 4. *Adaptation policy contributes to the occurrence of multiple equilibria.*

In order to assess the effect of adaptation capital on the environmental trap, we calculate the necessary condition for multiple equilibria (see Appendix (G) for details)

$$G_S(S) = R_{SS}(S) - \frac{h_{SS}(S) \psi(K_A)}{u_c(R(S))} + \frac{h_S(S) \psi(K_A) u_{cc}(R(S)) R_S(S)}{(u_c(R(S)))^2} - \underbrace{\left[\frac{\psi_{K_A}(K_A) h_S(S)}{u_c(R(S))} \right] \frac{dK_A}{dS}}_{=Z_1} > 0 \quad (19)$$

where the term Z_1 stands for the effect of adaptation capital on necessary condition for the environmental trap.

In order to assess the effect of this term, we should know how environmental quality changes the adaptation capital level at steady state. To do this, we write equation (18) at steady-state

$$Q_A^1(\delta K_A)(\rho + \delta) = -\psi_{K_A}(K_A) h(S) \quad (20)$$

which defines the function $K_A(S)$. The graph of $K_A(.)$ represents a curve in the (K_A, S) plane, denoting the steady-state curve and having the following economic intuition: right-hand side $-\psi_{K_A}(K_A) h(S)$ can be interpreted as the marginal benefit of adaptation capital and left-hand side $Q_A^1(\delta K_A)(\rho + \delta)$ is the marginal cost of adaptation capital. Optimal steady states are located on this curve. We can find the slope of the steady state curve by taking the total derivative of equation (20)

$$\frac{dK_A}{dS} = \frac{h_S(S)\,\psi_{K_A}(K_A)}{(\rho+\delta)\,Q_A^1(\delta K_A) + \psi_{K_A K_A}(K_A)\,h(S)} < 0 \tag{21}$$

Since we have $\frac{dK_A}{dS} < 0$, the term Z_1 is negative and this means that the possibility of multiple equilibria increases with adaptation capital.

The nominator in (21) can be considered as the variation of marginal benefit of adaptation with respect to environmental quality, which is a negative term. This means that the marginal benefit of adaptation is lower when the environmental quality level is higher. This also translates into a decrease of adaptation capital at steady state. The first and second terms in denominator represent the marginal cost of adaptation capital and concavity of penalty function with respect to adaptation capital. According to equation (21), a higher environmental quality requires a lower adaptation capital level at steady state. However, a more concave penalty function with respect to environmental quality means that adaptation capital is able to decrease much more the penalty rate. This results in a lower decrease of adaptation capital with an increase in environmental quality.

The negative slope indicates that when the environmental quality is higher, the economy needs less adaptation capital. This is plausible, since the probability of the catastrophic event is lower. Correspondingly, one may say that when the economy accumulates more adaptation capital, natural resources start to be overused.[15] A higher environmental quality level decreases the probability of a catastrophic event occurring. Then, the economy needs less adaptation capital.

We can also explain this result by looking at the catastrophic risk factor $h(S)(V(S, K_A) - \psi(K_A))$ in (16). When environmental quality increases, the weight of this component decreases due to a lower hazard rate, which means that there is less incentive to accumulate adaptation capital.

Another interpretation of this result can be as follows: Since agents expect to face less damage with an adaptation policy and can more easily bear the negative consequences of a catastrophic event, they tend to care less about the environmental quality.

How can adaptation policy trap an economy in a low equilibrium? The mechanism is as follows: when a policymaker starts to invest in adaptation capital, the environmental quality decreases as shown in (21) and the hazard rate increases. Since the preferences of low-income countries are directed towards the present[16], an increase in hazard rate due to adaptation capital accumulation amplifies the impatience of low-income countries. Then, these countries become less precautionary in exploiting more natural resources. It follows that the event risk amplifies again, with an increasing need for adaptation capital[17]. This mechanism, yielding a vicious cycle, explains why a developing country with a high level of marginal utility may get trapped into a low steady-state equilibrium by investing only in adaptation capital.

Indeed, the trade-off between present consumption and a catastrophic event becomes more significant with adaptation policy, since the catastrophic event risk increases. We prove this claim by an example in the following subsection.

An example with only adaptation policy

To prove the existence of the multiple equilibria in an economy with adaptation policy, we start with an example of an economy without any environmental policy that admits a unique equilibrium and use the

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[15]Zemel (2015) also finds a similar result.

[16]Recall from equation (7) that the multiple equilibria occur when agents are poor and not willing to postpone their consumption. Then, implementing adaptation policy increases the hazard rate.

[17]Since the catastrophic event risk increases, the marginal value of adaptation capital increases.

following usual functional specifications

$$R(S) = gS(1 - S) \tag{22}$$

$$h(S) = (1 - \bar{h}S) \tag{23}$$

$$u(c) = log(c) \tag{24}$$

$$\psi(K_A) = \bar{\psi}(1 - aK_A) \tag{25}$$

Differently from the previous example, we use a linear hazard function for the sake of analytical tractability. We show that a linear hazard function ensures a unique equilibrium for an economy without environmental policy. Then, we prove that an adaptation policy results in a multiple-equilibria economy.

Proposition 5. *Under the specification of functional forms (22), (23), (24) (i) An economy without an environmental policy admits a unique steady-state.*

(ii) When the economy invests in adaptation activity, multiplicity of equilibria occurs if the following condition on catastrophic damage $\bar{\psi}$ holds

$$\frac{\left[\left(4g^2 + 36g(g - \rho) - 27(g - \rho)^2\right) - \sqrt{\left(4g^2 + 36g(g - \rho) - 27(g - \rho)^2\right)^2 - 2^9 g^3 (g - \rho)}\right]}{8(g - \rho)g\left(\bar{h}\right)^2}$$
$$< \bar{\psi} < \frac{\left[\left(4g^2 + 36g(g - \rho) - 27(g - \rho)^2\right) + \sqrt{\left(4g^2 + 36g(g - \rho) - 27(g - \rho)^2\right)^2 - 2^9 g^3 (g - \rho)}\right]}{8(g - \rho)g\left(\bar{h}\right)^2} \tag{26}$$

Proof. See Appendix (H)

This example clearly shows that an economy which is initially not exposed to multiple equilibria faces the possibility of environmental traps under an adaptation policy if the condition (H.6) is ensured.

Indeed, a policy recommendation based on more adaptation capital could cause a multiplicity of steady states and trap an economy at lower equilibrium.

Consider a benchmark case where the economy without environmental policy admits a unique equilibrium. Then, when a social planner takes the benchmark economy and starts to invest in adaptation capital, the multiplicity of steady states occurs if the condition (26) is ensured.

3.2 An economy with mitigation policy only

We now show that an economy implementing mitigation policy could escape an environmental trap. The reason is as follows: since consumption comes from environmental assets, improving environmental quality (mitigation) increases the consumption level in the long run. Therefore, low income countries could have a lower level of marginal utility of consumption, implying that they could make more far-sighted decisions. As the catastrophic event probability decreases with mitigation policy, agents will be more patient and willing to postpone their consumption to the future.

Investing at rate M for mitigation improves the environmental quality. Then, in the presence of mitigation activity, environmental quality evolves according to

$$\dot{S}(t) = R(S(t)) + \Gamma(M(t)) - c(t) \tag{27}$$

where $\Gamma(M)$ represents the effects of mitigation activities such as reforestation, desalination of water stock, enhancing carbon sinks, etc. Mitigation is defined as a "human intervention to reduce the sources or enhance the sinks of greenhouse gases." (IPCC (2014), p.4) In this sense, reforestation can be considered as a means to enhance carbon sinks since forests allow for carbon sequestration.

The specification of the mitigation variable is similar to that in Chimeli and Braden (2005). Alternatively, function $\Gamma(M)$ can be considered as "environmental protection function". The expenditures for environmental protection may be directed not only toward pollution mitigation but also toward the protection of forests and recovery of degraded areas. Equivalently, mitigation activity can be seen as a means of improving environmental quality.

In order to keep the model as simple as possible, we choose to consider mitigation as a flow variable. We use the following assumption

A.4 The mitigation function given by $\Gamma(.) : \mathbb{R}_+ \to \mathbb{R}_+$ is twice continuously differentiable, with the following properties ; $\Gamma_M(M) > 0$, $\Gamma_{MM}(M) < 0$.

