# Quorum Rules and Shareholder Voting*

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September 13, 2011

**Abstract**

This paper completely characterizes the equilibria of a costly voting game where shareholders of a firm may vote for or against a proposed resolution, or not vote. Two types of shareholders are considered: unconditional who always vote and partisans who vote strategically. It is shown that the existence of equilibria crucially depends on the quorum rule and on the shareholding structure. Equilibria in favor of the resolution and against may co-exist. A high minimum quorum favors the occurrence of (1) equilibria where coalitions of small partisans of the proposed resolution vote and the resolution is adopted (2) an equilibrium against where no partisan votes. For smaller values of the minimum quorum, other equilibria for and against exist where coalitions of partisans vote. Such coalitions gather only large shareholders. Finally, the equilibrium outcome may or may not correspond to the preference of the dominant shareholder.

**Keywords**: General assembly, Strategic voting, Unconditionals, Quorum rule.

**JEL codes:** D72, G32, G34, K22

*Our research was financially supported by the Europlace Institute of Finance.

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# 1 Introduction

The annual meeting is the only regular occasion where all shareholders have the opportunity to express themselves directly on important issues regarding their company. The only other case in which they have a direct say is a tender offer, which is a rare and disruptive event in a company's life. Yet, shareholder meetings received relatively less attention than other governance mechanisms such as takeovers, boards of directors or management's compensation from legislators and academics. Still the power of shareholders may be quite important in some countries where they can sponsor resolutions in favor of the replacement of management (Charléty, Chevillon and Messaoudi 2009).

Voting models developed in public economics for large electorates usually assume that each elector has one vote, and that votes are simultaneous. One objective of these models is to explain the turnout in elections. In essence, with no voting cost, all electors should vote for their preferred alternative. On the other hand, even with a very small voting cost, most people should not vote since the probability to affect the outcome in large elections is infinitesimal. Feddersen and Pesendorfer (1996) and Feddersen (2004) explain why abstention may be strategic even when voting entails no cost: if some voters are better informed about the best alternative, it is rational to delegate the choice to the better informed and abstain. They also show that it may be rational for electors to vote against their private but imperfect information when they know that other voters know what is best.

Based on a version of Feddersen and Pesendorfer (1998), Maug and Rydqvist (2009) analyze the strategic voting of shareholders in annual meetings under the following assumptions: (1) each shareholder has one vote; (2) all shareholders have the same objective, i.e. select resolutions which increase profits; (3) information is asymmetric: although they share the same initial priors, they receive a private signal; (4) the only two possibilities are to vote for or against a resolution proposed by the management. In line with Feddersen and Pesendorfer, they show that shareholders may vote against their private information; they also derive an interesting result concerning the effects of majority voting rules. More stringent majority rules (for example a majority of 2/3 of votes rather than 1/2 to pass a resolution) induce more shareholders to vote in favour of the resolution. Indeed, understanding that a higher majority may prohibit the adoption of a good resolution, shareholders compensate this bias by voting more often for. As a result, their model predicts that the number of votes for increases with the required majority and that the adoption rate is independent of the rule in equilibrium.

The aforementioned model assumes all stockholders have one vote, share the same objective and that voting is costless. In practise, none of these hypotheses holds. In many countries the presence of large voting blocks besides smaller ones is the rule and shareholders differ in their voting power (Becht and Roell (1999) or Becht and Mayer (2002)). Also shareholders often have conflicting interests (e.g. the State versus hedge funds or employees).

Ritzberger (2005) analyzed voting in annual meetings when stockholders with different voting shares disagree on the resolutions, some being in favour of the proposal, others against (or equivalently for the status quo). Information is symmetric and voting entails a small cost. He concludes that there exists an equilibrium if and only if the largest (or dominant) shareholder is for the resolution (assuming that when nobody votes the status quo prevails). In equilibrium, only one shareholder votes, and the resolution is adopted. The equilibrium outcome therefore always corresponds to the dominant shareholder's preference. This result is easily explained. Since voting is costly, a shareholder votes only if: (1) his vote is necessary to obtain his preferred outcome; (2) no opponent shareholder may rock the result. This can only happen when the shareholder voting for the resolution (not necessarily the largest) commands more votes than any partisan of the status quo. It should be noted that, in equilibrium, the quorum is the share of the only voter, and the resolution passes with a majority of 100%. Thus the majority rule plays no role.

In line with Ritzberger, we analyze the case where shareholders who do not agree on the relevance of resolutions, differ in their voting power, incur a (small) voting cost and vote strategically. In addition to these partisans, we assume the presence of unconditional shareholders who always vote for or against the resolutions; or equivalently that voting is mandatory for some shareholders. Finally, as specified by law in most countries, or defined by the corporate charter, we suppose that a minimum number of shares must be present or represented (quorum rule) in assembly, and that a resolution must obtain a minimum of favourable votes (majority rule) to be adopted.