The mitigation function is assumed to be an increasing and concave function. Note that mitigation activity can be considered as a complement to the regeneration of environment. In an economy with mitigation activity, the expected payoff is

$$V(S) = \max_{\{c(t), M(t)\}} \int_0^\infty \left[u(c(t)) - Q^2(M) + h(S(t))[V(S(t))] - \bar{\psi} \right] \exp\left(-\int_0^t [\rho + h(S(\tau))] d\tau \right) dt \tag{28}$$

where $Q^2(M) = P_M M$ with the unit price for mitigation P_M is the linear cost function for mitigation that enters in the utility function as a social cost as in Zemel (2015) and Tsur and Zemel (2016). Since we assume a concave function for mitigation activity $\Gamma(M)$, the use of a linear cost function gives optimal interior solutions and simplifies calculations.

The optimal policy is to maximise (28) subject to (27) (see details in Appendix (K)). Optimal dynamics for consumption and mitigation are as follows

$$\dot{c} = -\frac{u_c(c)}{u_{cc}(c)} \left[R_S(S) - \rho - \frac{\bar{\psi} h_S(S)}{u_c(c)} \right] \tag{29}$$

$$\dot{M} = \frac{\Gamma_M(M)}{\Gamma_{MM}(M)} \left[R_S(S) - \rho - \frac{\bar{\psi} h_S(S)}{u_c(c)} \right] \tag{30}$$

In fact, mitigation activity renders the trade-off between present consumption and the probability of a catastrophic event occurring less binding. It follows that there are less chances for an economy to be trapped into a multiplicity of steady states.

Proposition 6. *Mitigation activity counteracts the multiplicity of equilibria.*

In order to understand how mitigation policy can avoid an environmental trap in a formal way, by using equations (29) and (30), we provide the necessary condition for multiple equilibria (Appendix (I) for details)

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$$G_S(S) = R_{SS}(S) - \frac{\bar{\psi}h_{SS}(S)}{u_c(c)} + \frac{\bar{\psi}h_S(S)u_{cc}R_S(S)}{(u_c(c))^2} + \underbrace{\frac{\bar{\psi}h_S(S)u_{cc}R_S(S)}{(u_c(c))^2}\frac{dM}{dS}}_{=Z_2} > 0 \tag{31}$$

where the term Z_2 represents the effect of mitigation on the necessary condition for multiple equilibria.

In order to understand the implications of mitigation policy on multiple equilibria condition, we look at how environmental quality level changes the steady state level of mitigation investment. To do that, we write the first order condition for mitigation activity which is (see equations (I.3) and (I.4) in Appendix (I))

$$u_c(R(S) + \Gamma(M))\Gamma_M(M) = Q_M^2(M) = P_M \tag{32}$$

and take the total derivative of (32) to find

$$\frac{dM}{dS} = -\frac{u_{cc}(c)R_S(S)\Gamma_M(M)}{u_{cc}(c)(\Gamma_M(M))^2 + u_c(c)\Gamma_{MM}(M)} < 0 \tag{33}$$

Since we have $\frac{dM}{dS} < 0$, the term Z_2 is negative and decreases $G_S(S)$. This implies that it is more difficult to ensure the multiple equilibria condition (31).

The economic intuition behind equation (33) is as follows: When environmental quality is higher, the social planner decreases mitigation. In other words, the economy needs less mitigation if environmental quality is higher.

To better understand how mitigation can avoid an environmental trap, suppose for a moment that mitigation activity could reduce the hazard rate to zero. It follows that the trade-off between present consumption and a catastrophic event, which causes multiple equilibria, disappears completely. Therefore, the multiplicity of steady state is not a possible outcome. Based on these elements, one can understand that mitigation activity leads to weakening the trade-off between present consumption and a catastrophic event.

Obviously, mitigation activity not only removes the economy from environmental trap but also increases the steady-state level of environmental quality as expected. Indeed, since consumption comes from natural resource rents, mitigation policy allows the economy to have a higher consumption level. Hence, impatience level of low-income countries decreases and they could postpone their consumption to the future as they can fulfill more easily the basic needs for survival. In a nutshell, a mitigation policy could break the vicious cycle of low consumption and environmental quality that can be triggered by an adaptation policy. Then, one can conclude that social planner should couple an adaptation policy with a mitigation policy in order to avoid a potential environmental trap.

An example with only mitigation policy

To prove that mitigation can rid the economy of multiple equilibria, we start with an example of a benchmark economy with multiple equilibria and use the following usual functional specifications. In order to have analytical results, we use the following linear functional forms;

$$R(S) = gS(1-S) \tag{34}$$

$$h(S) = \left(1 - \bar{h}S^2\right) \tag{35}$$

$$u(c) = log(c) \tag{36}$$

$$\Gamma(M) = M \tag{37}$$

$$Q^2(M) = P_M \frac{M^2}{2} \tag{38}$$

Another simplification only used for this example in order to ease the calculations is that the cost function appears in dynamic equation of environmental quality $\dot{S}$.

$$\dot{S} = R(S) + \Gamma(M) - Q^2(M) - c \tag{39}$$

For the sake of analytical tractability, we use a linear mitigation function and convex cost function for mitigation. We relax these simplifications in the numerical analysis to show that our result is robust, with more realistic functional forms. With the presence of mitigation activity, the function $G(S)$ is as follows (see Appendix (J))

$$G(S) = -2g\bar{\psi}\bar{h}S^3 + 2g\bar{\psi}\bar{h}S^2 + \left(\frac{\bar{\psi}\bar{h}}{P_M} - 2gS\right) + (g - \rho) = 0 \tag{40}$$

Proposition 7. *In an economy with mitigation policy, the multiplicity of steady states can be avoided.*
Proof. See Appendix (J)

When the economy implements mitigation policy, there is a lower number of parameter combination which causes multiple equilibria. As mentioned previously, the economic intuition relies on the fact that mitigation activity reduces the hazard rate, which weakens the trade-off between present consumption and catastrophic event risk.

3.3 Adaptation and mitigation : complementarity vs substitutability

An important issue regarding the adaptation and mitigation policies is to know whether adaptation and mitigation are complementary or substitute policies (chapter 18, IPCC (2007)). This is still an open debate in environmental economics literature. Buob and Stephan (2011) argue that the country's stage of development matters and high-income countries should invest both in adaptation and mitigation while low-income countries should only invest in adaptation. Bréchet et al. (2012) study the optimal mix of adaptation and mitigation and conclude that the substitutability between adaptation and mitigation depends the country's stage of development. A recent study by Tsur and Zemel (2015) focuses on the optimal mix of adaptation and mitigation under catastrophic event uncertainty but it remains agnostic about whether adaptation and mitigation be complementary or substitute policies. We seek to analyze this issue when an economy is subject to a catastrophic event risk. To this aim, we solve the model with both adaptation and mitigation policies. The social planner maximizes the following program

$$V(S, K_A) = \max_{\{c(t), A(t)\}} \int_0^\infty \left[u(c(t)) - Q^1(A(t)) - Q^2(M(t)) \right.$$
$$\left. + h(S(t)) \left[V(S(t), K_A(t)) \right] - \psi(K_A(t)) \right] \exp\left(-\int_0^t \left[\rho + h(S(\tau)) \right] d\tau \right) dt \quad (41)$$

subject to (15) and (27). (see Appendix K for calculations)

The steady state of the economy can be expressed in the following manner:

$$R_S(S) - \rho - \frac{\psi(K_A) h_S(S)}{V_S^B(S)} = 0 \tag{42}$$

$$\rho + \delta + \frac{h(S) \psi_{K_A}(K_A)}{Q_A^1(\delta K_A)} = 0 \tag{43}$$

$$A = \delta K_A \tag{44}$$

$$c = R(S) + \Gamma(M) \tag{45}$$

Using steady state equations, we can calculate the effect of mitigation activity on adaptation capital accumulation at steady state (see Appendix K for details)

$$\frac{dK_A}{dM} = -\frac{\overbrace{h_S(.) R_S^{-1}(.) \left(u_{cc}^{-1}(.) \left(-\frac{P_M}{(\Gamma_M(M))^2} \right) - \Gamma_M(M) \right) \frac{\psi_{K_A}(K_A)}{Q_A^1(\delta K_A)}}^{< 0}}{\underbrace{\frac{h(S) \psi_{K_A K_A}(K_A)}{Q_A^1(\delta K_A)} - \frac{h(S) \psi_{K_A}(K_A) Q_{AA}(\delta K_A) \delta}{(Q_A^1(\delta K_A))^2}}_{> 0}} > 0 \tag{46}$$

The equation (46) states that when the economy increases its mitigation activities, the adaptation investments increase as well if $R_S(S) > 0$. Then, adaptation and mitigation activity are complementary. However, note that this result is sensitive to how one models the adaptation and mitigation cost in the dynamic optimization program. Adaptation and mitigation can also be substitutes if adaptation cost enters in the resource constraint instead of a disutility (see equation (41)) in the following way

$$\dot{S} = R(S) + \Gamma(M) - c - Q^1(A) \tag{47}$$

We find[18]

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[18]For sake of space, we put all calculations in Appendix (K.2)

$$\frac{dK_A}{dM} = -\frac{\overbrace{h_S\left(.\right)R_S^{-1}\left(.\right)\left(u_{cc}^{-1}\left(.\right)\left(-\frac{P_M}{\left(\Gamma_M\left(M\right)\right)^2}\right)-\Gamma_M\left(M\right)\right)\frac{\psi_{K_A}\left(K_A\right)\Gamma_M\left(M\right)}{Q_A^1\left(\delta K_A\right)P_M}}^{<0}+\overbrace{\frac{h\left(.\right)\psi_{K_A}\left(K_A\right)\Gamma_{MM}\left(M\right)}{P_M Q^2\left(A\right)}}^{>0}}{\underbrace{\frac{h\left(.\right)\psi_{K_A K_A}\left(K_A\right)}{Q_A^2\left(\delta K_A\right)}-\frac{h\left(.\right)\psi_{K_A}\left(K_A\right)Q_{AA}\left(\delta K_A\right)\delta}{\left(Q_A^2\left(\delta K_A\right)\right)^2}+h_S\left(.\right)R_S^{-1}\left(.\right)\psi_{K_A}\left(K_A\right)\delta}_{>0}} \gtrless 0$$

$$(48)$$

The implications of modeling the cost of adaptation are neglected in the existing literature, which takes adaptation cost as a disutility (Zemel (2015); Tsur and Zemel (2015)). This example clearly shows that adaptation and mitigation can be either complementary or substitutes depending on how one models the adaptation cost. What is the economic intuition behind this result? Since adaptation investments are now financed by the rents stemming from natural resources, the economy may decrease its adaptation capital when investing in mitigation activity which aims at increasing the natural capital. Analyzing the equation (48), one may easily observe that the additional term $\frac{h(.)\psi_{K_A}(K_A)\Gamma_{MM}(M)}{P_M Q(A)}$ in the nominator relates to the marginal benefits of mitigation activity. When the marginal benefit of mitigation is sufficiently high, the economy can only invest in mitigation to avert the catastrophic event risk. Indeed, since adaptation is financed by natural resources, it contributes to the increase of the catastrophic event risk. Therefore, the economy prefers to avert the risk rather than adapting to a catastrophic damage. In a sense, we can also argue that investing less in adaptation is equivalent to mitigating more when adaptation is financed by natural resources.

If the marginal benefit of mitigation is not sufficiently high, the economy cannot decrease the catastrophic event probability. Then, the adaptation capital becomes necessary in order to deal with a catastrophic event.

4 Numerical analysis

The section aims at illustrating the theoretical findings we obtained for the examples with functional forms that we presented in the previous sections. We show the same results regarding the effects of adaptation and mitigation on the occurrence of multiple equilibria by relaxing the linearity assumptions on functional forms that we imposed for the analytical tractability.

For the numerical part, we calibrate the benchmark economy according to New Zealand economy. The reason for this choice relies on the increased frequency of catastrophic events in recent years such as floods and droughts (see EMDAT International Disaster Database (2015)).

By doing a calibration exercise, we produce a model that matches the EPI index and catastrophic event probability that is computed with real data for New Zealand [19].

In this section, we also seek to show numerically the effects of adaptation and mitigation policies that we explained in the text, on the occurrence of the multiplicity of equilibria.

We use the following functional specifications.

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[19]We refer to EMDAT International Disaster Database (2015).

Natural Regeneration Function : $R(S) = gS\left(1 - \frac{S}{\bar{S}}\right)$	Utility function : $u(c) = \frac{c^{1-\sigma} - c_{min}^{1-\sigma}}{1-\sigma}$
Source : Ren and Polasky (2014)	Source : Bommier et al. (2015)
$\bar{S}$ Carrying capacity of environment g Intrinsic growth rate of the resource stock	c_{min} Post-value consumption σ Degree of relative risk aversion
Catastrophic Damage : $\psi(K_A) = \bar{\psi}\left(\omega + (1-\omega)e^{-\gamma K_A}\right)$	Hazard Function : $h(S) = \frac{2\bar{h}}{1 + exp\left[\eta\left(s/\bar{S} - 1\right)\right]}$
Source : Bréchet et al. (2012)	Source : Ren and Polasky (2014)
$\bar{\psi}$ Damage rate without adaptation policy ω Lower bound of damage when $\psi(\infty)$ γ Elasticity of adaptation w.r.t to damage rate	$\bar{h}$ Upper bound for hazard rate η Endogeneity level of catastrophic event $\bar{S}$ Carrying capacity of environment
Mitigation function : $\Gamma(M) = M^{\alpha}$	Cost of adaptation investment : $Q_1(A) = \phi_A \frac{A^2}{2}$
Source : Kama and Pommeret (2016)	
α Elasticity of mitigation activity	ϕ_A Parameter for the change of marginal cost of adaptation

4.1 The economy with mitigation-only policy

We present the multiple equilibria economy without policy and the economy with mitigation policy. In order to understand the implications of mitigation on the economy with multiple equilibria, we first calibrate the benchmark model with multiple equilibria to match the features of New Zealand economy. We calibrate the damage parameter $\bar{\psi}$ and the intrinsic growth rate of the resource stock g such that the high steady-state level of environmental quality S matches the Environmental Performance Index (EPI) score of New Zealand[20] in 2018. We calibrate the important parameters of the hazard function (the endogeneity of the hazard η, upper bound for hazard rate $\bar{h}$) according to the estimates of New Zealand government [21] which shows that the probability of a flood in any 30 year period is measured as 45%. For the elasticity of substitution σ, we choose the conventional value used in the literature, which is 2 and the pure rate of time preference ρ is set to 0.015 as in Nordhaus (2008). We assume that the elasticity of mitigation and the unit price for mitigation activity are equal to 0.5 and to 250 respectively.

Parameter	Calibrated value
$\bar{S}$	1.01
g	0.062
$\bar{\psi}$	1315
$\bar{h}$	0.25
η	7.8
α	0.5
P_M	250

[20]The EPI index gives a score for each country between 0 and 100. Since we use a logistic growth function with carrying capacity set to 1 in our calibration, we have a value of S between 0 and 1. Hence, if a country have a score of 60, we take it as 0.60. The EPI score of New Zealand is 0.75 in 2018.

[21]https://www.ecan.govt.nz/your-region/your-environment/natural-hazards/floods/flood-probabilities/
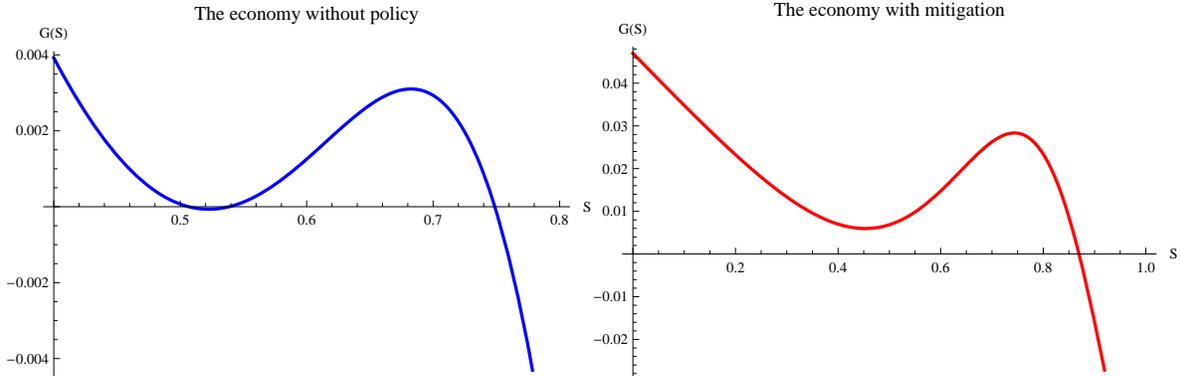
Figure 3: The benchmark economy vs the economy with mitigation

The most important message of Figure (3) is that mitigation activity enables one to avoid the occurrence of multiple equilibria as shown on the graphic on the right. The reason behind this result is that mitigation activity reduces the probability of a catastrophic event occurring. Consequently, the trade-off between present consumption and catastrophic event probability weakens, which allows an economy avoiding the multiplicity of steady states.

The Figure (3) shows also that mitigation activity not only avoids multiple equilibria but also increases environmental quality. It is easy to remark on the graphic on the right hand side of Figure (3) that the steady state level of environmental quality is higher than all three steady state levels of environmental quality on the graphic on the left.