We investigate the role of the quorum rule and the presence of unconditional shareholders under these hypotheses.

Our results differ substantively from Ritzberger's. In particular, we show the existence of equilibria where: (1) coalitions of shareholders for (and against) the resolution win even though the dominant shareholder is against (for); (2) no shareholder (except the unconditionals) votes and the resolution is not adopted even though the dominant shareholder is for the resolution. Moreover, for some shareholding structures and preferences, equilibria leading to a different outcome (adoption or rejection) may coexist.

Obviously, the existence and nature of the equilibria depend on the shareholding structure and the quorum rule. Let us take an example. Suppose a company has two large shareholders in favour of a resolution, holding respectively 20% and 15% of the voting shares, one large shareholder against the resolution with 30% of the voting shares. All the remaining shares are widely held and the unconditionals altogether represent a very small part of capital. Let the minimum quorum be 25%. The situation where only the two large shareholders partisan of the resolution vote in addition to the (negligeable) unconditionals is an equilibrium: if either one of the two shareholders does not vote, the minimum quorum does not hold and the resolution is not adopted; no opponent to the resolution should vote since he cannot (alone) change the outcome; and no other shareholder for should vote since he is satisfied with the outcome. However, the case where no partisan (for or against) votes and the resolution does not pass is also an equilibrium: holding less than the minimum quorum each, no shareholder for can change the outcome by voting, and the dominant shareholder who is a partisan of against is satisfied with the result. Interestingly, a low minimum quorum does not always favour the dominant shareholder when he supports against. For any minimum quorum below 20%, no equilibrium remains in the previous example so the outcome of the assembly becomes random.

Our results suggest that the choice of the minimum quorum is quite strategic. In some countries (the United States), the quorum is defined in the corporate charter and may be modified in assemblies. In other countries it is fixed by law.[1] Respondents to consultation of the European Commission in 2007 suggested that quorums should be reduced or abolished. According to our model, this potentially means reinforcing the power of the largest blockholder by precluding coalitions of smaller shareholders.

Also, the presence of unconditionals does not amount to lowering the minimum quorum. Actually with no minimum quorum, the largest partisan's preferred alternative is selected in some equilibria when the group of unconditional shareholders *opposed* to him is sufficiently larger than the group of unconditionals on his side; a relatively large unconditional electorate destroys the incentive to vote for partisans on the same side. The asymmetry between the two groups of unconditionals matters. Mandatory voting disclosure laws for institutionals which discourage them from staying away, as they must explain why to investors, may in this respect have unexpected effects on assemblies' results.

The paper is organized as follows. The model is presented in section 2. Preliminary results and equilibrium conditions are detailed in section 3. Equilibria are characterized and discussed in section 4. Section 5 concludes.

# 2 The model

The main role of annual meetings is to adopt or reject resolutions on the agenda which are usually sponsored by the board of directors and sometimes

[1] Actually in the United States, under NASDAQ rules, companies are required to have a minimum quorum of 33 1/3% of the holders of common stock for shareholder meetings. In Europe, the minimum quorum varies across countries (no minimum quorum in Germany, 20% of capital in France for AGM, 2 shareholders regardless of the level of their aggregate share ownership in the UK...) and may be different for AGM and EGM.

by one or several shareholders. We analyze the result of corporate assemblies in a context where unanimity among shareholders does not hold and in the presence of large voting blocks besides smaller ones.

More precisely, we assume there exist two types of shareholders:

The first group, the *unconditionals*, always vote whatever their anticipations about the outcome. Some of them (representing altogether $\alpha_U^F$ of voting rights) systematically vote for, while others (representing altogether $\alpha_U^A$ of voting rights) systematically vote against the proposed resolution. Indeed, typically some shareholders are close to the management for different reasons and support their propositions. For example, mutual funds often support management of client firms in which they have a stake.[2] The contrary (systematic opposition) may also exist; obviously, when resolutions are sponsored by active shareholders and directed against the management,[3] those close to management vote systematically against. Unconditionals may also account for mandatory voting[4] and the presence of ethical voters who consider voting as a duty.

Other shareholders are either *partisans* of for ($F$) or against ($A$) who vote strategically. Maug and Rydqvist (2009) assume that the common objective of shareholders is to adopt resolutions increasing the value of the company. Indeed, even when they care only about value, shareholders may disagree on the relevance of resolutions in several instances. When a stockholder holds simultaneously shares in two business related companies, he may be favorable to a value decreasing resolution in a company that has a positive impact on the value on his holdings in the other while other shareholders disagree with the resolution.[5] Other types of private bene-

[2] For example Davis and Kim (2007) document "a positive relation between business ties and the propensity to vote with management".

[3] In France, for example, shareholders representing at least 5% of voting rights may propose an entire new board of directors.

[4] Introducing compulsory voting for some shareholders is sometines mentioned (see the Wall Street Journal, 9 September 2009). It formally exists in politics but takes the form of having non voters pay a fine. One could argue that not voting for institutionals imposes a cost on them since they must explain why to their clients.

[5] Matvos and Ostrovsky (2006) show for example that "in mergers with negative ac-

fits (ethical considerations,...) may explain opposite views. Partisan shareholders incur a small voting cost and vote strategically, contrary to the unconditionals. We represent a partisan supporting $K$ ($K = F, A$) by his share of voting rights denoted $\alpha_i^K$. There are $N_F$ partisans of $F$ belonging to $\mathcal{P}_F=\{\alpha_1^F, \alpha_2^F, \alpha_3^F, ..., \alpha_{N_F}^F\}$ and $N_A$ partisans of $A$ belonging to $\mathcal{P}_A=\{\alpha_1^A, \alpha_2^A, \alpha_3^A, ..., \alpha_{N_A}^A\}$; $\alpha_1^K \geq \alpha_2^K \geq \alpha_3^K \geq ... \geq \alpha_{N_K}^K > 0$.

In practice, shareholders have four possibilities: if they vote, they may approve (vote for), disapprove (vote against) or abstain; they may also decide not to participate to the vote. We exclude the possibility of voting and abstaining for partisans since, with a small voting cost, abstaining may not change the outcome and is always strictly dominated by either voting according to one's preference or not voting.[6] We therefore consider only three possible strategies for shareholders: vote for ($F$) the proposed resolution, or against ($A$) the resolution, or not vote (except for the unconditional supporters who always vote for or against).

Finally, two conditions must be verified for a resolution to be adopted. First, a minimum number of shareholders must be present or represented (quorum rule) in the assembly. We call $Q$ the minimum proportion of shareholders that must vote for a resolution to be adopted. When the minimum quorum $Q$ is not reached, the resolution cannot pass. Second, the resolution must obtain a minimum of favourable votes (majority rule). We assume that a resolution cannot pass unless $F$ represents strictly more than 50% of the votes. If either the quorum rule or the majority rule do not hold, the resolution does not pass and $A$ prevails. Thus the result of an insufficient quorum is considered to be equivalent to a valid vote against, and may be interpreted as the "Status Quo".

We model the assembly as a simultaneous game in which each partisan

quirer announcement returns, cross-owners are significantly more likely to vote for the merger". Charléty, Fagart and Souam (2009) endogenize such private benefits in the case of horizontal partial acquisitions.

[6] The same conclusion would apply to indifferent shareholders whom, for this reason, we do not consider.

decides to vote or not based on his anticipations about the other partisans' strategies. All shareholders are assumed to know all others' voting shares and preferences (information is perfect). We look for the pure strategy Nash equilibria of this game. Throughout the paper, equilibrium $\mathcal{F}$ (resp $\mathcal{A}$) refers to a pure strategy Nash equilibrium in which $F$ (resp. $A$) passes. "Inexistence" refers to a situation where no pure strategy equilibrium exists.[7]

We investigate the role played by the presence of unconditionals and the quorum rule on the strategies adopted by partisans and the result of the assembly under these hypotheses.

# 3 Partisans' equilibrium strategies and equilibrium conditions

In this section we present in a first step some preliminary results regarding the partisans' equilibrium strategies. These considerations enable us to restrict in a second step the examination of the Nash equilibrium conditions to the only admissible strategies.

## Partisans' equilibrium strategies

By assumption, the group of unconditionals $\alpha_U^A$ (resp. $\alpha_U^F$) always votes against (resp. for).

Since voting is slightly costly, the best outcome for any partisan in favour of the resolution $\alpha_i^F \in \mathcal{P}^F$ (resp. any partisan against $\alpha_i^A \in \mathcal{P}^A$) is $F$ (resp. $A$) without participation, the second best is $F$ (resp. $A$) with participation, which is better than $A$ (resp. $F$) without participation, and the worst is outcome $A$ (resp. $F$) with participation.

[7] When no pure strategy Nash equilibrium exists, the outcome of the meeting cannot be predicted. Since the game is finite, we know (Nash) that a mixed strategy equilibrium always exists. The outcome of the assembly is random in that case.

Consequently, two properties hold in equilibrium:

**(P1) No partisan of $A$ (resp. $F$) votes in equilibrium $\mathcal{F}$ (resp. $\mathcal{A}$).**

Indeed suppose a non-voting partisan of against $\alpha_i^A$ (resp. $\alpha_i^F$ ) expects $F$ (resp. $A$) to result from the assembly. If, given the others' actions, he can rock the result, his best strategy is to vote against (resp. for), which means that the initial set of shareholders' actions was not an equilibrium. Therefore in equilibrium $\mathcal{F}$ (resp. $\mathcal{A}$) no partisan of $A$ (resp. $F$) is able to rock the result and since voting is costly does not vote.