4.2 The economy with adaptation-only policy

We present the benchmark model without policy and the economy with adaptation policy. For this numerical exercise, we always calibrate the benchmark economy according to New Zealand economy. As in the previous section, we calibrate the damage parameter $\bar{\psi}$ and the intrinsic growth rate of the resource stock g such that the steady-state level of environmental quality S equals the Environmental Performance Index (EPI) score of New Zealand in 2018.

Parameter	Calibrated value
$\bar{S}$	1
g	0.038
$\bar{\psi}$	30000
$\bar{h}$	0.23
η	8
ω	0.95
γ	0.9
ϕ_A	1

As in the previous numerical exercise for mitigation, we set the elasticity of substitution σ to 2 and the pure rate of time preference ρ to 0.015 as in Nordhaus (2008). Unfortunately, we do not have any available data for the adaptation capital. Therefore, we set parameters ω to 0.95 and γ to 0.9.
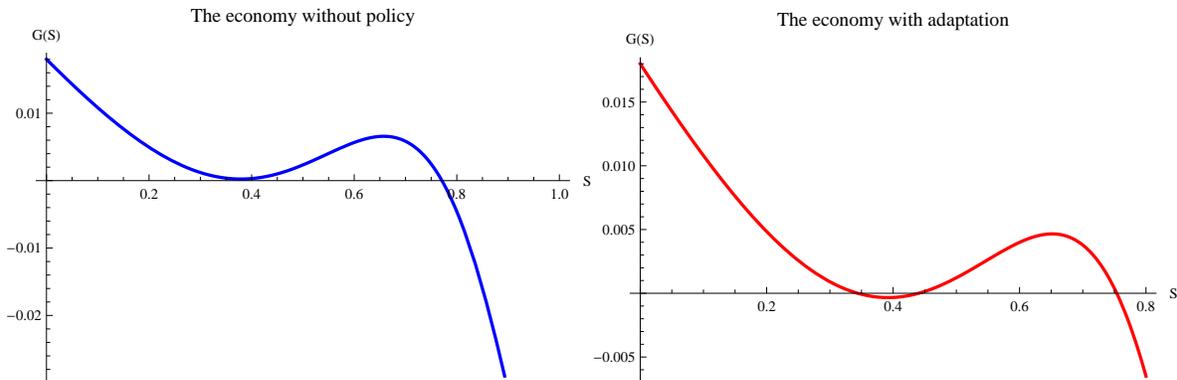
Figure 4: The benchmark economy vs the economy with adaptation

Figure (4) clearly shows that the economy can find itself in a multiple equilibria by only investing in adaptation capital. This result relies on the fact that adaptation capital decreases environmental quality (see equation (21)). As a result, the trade-off between present consumption and catastrophic event probability becomes more binding, which is the source of the multiplicity of steady states.

Then, an important policy-based message of this result is that adaptation policy should be coupled with mitigation policy in order to avoid environmental traps.

5 Conclusion

In this paper, we analyzed the effect of adaptation and mitigation policies on environmental traps in an economy subject to catastrophic event risk. The contribution of the study is to offer a new explanation for environmental traps by the catastrophic event probability and understand how environmental policy plays an important role in causing or avoiding development traps. We believe that this new perspective also provides interesting pointers to policymakers regarding the opposite effects of adaptation and mitigation policies. Our main results show that adaptation policy can lead the economy into an environmental trap, whereas mitigation helps to avoid an environmental trap. We show that a new trade-off appears between adaptation and mitigation with respect to their effect on environmental traps, other than the trade-off between adaptation and mitigation over time mentioned in numerous studies (see Zemel (2015), Tsur and Zemel (2015), Bréchet et al. (2012)). The fact that adaptation policy could cause an environmental trap does not mean that social planner should not invest in adaptation activity. On the contrary, since it is impossible to eliminate completely the hazardous event risk, she should invest in adaptation capital but should couple this policy with mitigation activity to avoid the adverse effects of adaptation policy. This is because mitigation activity weakens the trade-off between present consumption and catastrophic events, by improving the environmental quality.

Future research could test the model using empirical methods. Currently, this is very challenging since there are no available data on adaptation investments, but it is very desirable and also part of our future research agenda.

Appendix

A   Derivation of (7).

To solve the maximization problem, we write the Hamilton-Jacobi-Bellman equation.

$$\rho V^B (S) = \max_c \left\{ u (c) + V_S^B (S) (R (S) - c) - h (S) \left(V^B (S) - \varphi (S) \right) \right\} \tag{A.1}$$

where $V^B (S)$ is the value of the maximization program before the event. As also stated in the text, the value of the problem after the event is as follows:

$$\varphi (S) = V^B (S) - \bar{\psi} \tag{A.2}$$

The economy is exposed to an inflicted damage after the event. The first-order condition is given by

$$u_c (c) = V_S^B (S) \tag{A.3}$$

Computing the derivative of (A.1) with respect to the environmental quality stock S yields

$$\rho V_S^B (S) = -\bar{\psi} h_S (S) + V_{SS}^B (S) (R (S) - c) + V_S^B (S) R_S (S) \tag{A.4}$$

Differentiating the equation (A.3) and using equations (A.3) and (A.4) gives

$$\frac{u_{cc} (c)}{u_c (c)} \dot{c} = \frac{V_{SS}^B (S)}{V_S^B (S)} \dot{S} \tag{A.5}$$

$$\rho = -\frac{\bar{\psi} h_S (S)}{V_S^B (S)} + \frac{V_{SS}^B (S)}{V_S^B (S)} \dot{S} + R_S (S) \tag{A.6}$$

Arranging equations (A.5) and (A.6) gives the Keynes-Ramsey rule

$$\dot{c} = -\frac{u_c (c)}{u_{cc} (c)} \left[R_S (S) - \rho - \frac{\bar{\psi} h_S (S)}{u_c (c)} \right]$$

B   Proof of Proposition 1

The first part of the proof starts with an analysis of the limits of a function (let this function be $G (S)$) that describes the steady state of the economy by a single equation in the long run. Then, in the second part, we focus on the form of $G (S)$ and related necessary conditions for the existence of multiple equilibria.

(a) In a sense, the function $G (S)$ can be considered as the equation $\dot{c} = 0$ in terms of S at the steady state equilibrium. Writing equations $\dot{c} = 0$ and $\dot{S} = 0$;

$$R_S (S) - \rho - \frac{\bar{\psi} h_S (S)}{u_c (c)} = 0 \tag{B.1}$$

$$R (S) - c = 0 \tag{B.2}$$

Plugging equation (B.2) in (B.1) yields

$$G\left(S\right) = R_S\left(S\right) - \rho - \frac{\bar{\psi}h_S\left(S\right)}{u_c\left(R\left(S\right)\right)} \tag{B.3}$$

Note that we limit our analysis between $S \in \left[0, \bar{S}\right]$. It is possible to notice that the function $G\left(S\right)$ starts with a positive value and has negative values when S approaches $\bar{S}$. In this framework, $\bar{S}$ is the level of environmental quality level where, the consumption level is equal zero. With these information, it is easy to verify $\lim_{S \to 0} G\left(S\right) = \infty$ and $\lim_{S \to \bar{S}} G\left(S\right) = z < 0$.

(b) In this part, we show the necessary conditions for the existence of multiple equilibria, which also allows to represent the function $G\left(S\right)$;
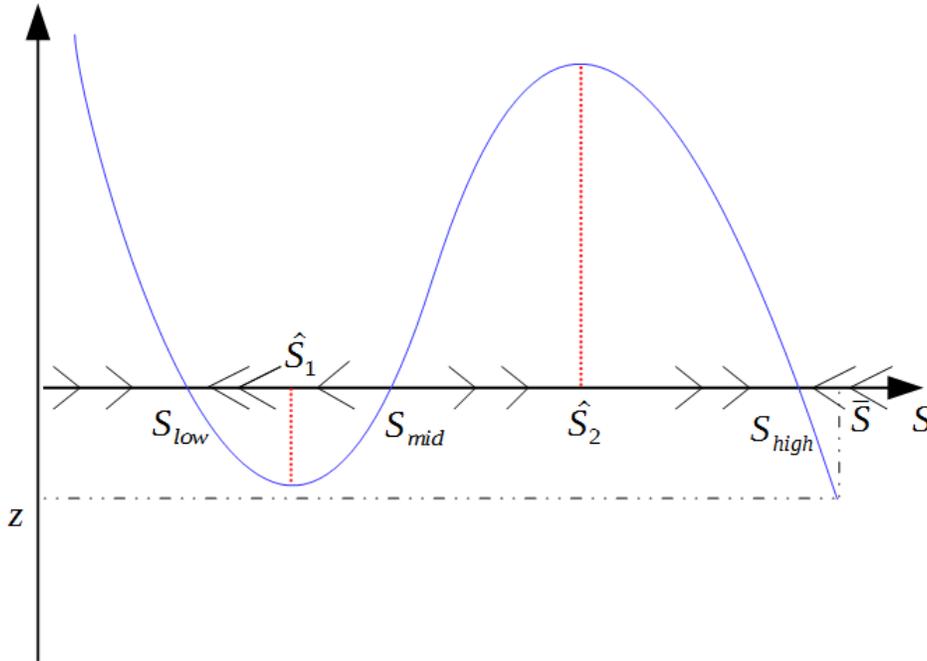

Figure 5: $G\left(S\right)$ function with uncertainty

The sufficient condition for the existence of S_{low} is that $\exists S < \widetilde{S}$ [22] which ensures (i) $G_S\left(S\right) > 0$ and (ii) $G\left(S\right) < 0$. Unless the condition (i) $G_S\left(S\right) > 0$ is satisfied, $G\left(S\right)$ starting from an arbitrary positive value may cross x-axis just one more time and converge to z, which results in a unique steady state equilibrium. Additionally, the condition (ii) $G\left(S\right) < 0$ is also necessary to ensure that $G\left(S\right)$ crosses the x-axis by S_{low} at least once.