**(P2) In equilibrium $\mathcal{F}$ (resp. $\mathcal{A}$), a partisan for $\alpha_i^F \in P^F$ (resp. a partisan against $\alpha_i^A \in P^A$) participates to the vote if and only if he is pivotal, i.e. his vote is necessary to obtain his preferred outcome.**

Effectively suppose a voting partisan of for $\alpha_i^F$ (resp. $\alpha_i^A$ ) expects the assembly to decide $F$ (resp. $A$). If, given the others' actions, the result of the assembly remains $F$ (resp. $A$) if he does not vote, his best strategy is to abstain from voting since voting is costly; this means that the initial set of shareholders' actions was not an equilibrium. Therefore in equilibrium $\mathcal{F}$ (resp. $\mathcal{A}$) no partisan participates when his preferred outcome $F$ (resp. $A$) emerges without his vote. In other words, with a small voting cost, a partisan votes in equilibrium only if he anticipates that his vote is useful (if he is pivotal): he does not vote when his preferred outcome obtains without his vote, nor does he vote when his vote cannnot change the outcome he dislikes.

## Equilibrium conditions

We successively examine under which conditions the (Nash equilibrium) result of the assembly is to adopt (equilibrium for or $\mathcal{F}$) or reject (equilibrium against or $\mathcal{A}$) the resolution.

**Equilibrium $\mathcal{F}$**

Let $V^F \subset \mathcal{P}^F$ represent the set of partisans who vote for in equilibria $\mathcal{F}$.

For an equilibrium $\mathcal{F}$ to exist, four conditions must hold (from (P1) no partisan of $A$ ever votes in equilibrium $\mathcal{F}$ therefore $\alpha_U^A$ represents all votes against):

(F1) Shareholders in favor of $F$ must represent a (strict) majority (**simple majority rule**):

$$\sum_{V^F} \alpha_i^F + \alpha_U^F > \alpha_U^A$$

(F2) The minimum quorum must be reached (**quorum rule**):

$$\sum_{V^F} \alpha_i^F + \alpha_U^F + \alpha_U^A \geq Q$$

(F3) The winning coalition may not be overturned by any partisan of $A$, therefore by the largest $\alpha_1^A$ (**non contestability condition** for a Nash Equilibrium):

$$\sum_{V^F} \alpha_i^F + \alpha_U^F > \alpha_U^A + \alpha_1^A$$

(F4) All voting partisans $\alpha_j^F$ of the coalition must be pivotal (**pivotal voting partisan condition** for a Nash Equilibrium):

$$\text{for any } \alpha_j^F, \text{ either (a) } \sum_{V^F} \alpha_i^F + \alpha_U^F - \alpha_j^F \leq \alpha_U^A$$

$$\text{or (b)} \sum_{V^F} \alpha_i^F + \alpha_U^F + \alpha_U^A - \alpha_j^F < Q$$

Against obtains the majority (ties are in favor of the Status Quo which is equivalent to against) or the minimum quorum is not reached (in which case the Status Quo prevails) if any $\alpha_j^F \in V^F$ does not participate. Condition (F4) can be rewritten as:

$$\sum_{V^F} \alpha_i^F + \alpha_U^F - \underset{V^F}{Min} \alpha_j^F \leq Max(\alpha_U^A, Q^- - \alpha_U^A)$$

where $Q^-$ represents the minimum quorum minus one vote.

Since (F3) implies (F1), only the three conditions (F2) to (F4) remain for the existence of an equilibrium $\mathcal{F}$. They are summarized in (F5):

$$Max(\alpha_U^A+\alpha_1^A, Q^- - \alpha_U^A) - \underset{V^F}{Min}\,\alpha_j^F < \sum_{V^F} \alpha_i^F + \alpha_U^F - \underset{V^F}{Min}\,\alpha_j^F \leq Max(\alpha_U^A, Q^- - \alpha_U^A)$$

**Remarks and additional assumption:**

**Remark 1** *First note that (F2) can never hold if* $\sum_{\mathcal{P}^F}\alpha_i^F + \alpha_U^F + \alpha_U^A < Q$, *similarly (F3) can never hold if* $\sum_{\mathcal{P}^F}\alpha_i^F + \alpha_U^F \leq \alpha_1^A + \alpha_U^A$.

We therefore assume that $\sum_{\mathcal{P}^F}\alpha_i^F + \alpha_U^F > Max(\alpha_1^F + \alpha_U^F, Q^-)$ for non triviality.

**Remark 2** *If unconditional shareholders against the resolution represent more than 50% of the minimum quorum, any equilibrium voting coalition for must gather only partisans larger than the largest partisan against. This is not necessarily the case when unconditionals against represent strictly less than 50% of the minimum quorum.*

Indeed, when $2\alpha_U^A \geq Q$, the pivotal partisan condition amounts to (F4a) which together with (F3) imply:

$$\underset{V^F}{Min}(\alpha_i^F) > \alpha_1^A$$

**Equilibrium $\mathcal{A}$**

$\mathcal{A}$ prevails whenever voters against obtain a majority (first type) or the minimum quorum is not reached (second type).