Recall that $\hat{S}_1$ and $\hat{S}_2$ are the points of inflection. After these points, G_S changes sign. Understanding these critical points is important to determine the directions of arrows for the phase diagram analysis on plane $\left(S, c\right)$.

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[22]Note that $\widetilde{S} > S_{low}$.

The necessary condition for the existence of S_{high} is that $G\left(\widetilde{S}\right) > 0$ for $\exists \widetilde{S} < \bar{S}$. If this condition does not hold, $G(S)$ does not cross x-axis for the second time and converges to z without changing sign. Then, there exists a unique equilibrium. Once the condition $G\left(\widetilde{S}\right) > 0$ is satisfied, we notice that the $G(S)$ crosses unambiguously x-axis by S_{mid} and after converges to z when S approaches $\bar{S}$. With these necessary conditions, we prove the existence of three steady states, one being unstable and two others being stable.

When there is no endogenous occurrence probability, the necessary condition (8) reduces to $R_{SS}(S) > 0$, which makes multiple equilibria an impossible outcome. Therefore, the model reduces to a standard neoclassical growth model. This completes the proof.

C   Slope of the steady-state curve

Using equation (B.3) and implicit function theorem, we can find the slope of $\dot{c} = 0$ line.

$$\left.\frac{dc}{dS}\right|_{\dot{c}=0} = \frac{R_{SS}(S) - \frac{\bar{\psi}h_{SS}(S)}{u_c(R(S))}}{-\frac{u_{cc}(c)h_S(S)\bar{\psi}}{(u_c(c))^2}} \tag{C.1}$$

We can remark that the denominator is always negative. The sign of the necessary condition (8) is crucial in order to determine the sign of $\left.\frac{dc}{dS}\right|_{\dot{c}=0}$. Since the nominator is quite similar to condition (8), it is easy to notice that nominator is unambiguously negative within zone A and C when $G_S(S) < 0$, which makes the slope of $\dot{c} = 0$ line positive at these areas. Within the zone B, since $G_S(S) > 0$, we cannot determine if the nominator is positive or negative. This is the reason why we draw two different phase diagrams.

D   Proof of Lemma 1

The differential system describing the economy can be written as follows;

$$\begin{bmatrix} \dot{c} \\ \dot{S} \end{bmatrix} = \begin{bmatrix} \frac{d\dot{c}}{dc} & \frac{d\dot{c}}{dS} \\ \frac{d\dot{S}}{dc} & \frac{d\dot{S}}{dS} \end{bmatrix}_{\dot{c}=0,\dot{S}=0} \begin{bmatrix} c - c^* \\ S - S^* \end{bmatrix}$$

$$\frac{d\dot{c}}{dc} = \rho - R_S(S) \tag{D.1}$$

$$\frac{d\dot{c}}{dS} = -\frac{u_c(c)}{u_{cc}(c)}\left[R_{SS}(S) - \frac{\bar{\psi}h_{SS}(S)}{u_c(c)}\right] \tag{D.2}$$

$$\frac{d\dot{S}}{dc} = -1 \tag{D.3}$$

$$\frac{d\dot{S}}{dS} = R_S(S) \tag{D.4}$$

We know that for a saddle-stable path system, it is necessary to have one positive and one negative eigenvalue, denoted $\mu_{1,2}$. As the $Tr(J) = \mu_1 + \mu_2$ and $Det(J) = \mu_1\mu_2$. It is sufficient to show that $Tr(J) > 0$ and $Det(J) < 0$. It is easy to see that $Tr(J) = \rho > 0$ and by arranging the terms for the

determinant, we can see that determinant reduces to the multiple steady state condition $G(S)$. We conclude that $Det(J)$ is negative if

$$G_S(S) = R_{SS}(S) - \rho - \frac{\bar{\psi} h_{SS}(S)}{u_c(R(S))} + \frac{\bar{\psi} u_{cc}(R(S))}{(u_c(R(S)))^2} < 0 \tag{D.5}$$

$$det(J) = (\rho - R_S(S)) R_S(S) - \frac{u_c(c)}{u_{cc}(c)} \left[R_{SS}(S) - \frac{\bar{\psi} h_{SS}(S)}{u_c(c)} \right] \tag{D.6}$$

Complex dynamics arise if $(Tr(J))^2 - 4Det(J) < 0$.

$$\rho^2 < 4 \left[(\rho - R_S(S)) R_S(S) - \frac{u_c(c)}{u_{cc}(c)} \left[R_{SS}(S) - \frac{\bar{\psi} h_{SS}(S)}{u_c(c)} \right] \right] \tag{D.7}$$

As $Det(J)$ is shown to be negative for low and high steady states, this prevents these two steady states to have complex dynamics. However, for the middle steady state, there is a possibility to have complex dynamics if the condition (D.7) above holds.

E   Proof of Proposition 2

The aim of the proof is to show mathematically that all steady state values of environmental quality S take higher values than the unique steady state of the model without uncertainty. This means that the economy becomes always precautionary when exposed to risk relative to an economy without any catastrophic events. Suppose that the condition (14) holds for the existence of three positive real roots. The function $G(S)$ with risk and the function without risk $G^{WR}(S)$ are as follows:

$$G(S) = R_S(S) - \rho - \frac{\bar{\psi} h_S(S)}{u_c(c)} = 0 \tag{E.1}$$

$$G^{WR}(S) = R_S - \rho \tag{E.2}$$

With functional forms in section (2.2), we have

$$G(S) = -2g\bar{\psi}\bar{h}S^3 + 2g\bar{\psi}\bar{h}S^2 - 2gS + (g - \rho) = 0 \tag{E.3}$$

$$G^{WR}(S) = -2gS + (g - \rho) = 0 \tag{E.4}$$

Since we use a logistic function for natural resource regeneration, we have $S \in [0;1]$. We know that $G(0) = G^{WR}(0) = (g - \rho)$. This means that two functions start from the same point. It is easy to remark that functions $G(S)$ and $G^{WR}(S)$ never intersect, since $G(S) \neq G^{WR}(S)$ other than the point where S equals 0.

After these information, we can easily show that all three roots of function $G(S)$ take higher values than the unique root of function $G^{WR}(S)$. The additional term $-2g\bar{\psi}\bar{h}S^3 + 2g\bar{\psi}\bar{h}S^2$ in $G(S)$ is always a positive term when $S \in [0;1]$. This means that $G(S) > G^{WR}(S)$ for $S \in [0;1]$. This completes the proof.

F Proof of Proposition 3

The third degree polynomial equation has the following form

$$a_1 x^3 + b_1 x^2 + c_1 x + d_1 = 0 \tag{F.1}$$

With the functional forms given in the text, we remark that terms $b_1 = -a_1$ with $a_1 = -2g\bar{\psi}\bar{h}$, $c_1 = -2g$ and $d_1 = g - \rho$. This simplifies the proof of the existence for three positive real roots. The discriminant of the cubic equation is as follows:

$$\Delta = 18a_1 b_1 c_1 d_1 - 4b_1^3 d + b_1^2 c_1^2 - 4_1 ac_1^3 - 27a_1^2 d_1^2 \tag{F.2}$$

- $\Delta > 0$, the equation (F.1) has three distinct real roots.

- $\Delta = 0$, the equation (F.1) has multiple roots and all three roots are real.

- $\Delta < 0$, the equation (F.1) has one real root and two non-real, complex roots.