**First type** Let $V^A \subset \mathcal{P}^A$ represent the set of partisans who vote against in equilibrium $\mathcal{A}$ in the first case.

For an equilibrium $\mathcal{A}$ to exist, four conditions must hold (from (P1) no partisan of $F$ ever votes in equilibrium $\mathcal{A}$ therefore $\alpha_U^F$ represents all votes for):

(A1) Shareholders in favor of $A$ must represent a majority (**simple majority rule**):

$$\sum_{V^A} \alpha_i^A + \alpha_U^A \geq \alpha_U^F$$

(A2) The winning coalition may not be overturned by any partisan of $F$, therefore by the largest $\alpha_1^F$ (**non contestability condition** for a Nash Equilibrium):

$$\sum_{V^A} \alpha_i^A + \alpha_U^A \geq \alpha_U^F + \alpha_1^F$$

(A3) All voting partisans $\alpha_j^A$ of the coalition must be pivotal (**pivotal voting partisan condition** for a Nash Equilibrium):

$$\text{(a)} \sum_{V^A} \alpha_i^A + \alpha_U^A - \underset{V^A}{Min}(\alpha_j^A) < \alpha_U^F$$

$$\text{and (b)} \sum_{V^A} \alpha_i^A + \alpha_U^F + \alpha_U^A - \underset{V^A}{Max}(\alpha_j^A) \geq Q$$

If any $\alpha_j^A \in V^A$ does not participate, for obtains a strict majority (a) (ties are in favor of the Status Quo which is equivalent to against) and the minimum quorum is still reached (b).[8]

Since (A2) implies (A1), only the three conditions (A2), (A3a) and (A3b) remain for the existence of an equilibrium $\mathcal{A}$ in which some partisans of A

[8] Condition (A3b) guarantees that the same result (against when the minimum quorum is not reached) cannot be obtained without the vote of any, therefore the largest shareholder in the voting coalition (P2).

vote. These conditions together are summarized in (A4):

$$Max\left\{\alpha_U^F+\alpha_1^F, Q-\alpha_U^F+\underset{V^A}{Max}(\alpha_j^A)\right\}-\underset{V^A}{Min}(\alpha_j^A)\leq\sum_{V^A}\alpha_i^A+\alpha_U^A-\underset{V^A}{Min}(\alpha_j^A)<\alpha_U^F$$

**Remarks and additional assumptions**

**Remark 3** *(A2) can never hold when* $\sum\limits_{\mathcal{P}^A}\alpha_i^A+\alpha_U^A<\alpha_1^F+\alpha_U^F$ *since any possible coalition of shareholders against could then be challenged by the largest partisan for.*

We therefore assume that $\sum\limits_{\mathcal{P}^A}\alpha_i^A+\alpha_U^A\geq\alpha_1^F+\alpha_U^F$ for non triviality.

**Remark 4** *If an equilibrium voting coalition against exists, it gathers only partisans larger than the largest partisan for.*

This follows from (A2) and (A3a) which together imply:

$$\underset{V^A}{Min}(\alpha_i^A)>\alpha_1^F$$

**Remark 5** *A voting coalition of partisans against may exist only if the group of unconditionals for represents at least 50% of the minimum quorum and is larger than the group of unconditionals against.*

As indeed (A3a) and (A3b) imply:

$$\begin{aligned}\alpha_U^F &> \alpha_U^A\\ 2\alpha_U^F &> Q\end{aligned}$$

**Remark 6** *The total of votes non marginally exceeds the minimum quorum in equilibrium, the difference represents at least the votes of the largest partisan.*

Since from (A3b) and remark 4:

$$\sum_{V^A}\alpha_i^A+\alpha_U^F+\alpha_U^A-Q\geq\underset{V^A}{Max}(\alpha_j^A)>\alpha_1^F$$

**Second type** In the second case where equilibrium $\mathcal{A}$ prevails because the minimum quorum is not reached, from (P1) and (P2), no partisan votes. The necessary and sufficient conditions are therefore:

(A'1) The minimum quorum is not reached:

$$\alpha_U^F + \alpha_U^A < Q$$

(A'2) No partisan of $F$ (therefore the largest $\alpha_1^F$) may rock the result, (**non contestability condition** for a Nash Equilibrium):

$$\begin{aligned} \alpha_U^F + \alpha_U^A + \alpha_1^F &< Q \\ \text{or } \alpha_U^F + \alpha_1^F &\leq \alpha_U^A \\ \text{which are summarized as } \alpha_U^F + \alpha_U^A + \alpha_1^F &\leq Max(Q^-, 2\alpha_U^A) \end{aligned}$$

# 4 Equilibrium analysis

The objective of this section is to characterize the equilibria of the voting game. In particular, we predict whether (at least) an equilibrium for, an equilibrium against, or both types co-exist for different shareholding structures. The presence of unconditionals and the minimum quorum turn out to play a crucial role and two cases emerge naturally from the analysis. In the first case, $Q$ is relatively low so that the unconditionals alone represent more than the minimum quorum. In the second case, the quorum rule cannot be met unless some partisans vote.

## Large unconditionals ($\alpha_U^F + \alpha_U^A \geq Q$)

In this sub-section, we consider the case where the unconditionals have enough voting rights to reach the minimum quorum; either $Q$ is small, or the unconditionals represent a large share of total voting rights.

Let $\Delta^A$ represent the set of voting coalitions of partisans against that cannot be challenged, i.e. $\Delta^A = \{V^A \subset \mathcal{P}^A \mid \sum_{V^A} \alpha_i^A + \alpha_U^A \geq \alpha_1^F + \alpha_U^F\}$ and define:

$$\begin{aligned} V_m^A &= \underset{V^A \in \Delta^A}{Arg\ Min} \left\{ \sum_{V^A} \alpha_i^A - \underset{V^A}{Min}(\alpha_j^A) \right\} \\ \text{and } \alpha_m^A &= \sum_{V_m^A} \alpha_i^A - \underset{V_m^A}{Min}(\alpha_j^A) \end{aligned}$$

$\alpha_m^A$ represents the minimum, among all coalitions against that cannot be challenged, of total partisan votes against less the share of the smallest partisan of the coalition.

$\alpha_m^F$ is defined symmetrically: $\Delta^F$ represents the set of coalitions of partisans for that cannot be challenged, i.e. $\Delta^F = \{V^F \subset \mathcal{P}^F \mid \sum_{V^F} \alpha_i^F + \alpha_U^F > \alpha_1^A + \alpha_U^A\}$ and

$$\begin{aligned} V_m^F &= \underset{V^F \in \Delta^F}{Arg\ Min} \left\{ \sum_{V^F} \alpha_i^F - \underset{VF}{Min}(\alpha_j^F) \right\} \\ \text{and } \alpha_m^F &= \sum_{V_m^F} \alpha_i^F - \underset{V_m^F}{Min}(\alpha_j^F) \end{aligned}$$

**Proposition 1** *The equilibria of the game are given by :*

(1) When $\alpha_U^A < \alpha_U^F$, a unique equilibrium $\mathcal{F}$ exists where no partisan votes if and only if $\alpha_1^A < \alpha_U^F - \alpha_U^A$.

There exists at least one equilibrium $\mathcal{A}$ where some partisan(s) against vote(s) if and only if $\alpha_m^A < \alpha_U^F - \alpha_U^A$.

(2) Symmetrically when $\alpha_U^A \geq \alpha_U^F$, a unique equilibrium $\mathcal{A}$ exists where no partisan votes if and only if $\alpha_1^F \leq \alpha_U^A - \alpha_U^F$.

There exists at least one equilibrium $\mathcal{F}$ if and only if $\alpha_m^F \leq \alpha_U^A - \alpha_U^F$.

**Proof.** See appendix. ■

Therefore when the largest unconditional group (for example $\alpha_U^F$) holds a stake larger than the sum of the shares of the other unconditional and of the largest opponent (for example, $\alpha_U^F > \alpha_1^A + \alpha_U^A$), there always exists an equilibrium where this largest unconditional "controls" the assembly (under the simple majority rule) since no opposed partisan (say supporting $A$) may change the result and partisans of the same side (say supporting $F$) obtain their preferred outcome without voting.

However, even in these situations where the dominant unconditional group cannot be challenged, there may also exist equilibria where coalitions of the opposed outcome (say $A$) win. These coalitions have to be large enough to prevent opposition from the largest partisan of the opposite side (they belong to $\Delta^A$); and they must be composed of large enough partisans who otherwise might be tempted not to vote: the smallest partisan of the voting coalition (say against) must be larger than the largest opposed partisan (say $\alpha_1^F$) as stated in the previous section (remark 4).[9] The following example illustrates this case.

**Example 1** *Co-existence of a unique equilibrium $\mathcal{F}$ and multiple coalitions supporting equilibrium $\mathcal{A}$* : $Q = 25\%, \alpha_U^F = 15\%, \alpha_U^A = 10\%, \alpha_1^F = 1\%, \mathcal{P}^A =$

$(4\%, 3\%, 2\%, 2\%, 1\%, ...)$.

Equilibrium $\mathcal{F}$ where only the unconditionals vote exists, and there also exist equilibria $\mathcal{A}$: $\Delta^A$ is the set of coalitions against representing at least 6% of voting rights, $V_m^A \in \{\{4\%, 3\%\}, \{4\%, 2\%\}\}$, $\alpha_m^A = 4\% < 15\% - 10\%$, $V^A = \{3\%, 2\%, 2\%\}$ is another equilibrium voting coalition.