Since we have $b = -a$, we can reformulate the discriminant (F.2) in the following way

$$\Delta = -b_1 \left[4d_1 b_1^2 - \left(c_1^2 - 18c_1 d_1 - 27d_1^2 \right) b_1 - 4c_1^3 \right] \tag{F.3}$$

Then, we have a second degree equation to be solved for the value of b. The discriminant of this second degree equation is $\Delta_1 = \left(c^2 - 18cd - 27d^2 \right)^2 - 64dc^3$. The equation (F.3) is written as

$$\Delta = \underbrace{-b_1}_{<0} \left[b_1 - \left(\frac{\left(c_1^2 - 18c_1 d_1 - 27d_1^2 \right) + \sqrt{\left(c_1^2 - 18c_1 d_1 - 27d_1^2 \right)^2 + 64d_1 c_1^3}}{8d_1} \right) \right]$$
$$\left[b_1 - \left(\frac{\left(c_1^2 - 18c_1 d_1 - 27d_1^2 \right) - \sqrt{\left(c_1^2 - 18c_1 d_1 - 27d_1^2 \right)^2 + 64d_1 c_1^3}}{8d_1} \right) \right] \tag{F.4}$$

Then, by assuming $\Delta_1 > 0$, the discriminant Δ has a positive value if

$$\underbrace{\left(\frac{\left(c_1^2 - 18c_1 d_1 - 27d_1^2 \right) - \sqrt{\left(c_1^2 - 18c_1 d_1 - 27d_1^2 \right)^2 + 64d_1 c_1^3}}{8d_1} \right)}_{L_E}$$
$$< b_1 < \underbrace{\left(\frac{\left(c_1^2 - 18c_1 d_1 - 27d_1^2 \right) + \sqrt{\left(c_1^2 - 18c_1 d_1 - 27d_1^2 \right)^2 + 64d_1 c_1^3}}{8d_1} \right)}_{U_E} \tag{F.5}$$

where L_E and U_E are lower and upper extremes of the inequality (F.5). By replacing the terms a_1, b_1, c_1, d_1 by their corresponding values, the condition (F.5) becomes

$$\frac{\left(4g^2 + 36g\left(g - \rho\right) - 27\left(g - \rho\right)^2\right) - \sqrt{\left(4g^2 + 36g\left(g - \rho\right) - 27\left(g - \rho\right)^2\right)^2 - 2^9\left(g - \rho\right)}}{8\left(g - \rho\right)g\bar{h}}$$

$$< \bar{\psi} < \frac{\left(4g^2 + 36g\left(g - \rho\right) - 27\left(g - \rho\right)^2\right) + \sqrt{\left(4g^2 + 36g\left(g - \rho\right) - 27\left(g - \rho\right)^2\right)^2 - 2^9\left(g - \rho\right)}}{8\left(g - \rho\right)g\bar{h}} \quad \text{(F.6)}$$

However, the positive discriminant Δ proves only the existence of real roots and does not give an idea if these roots are positive or not. For this purpose, we use the Descartes rule. To see if the equation $G\left(S\right) = -2g\bar{\psi}\bar{h}S^3 + 2g\bar{\psi}\bar{h}S^2 - 2gS + \left(g - \rho\right) = 0$ has three positive real roots, we write $G\left(-S\right)$

$$G\left(-S\right) = -2g\bar{\psi}\bar{h}\left(-S\right)^3 + 2g\bar{\psi}\bar{h}\left(-S\right)^2 - 2g\left(-S\right) + \left(g - \rho\right) = 0 \quad \text{(F.7)}$$

This yields

$$G\left(-S\right) = 2g\bar{\psi}\bar{h}S^3 + 2g\bar{\psi}\bar{h}S^2 + 2gS + \left(g - \rho\right) = 0 \quad \text{(F.8)}$$

By assuming $g - \rho > 0$, we observe that there is no sign change. This proves all roots are positive for $G\left(S\right)$. This completes the proof.

G The economy with only adaptation

The Hamilton-Jacobi-Bellman equation of the economy with only adaptation policy is as follows

$$\rho V^B\left(S, K_A\right) = \max_{c,A}\left\{u\left(c\right) - Q^1\left(A\right) + V_S^B\left(S, K_A\right)\left(R\left(S\right) - c\right)\right.$$
$$\left. + V_{K_A}^B\left(S, K_A\right)\left(A - \delta K_A\right) - h\left(S\right)\left(V^B\left(S, K_A\right) - \varphi\left(S, K_A\right)\right)\right\} \quad \text{(G.1)}$$

First order conditions for an internal optimal solution give

$$u_c = V_S^B \quad \text{(G.2)}$$

$$V_{K_A}^B = Q_A^1 V_S^B \quad \text{(G.3)}$$

Differentiating (G.1) with respect to S and K_A gives

$$\rho V_S^B = V_{SS}^B\left(R\left(S\right) - c\right) + V_S^B R_S + V_{K_A S}\left(A - \delta K_A\right) - h_S\left(S\right)\psi\left(K_A\right) \quad \text{(G.4)}$$

where $V_{K_A S} = \frac{\partial^2 V}{\partial K_A \partial S}$

$$\rho V_{K_A}^B = V_{SK_A}^B\left(R\left(S\right) - c\right) + V_S^B R_S + V_{K_A K_A}\left(A - \delta K_A\right) - h\left(S\right)\psi_{K_A}\left(K_A\right) - \delta V_{K_A}^B \quad \text{(G.5)}$$

The optimal dynamics of consumption and adaptation investment are

$$\dot{c}^{RE} = -\frac{u_c\left(c\right)}{u_{cc}\left(c\right)}\left[R_S\left(S\right) - \rho - \frac{\psi\left(K_A\right)h_S\left(S\right)}{u_c\left(c\right)}\right] \tag{G.6}$$

$$\dot{A} = \frac{Q_A^1\left(A\right)}{Q_{AA}^1\left(A\right)}\left[\left(\rho + \delta\right) + \frac{h\left(S\right)\psi_{K_A}\left(K_A\right)}{Q_A}\right] \tag{G.7}$$

Using $A = \delta K_A$, at the steady state, we have

$$Q_A^1\left(\rho + \delta\right) + h\left(S\right)\psi_{K_A}\left(K_A\right) = 0 \tag{G.8}$$

$$R_S\left(S\right) - \rho - \frac{\psi\left(K_A\right)h_S\left(S\right)}{u_c\left(c\right)} = 0 \tag{G.9}$$

Using the equations (G.6) and $R\left(S\right) = c$, we have

$$G\left(S\right) = R_S\left(S\right) - \rho - \frac{\psi\left(K_A\right)h_S\left(S\right)}{u_c\left(R\left(S\right)\right)} \tag{G.10}$$

Differentiating the equation (G.10) with respect to S yields

$$G_S\left(S\right) = R_{SS}\left(S\right) - \frac{h_{SS}\left(S\right)\psi\left(K_A\right)}{u_c\left(R\left(S\right)\right)} + \frac{h_S\left(S\right)\psi\left(K_A\right)u_{cc}\left(R\left(S\right)\right)R_S\left(S\right)}{\left(u_c\left(R\left(S\right)\right)\right)^2} - \underbrace{\left[\frac{\psi_{K_A}\left(K_A\right)h_S\left(S\right)}{u_c\left(R\left(S\right)\right)}\right]\frac{dK_A}{dS}}_{=Z_1 > 0} > 0$$

It is also possible to write the equation (G.10) by using the functional forms in (2.2). Using the equations (G.6), (G.7) and functional forms presented in the section (2.2), the function $G\left(S\right)$ that describes the steady-state of the economy becomes

$$G\left(S\right) = \underbrace{g\left(1 - 2S\right) - \rho + \frac{\bar{\psi}\bar{h}}{u_c}}_{Benchmark} + \underbrace{\frac{a\bar{\psi}\left(1 - \bar{h}S\right)}{\phi\delta\left(\rho + \delta\right)u_c}}_{Adaptation} \tag{G.11}$$

The first part for the expression (G.11) is the same as in the benchmark model. The additional last term comes with the adaptation policy. After arranging terms, the equation (G.11) writes

$$G\left(S\right) = -\bar{h}gzS^3 + \left(\left(g + \bar{h}g\right)z - \bar{\psi}\bar{h}g\right)S^2 + \left(\bar{\psi}\bar{h}g - 2g - zg\right)S + \left(g - \rho\right) = 0 \tag{G.12}$$

H   Proof of Proposition 5 (Adaptation)

The equation (7) combined with (1) at the steady state can be reformulated as

$$G\left(S\right) = -\bar{\psi}\bar{h}gS^2 + \left(\bar{\psi}\bar{h}g - 2g\right)S + \left(g - \rho\right) = 0 \tag{H.1}$$