Interestingly, when the largest group of unconditionals (for example $\alpha_U^F$) represents less votes than the coalition of the opposed largest partisan and unconditionals (for example, $\alpha_U^F \leq \alpha_1^A + \alpha_U^A$), the result of the assembly is always the preferred outcome of the *smallest* unconditional (say for) when an equilibrium exists.

[9] Symmetrically, based on remark 2, winning coalitions for necessarily gather partisans larger than $\alpha_1^A$ when $\alpha_U^A \geq \alpha_U^F$ since condition $2\alpha_U^A > Q$ holds with large unconditionals ($\alpha_U^A + \alpha_U^F \geq Q$).

Suppose now $\alpha_U^F > \alpha_U^A$ but $\alpha_U^F \leq \alpha_1^A + \alpha_U^A$ so that the largest unconditional for may not control the assembly. Start from a hypothetical situation where only the unconditionals vote. This is clearly not an equilibrium since $\alpha_1^A$ has an incentive to vote and rock the result as $\alpha_U^F \leq \alpha_1^A + \alpha_U^A$. If $\alpha_1^A + \alpha_U^A \geq \alpha_1^F + \alpha_U^F$, this new situation where only the largest partisan of $A$ and the unconditionals vote against is an equilibrium. No partisan for considers voting since he alone cannot change the outcome of the assembly. Nor does any other partisan against since his favourite result already obtains. The same reasoning applies to the symmetric case (equilibrium for). This could be interpreted as a *generalized dominant shareholder rule* where the "dominant shareholder" is the largest coalition of the group of unconditionals and largest partisan on the same side. This equilibrium may not be unique. For instance, if $\alpha_2^A + \alpha_U^A \geq \alpha_1^F + \alpha_U^F$, the game has a second equilibrium in which only $\alpha_2^A$ and the unconditionals vote. A coalition of partisans of $A$, say $V^A \subset \mathcal{P}^A$, might also vote in equilibrium if it gathers enough voting rights to overcome the dominant shareholder for (i.e $\sum_{V^A} \alpha_i^A + \alpha_U^A \geq \alpha_1^F + \alpha_U^F$ ) and any member of the coalition is pivotal (i.e $\sum_{V^A} \alpha_i^A - \alpha_j^A + \alpha_U^A < \alpha_U^F$ for all $\alpha_j^A \in V^A$). Multiple equilibria are therefore feasible, however the result of the assembly is identical (the generalized dominant shareholder rule holds). Again an equilibrium voting coalition necessarily gathers voters whose voting rights are high enough (that is $\alpha_j^A > \alpha_1^F$ for all $\alpha_j^A \in V^A$ as showed in the previous section).

There exists no equilibrium if the largest group of unconditionals is "too small" (for example $\alpha_U^F > \alpha_U^A$ but $\alpha_U^F \leq \alpha_U^A + \alpha_1^A$) as well as all partisans opposed to this group ($\alpha_i^A < \alpha_1^F$ for all $i$). In the case where the game has no pure strategy equilibrium the outcome of the assembly is random from an ex ante point of view.

It is worth noting that partisans (possibly) vote only when they are opposed to the largest group of unconditionals. Indeed larger unconditionals unambiguously destroy the incentive for shareholders on the same side to vote. Suppose the group of unconditionals for dominates ($\alpha_U^F > \alpha_U^A$) and all shareholders anticipate that only the unconditionals vote. Partisans of $F$

then find it useless to vote. Only partisans of $A$, opposed here to the largest group of unconditionals for have an incentive to vote if they can rock the result. Equilibrium coalitions are always formed against the largest group of unconditionals. Overall, holding the largest share of voting rights does automatically ensure a shareholder that the decision of the assembly corresponds to his preference.

The following example illustrates these points:

**Example 2** *Suppose* $Q = 25\%$ *and:*

$$\begin{array}{rcl} \alpha_1^F & = & 5\% \text{ and } \alpha_1^A = 10\% \\ \alpha_i^A & < & 5\% \text{ for all } i > 1 \\ \text{Case 1:} & & \alpha_U^F = 15\% \text{ and } \alpha_U^A = 11\% \\ \text{Case 2:} & & \alpha_U^F = 11\% \text{ and } \alpha_U^A = 15\% \end{array}$$

In both cases, note that $\alpha_1^A + \alpha_U^A \geq \alpha_1^F + \alpha_U^F$. There exists a unique equilibrium $\mathcal{A}$ where $\alpha_1^A$ is the only partisan who votes in *Case 1* despite the fact that the largest shareholder $\alpha_U^F$ is for the resolution. The outcome of the assembly is random (no equilibrium) in *Case 2*.

### Coalitions, the role of unconditional shareholders and shareholding structure

With large unconditionals, the minimum quorum constraint is not binding, and Proposition 1 holds for $Q = 0$. The presence of the two groups of unconditionals and specifically the asymmetry between them explain the emergence of equilibrium coalitions in the assembly. More precisely, the relative weight of unconditionals for and against must be high enough.