It is easy to show that this equation has one positive and one negative real root when $g - \rho > 0$:

$$S_{1,2} = \frac{\left(\bar{\psi}\bar{h}g - 2g\right) \pm \sqrt{\left(\bar{\psi}\bar{h}g - 2g\right)^2 + 4\bar{\psi}\bar{h}g\left(g - \rho\right)}}{2\bar{\psi}\bar{h}g} \tag{H.2}$$

We exclude the negative root since it has no economic meaning. Consequently, the economy has a unique equilibrium without any possibility of multiple equilibria. What happens if this economy starts to invest in adaptation capital? For the sake of analytical tractability, we use a linear hazard and adaptation function. In the numerical analysis, we relax the linearity assumption on functional forms and use more general functional forms to show the robustness of our results.

$$\psi(K_A) = \bar{\psi} f(K_A) = \bar{\psi}(1 - aK_A) \tag{H.3}$$

$$Q(A) = \phi \frac{A^2}{2} \tag{H.4}$$

where a shows at which extent the adaptation capital is able to decrease the vulnerability against the inflicted damage $\bar{\psi}$. ϕ stands for a scale parameter for the adaptation cost function. The benchmark economy with an adaptation policy ends up with the following $G(S)$ (see Appendix G)

$$G(S) = -\bar{h}gzS^3 + \left((g + \bar{h}g)z - \bar{\psi}\bar{h}g\right)S^2 + \left(\bar{\psi}\bar{h}g - 2g - zg\right)S + (g - \rho) = 0 \tag{H.5}$$

where $z = \frac{\bar{h}(a\bar{\psi})^2}{\phi\delta(\rho+\delta)}$. For analytical tractability, we suppose that $\bar{\psi} = \frac{z}{h}$. Then, the equation (H.5) can be reformulated in a simplified form

$$G(S) = -\bar{h}gzS^3 + \bar{h}gzS^2 - 2gS + (g - \rho) = 0$$

This equation is similar to (13) which can be shown to have three positive real roots. Following the same method for the proof of Proposition 3., the condition to have multiple equilibria in the benchmark economy augmented by the adaptation investments is as follows[23]

$$\frac{\left[\left(4g^2 + 36g(g-\rho) - 27(g-\rho)^2\right) - \sqrt{\left(4g^2 + 36g(g-\rho) - 27(g-\rho)^2\right)^2 - 2^9g^3(g-\rho)}\right]}{8(g-\rho)g\left(\bar{h}\right)^2}$$

$$< \bar{\psi} < \frac{\left[\left(4g^2 + 36g(g-\rho) - 27(g-\rho)^2\right) + \sqrt{\left(4g^2 + 36g(g-\rho) - 27(g-\rho)^2\right)^2 - 2^9g^3(g-\rho)}\right]}{8(g-\rho)g\left(\bar{h}\right)^2} \tag{H.6}$$

I   The economy with only mitigation

We write the Hamilton-Jacobi-Bellman equation for the economy with a mitigation activity.

$$\rho V^B(S) = \max_{c,M}\left\{u(c) - Q^2(M) + V_S^B(S)(R(S) + \Gamma(M) - c) - h(S)\left(V^B(S) - \varphi(S)\right)\right\} \tag{I.1}$$

where $V^B(S)$ is the value of the maximisation program before the event. As also stated in the text, the value of the problem after the event is as follows:

$$\varphi(S) = V^B(S) - \bar{\psi} \tag{I.2}$$

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[23]The proof is available upon request.

The economy is exposed to an inflicted damage after the event. The first-order conditions are given by

$$u_c(c) = V_S^B(S) \tag{I.3}$$

$$Q_M^2 = V_S^B(S)\Gamma_M(M) \tag{I.4}$$

Calculating the derivative of (I.1) with respect to the environmental quality stock S yields

$$\rho V_S^B(S) = -\bar{\psi}h_S(S) + V_{SS}^B(S)(R(S) + \Gamma(M) - c) + V_S^B(S)R_S(S) \tag{I.5}$$

Differentiating the equation (I.3) and using equations (I.3) and (I.5) gives

$$\frac{u_{cc}(c)}{u_c(c)}\dot{c} = \frac{V_{SS}^B(S)}{V_S^B(S)}\dot{S} \tag{I.6}$$

$$\rho = -\frac{\bar{\psi}h_S(S)}{V_S^B(S)} + \frac{V_{SS}^B(S)}{V_S^B(S)}\dot{S} + R_S(S) \tag{I.7}$$

Arranging equations (I.6) and (I.7) gives the Keynes-Ramsey rule

$$\dot{c} = -\frac{u_c(c)}{u_{cc}(c)}\left[R_S(S) - \rho - \frac{\bar{\psi}h_S(S)}{u_c(c)}\right]$$

At the steady-state, we have

$$G(S) = R_S(S) - \rho - \frac{\bar{\psi}h_S(S)}{u_c(R(S) + \Gamma(M))} \tag{I.8}$$

Note that the optimal steady state level of mitigation M depends on S. Differentiating the equation (I.8) with respect to S yields

$$G_S(S) = R_{SS}(S) - \frac{\bar{\psi}h_{SS}(S)}{u_c(c)} + \frac{\bar{\psi}h_S(S)u_{cc}}{(u_c(c))^2}\left(R_S(S) + \Gamma_M(M)\frac{dM}{dS}\right)$$

J   Proof of Proposition 7 (Mitigation)

We start the proof by solving the following optimization problem with mitigation policy. Similar to the resolution of the benchmark economy, we write the Hamilton-Jacobi-Bellman equation

$$\rho V^B(S) = \max_{c,M}\left\{u(c) + V_S^B(S)(R(S) + \Gamma(M) - Q_2(M) - c) - h(S)\left(V^B(S) - \varphi(S)\right)\right\} \tag{J.1}$$

where $V^B(S)$ is the value of the maximisation program before the event. The economy is exposed to a constant inflicted damage after the occurrence of the event. As also stated in the text, the value of the problem after the event is as follows

$$\varphi(S) = V^B(S) - \bar{\psi} \tag{J.2}$$

The first-order conditions are given by

$$u_c(c) = V_S^B(S) \tag{J.3}$$

$$\Gamma_M(M) = Q_M^2(M) = P_M \tag{J.4}$$

Using the envelop theorem, we have

$$\rho V_S^B(S) = -\bar{\psi}h_S(S) + V_{SS}^B(S)(R(S) + \Gamma(M) - Q_2(M) - c) + V_S^B(S)R_S(S) \tag{J.5}$$

Differentiating the equation (J.3) and using equations (J.3) and (J.5) gives

$$\frac{u_{cc}(c)}{u_c(c)}\dot{c} = \frac{V_{SS}^B(S)}{V_S^B(S)}\dot{S} \tag{J.6}$$

$$\rho = -\frac{\bar{\psi}h_S(S)}{V_S^B(S)} + \frac{V_{SS}^B(S)}{V_S^B(S)}\dot{S} + R_S(S) \tag{J.7}$$

Arranging equations (J.6) and (J.7) gives the Keynes-Ramsey rule

$$\dot{c} = -\frac{u_c(c)}{u_{cc}(c)}\left[R_S(S) - \rho - \frac{\bar{\psi}h_S(S)}{u_c(c)}\right]$$

At the steady state, we have

$$G(S) = R_S(S) - \rho - \frac{\bar{\psi}h_S(S)}{u_c(R(S) + \Gamma(M) - Q_2(M))} = 0$$

Since we use linear specifications for analytical tractability, we have the optimum level of mitigation investment as $M^* = \frac{1}{P_M}$. Then, we can reformulate the function $G(S)$

$$G(S) = -2g\bar{\psi}\bar{h}S^3 + 2g\bar{\psi}\bar{h}S^2 + \left(\frac{\bar{\psi}\bar{h}}{P_M} - 2gS\right) + (g - \rho) = 0 \tag{J.8}$$

where $a_1 = -2g\bar{\psi}\bar{h}$, $b_1 = 2g\bar{\psi}\bar{h}$, $c_1 = \frac{\bar{\psi}\bar{h}}{P_M} - 2gS$ and $d_1 = g - \rho$. The equation (J.8) is similar to $G(S)$ of the benchmark economy without an environmental policy. The only term that is different in the economy with mitigation policy with respect to the benchmark economy is the term c_1 of the equation (F.1). The term c_1 is higher in the case with mitigation policy with respect to the benchmark economy case.

We look at the effect of a higher value c_1 on the upper and lower extremes of the inequality (F.5). By assuming $c_1 < 0$ and $d_1 > 0$, the derivative of the upper and lower extremes are as follows

$$\frac{\partial L_E}{\partial c_1} = \frac{1}{2}\frac{\overbrace{(c_1^2 - 18c_1d_1 - 27d_1^2)}^{>0}\overbrace{(2c_1 - 18d_1)}^{<0}}{\left((c_1^2 - 18c_1d_1 - 27d_1^2)^2\right)^{\frac{1}{2}}} - \frac{1}{2}\left(\frac{\overbrace{(c_1^2 - 18c_1d_1 - 27d_1^2)}^{>0}\overbrace{(2c_1 - 18d_1)}^{<0} + 3.64d_1c_1^2}{\left((c_1^2 - 18c_1d_1 - 27d_1^2)^2 + 64d_1c_1^3\right)^{\frac{1}{2}}}\right) > 0$$

$$\frac{\partial U_E}{\partial c_1} = \frac{1}{2} \frac{\overbrace{(c_1^2 - 18c_1 d_1 - 27d_1^2)}^{>0} \overbrace{(2c_1 - 18d_1)}^{<0}}{\left((c_1^2 - 18c_1 d_1 - 27d_1^2)^2 \right)^{\frac{1}{2}}} + \frac{1}{2} \left(\frac{\overbrace{(c_1^2 - 18c_1 d_1 - 27d_1^2)}^{>0} \overbrace{(2c_1 - 18d_1)}^{<0} + 3.64 d_1 c_1^2}{\left((c_1^2 - 18c_1 d_1 - 27d_1^2)^2 + 64 d_1 c_1^3 \right)^{\frac{1}{2}}} \right) < 0$$

We observe that the occurrence of multiple equilibria with mitigation policy is less likely, since there is a smaller range of values for damage $\bar{\psi}$ that causes multiple equilibria.

K   The economy with adaptation and mitigation

K.1   Adaptation cost as a social disutility

We write the Hamilton-Jacobi-Bellman equation for the economy implementing both adaptation and mitigation policies.

$$\rho V^B(S, K_A) = \max_{c,A} \left\{ u(c) - Q^1(A) - Q^2(M) + V_S^B(S, K_A)(R(S) + \Gamma(M) - c) \right.$$
$$\left. + V_{K_A}^B(S, K_A)(A - \delta K_A) - h(S)\left(V^B(S) - \varphi(S, K_A)\right) \right\} \quad \text{(K.1)}$$

where $V^B(S)$ is the value of the maximisation program before the event. As also stated in the text, the value of the problem after the event is as follows:

$$\varphi(S, K_A) = V^B(S, K_A) - \psi(K_A) \quad \text{(K.2)}$$

The economy is exposed to an inflicted damage after the event. The first-order conditions are given by

$$u_c(c) = V_S^B(S, K_A) \quad \text{(K.3)}$$

$$Q_A^1(A) = V_{K_A}^B(S, K_A) \quad \text{(K.4)}$$

$$Q_M^2(M) = V_S^B(S, K_A) \quad \text{(K.5)}$$

Computing the derivative of (K.1) with respect to the environmental quality stock S and adaptation capital K_A yields

$$\rho V_S^B(S, K_A) = -h_S(S)\psi(K_A) + V_{SS}^B(S, K_A)\left(R(S) + \Gamma(M) - Q^1(A) - c\right)$$
$$+ V_{K_A S}^B(S, K_A)(A - \delta K_A) + V_S^B(S, K_A) R_S(S) \quad \text{(K.6)}$$

$$\rho V_{K_A}^B (S, K_A) = V_{SK_A}^B (S, K_A) \left(R(S) + \Gamma(M) - Q^1(A) - c \right)$$
$$+ V_{K_A K_A}^B (S, K_A) (A - \delta K_A) - \delta V_{K_A}^B (S, K_A) - h(S) \psi_{K_A} (K_A) \quad \text{(K.7)}$$

Differentiating the equation (K.3) and using equations (K.3) and (K.6) gives

$$\frac{u_{cc}(c)}{u_c(c)} \dot{c} = \frac{V_{SS}^B (S, K_A)}{V_S^B (S, K_A)} \dot{S} \tag{K.8}$$

$$\frac{Q_{AA}^1(A)}{Q_A^1(A)} \dot{A} = \frac{V_{K_A K_A}^B (S, K_A)}{V_{K_A}^B (S, K_A)} \dot{K}_A \tag{K.9}$$

Using the equations (K.6) and (K.7), at the steady-state, we have

$$R_S(S) - \rho - \frac{\psi(K_A) h_S(S)}{V_S^B(S)} = 0 \tag{K.10}$$

$$\rho + \delta + \frac{h(S) \psi_{K_A}(K_A)}{Q_A^1(\delta K_A)} = 0 \tag{K.11}$$

Using the first order conditions (K.3), (K.4), (K.5) and $R(S) + \Gamma(M) - Q^1(A) = c$, we write inverse functions as

$$R^{-1}(c - \Gamma(M)) = S \tag{K.12}$$

$$c = u_c^{-1} \left(\frac{P_M}{\Gamma_M(M)} \right) \tag{K.13}$$

Plugging the equations (K.12) and (K.13) in (K.11), we have

$$\rho + \delta + \frac{h \left(R^{-1} \left(u_c^{-1} \left(\frac{P_M}{\Gamma_M(M)} \right) - \Gamma(M) \right) \right) \psi_{K_A}(K_A)}{Q_A^1(\delta K_A)} = 0 \tag{K.14}$$

Using the implicit function theorem for (K.14) yields the equation (46).

K.2 Adaptation cost in resource constraint

In this section, we model the adaptation cost in resource constraint. The Hamilton-Jacobi-Bellman equation is as follows

$$\rho V^B(S, K_A) = \max_{c, A} \left\{ u(c) - Q^2(M) + V_S^B(S, K_A) \left(R(S) + \Gamma(M) - Q^1(A) - c \right) \right.$$
$$\left. + V_{K_A}^B (S, K_A) (A - \delta K_A) - h(S) \left(V^B(S) - \varphi(S, K_A) \right) \right\} \quad \text{(K.15)}$$

The first-order conditions are given by

$$u_c(c) = V_S^B(S, K_A) \tag{K.16}$$

$$Q_A^1(A) V_S^B(S, K_A) = V_{K_A}^B(S, K_A) \tag{K.17}$$

$$Q_M^2(M) = V_S^B(S, K_A) \tag{K.18}$$

Computing the derivative of K.15 with respect to the environmental quality stock $S(t)$ and adaptation capital K_A yields

$$\rho V_S^B(S, K_A) = -h_S(S) \psi(K_A) + V_{SS}^B(S, K_A) \left(R(S) + \Gamma(M) - c - Q^1(A)\right)$$
$$+ V_{K_A S}^B(S, K_A) (A - \delta K_A) + V_S^B(S, K_A) R_S(S) \tag{K.19}$$

$$\rho V_{K_A}^B(S, K_A) = V_{SK_A}^B(S, K_A) \left(R(S) + \Gamma(M) - c - Q_A^1(A)\right)$$
$$+ V_{K_A K_A}^B(S, K_A)(A - \delta K_A) - \delta V_{K_A}^B(S, K_A) - h(S)\psi_{K_A}(K_A) \tag{K.20}$$

Differentiating the equation (K.16) and using equations (K.16) and (K.19) gives

$$\frac{u_{cc}(c)}{u_c(c)} \dot{c} = \frac{V_{SS}^B(S, K_A)}{V_S^B(S, K_A)} \dot{S} \tag{K.21}$$

$$\frac{Q_{AA}^1(A)}{Q_A^1(A)} \dot{A} = \frac{V_{K_A K_A}^B(S, K_A)}{V_{K_A}^B(S, K_A)} \dot{K}_A$$

Using the equations (K.19) and (K.20), at the steady-state, we have

$$R_S(S) - \rho - \frac{\psi(K_A) h_S(S)}{V_S^B(S)} = 0 \tag{K.22}$$

$$\rho + \delta + \frac{h(S) \psi_{K_A}(K_A)}{Q_A^1(A) u_c(c)} = 0 \tag{K.23}$$

Using the first order conditions (K.16), (K.17), (K.18) and $R(S) + \Gamma(M) = c$, we write inverse functions as

$$R^{-1}\left(c + Q^1(A) - \Gamma(M)\right) = S \tag{K.24}$$

$$c = u_c^{-1}\left(\frac{P_M}{\Gamma_M(M)}\right) \tag{K.25}$$

Plugging (K.24) and (K.25) in (K.23), we have

$$\rho + \delta + \frac{h\left(R^{-1}\left(u_c^{-1}\left(\frac{P_M}{\Gamma_M(M)}\right) - \Gamma(M) + Q^1(A)\right)\right)\psi_{K_A}(K_A)}{Q_A(\delta K_A) u_c(c)} = 0 \tag{K.26}$$

In order to see whether adaptation and mitigation are complementary or substitutes, we compute the derivatives of the implicit function (K.26) which yield the equation (48).

Acknowledgments

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