**Corollary 1** *When* $\alpha_U^F = \alpha_U^A$, *there exists an equilibrium for if and only if* $\alpha_1^F > \alpha_1^A$, *no equilibrium exists when* $\alpha_1^F \leq \alpha_1^A$.

**Proof.** According to Proposition 1 (2), there exists an equilibrium $\mathcal{F}$ if and only if

$$\alpha_m^F = \sum_{V_m^F} \alpha_i^F - \underset{V_m^F}{Min}(\alpha_j^F) \leq \alpha_U^A - \alpha_U^F = 0.$$

therefore at most one partisan shareholder in favour of the resolution votes in equilibrium. Since this voting partisan for $\alpha_i^F$cannot be contested, $\alpha_i^F + \alpha_U^F > \alpha_1^A + \alpha_U^A$ or $\alpha_i^F > \alpha_1^A$ here which implies $\alpha_1^F > \alpha_1^A$. Moreover, the condition above is verified for $\alpha_1^F > \alpha_1^A$. Thus there exists an $\mathcal{F}$ equilibrium iff $\alpha_1^F > \alpha_1^A$. The unique possible $\mathcal{A}$ equilibrium is degenerate and exists when there are no partisans for which we do not consider. ■

This is equivalent to the result of Ritzberger (2005) who analyzes a voting game with only partisans (all shareholders are strategic) and no minimum quorum. Only one partisan for whose share is large enough and therefore cannot be successfully contested participates to the vote. Otherwise no equilibrium exists.

Consider the more interesting case where the largest group of unconditionals is for the proposed resolution or $\alpha_U^F > \alpha_U^A$ (the case $\alpha_U^A > \alpha_U^F$ is symmetric). Suppose there exists an equilibrium coalition against $V_m^A$ with more than one member that includes the largest partisan $\alpha_1^A$. Therefore (from Proposition 1) $\alpha_m^A = \sum_{V_m^A} \alpha_i^A - \underset{V_m^F}{Min}(\alpha_j^A) \leq \alpha_U^F - \alpha_U^A$, moreover$\alpha_1^A < \alpha_m^A$ since $\alpha_1^A$ is in the coalition. As a consequence $\alpha_1^A < \alpha_U^F - \alpha_U^A$ which implies that an equilibrium $\mathcal{F}$ exists simultaneously.

## Small Unconditionals ($Q > \alpha_U^A + \alpha_U^F$)

We now consider the case where the unconditionals alone do not have enough voting rights to reach the minimum quorum. Recall $A$ is selected when the minimum quorum is not reached ($A$ is the status quo).

The following proposition gives the conditions under which there exist equilibria $\mathcal{A}$ and $\mathcal{F}$.

Consider all coalitions of partisans of A that cannot be challenged and for which the minimum quorum is still reached whenever a member leaves. $\Omega^A = \{V^A \subset \mathcal{P}^A \mid \sum_{V^A} \alpha_i^A + \alpha_U^F + \alpha_U^A - Max(\alpha_j^A) \geq Q$ and $\sum_{V^A} \alpha_i^A + \alpha_U^A \geq \alpha_U^F + \alpha_1^F\}$. Let $V_M^A = Arg \underset{V^A \in \Omega^A}{Min} \left\{ \sum_{V^A} \alpha_i^A - \underset{V^A}{Min}(\alpha_j^A) \right\}$ and $\alpha_M^A = \sum_{V_M^A} \alpha_i^A - \underset{V_M^A}{Min}(\alpha_j^A)$.

**Proposition 2** *The equilibria of the game are given by:*

*(1) An equilibrium $\mathcal{F}$ exists if and only if $Q > Q_m^F = \alpha_m^F + \alpha_U^F + \alpha_U^A$. Some partisans of F necessarily vote in equilibrium.*

*(2) A non voting equilibrium $\mathcal{A}$ exists if and only if $Q > Q^A = \alpha_1^F + \alpha_U^F + \alpha_U^A$.*

*Case (i): $\alpha_U^A < \alpha_U^F$.*

*When $Q < 2\alpha_U^F$, a voting equilibrium exists if and only if $\alpha_M^A < \alpha_U^F$. When $Q \geq 2\alpha_U^F$, there exists no voting equilibrium coalition against.*

*Case (ii): $\alpha_U^A \geq \alpha_U^F$.*

*There exists no equilibrium $\mathcal{A}$ against where partisans vote.*

• Equilibrium $\mathcal{A}$

Let us first consider the non voting equilibrium $\mathcal{A}$ where partisans do not vote. $A$ passes since the status quo prevails when the minimum quorum in not reached. This is the case here since unconditionals are small. Suppose all partisans anticipate that no partisan votes. A partisan of $A$ clearly should not vote since his preferred outcome prevails. And if $Q > Q^A = \alpha_1^F + \alpha_U^F + \alpha_U^A$, no partisan in $\mathcal{P}^F$ has alone enough voting rights to satisfy the quorum rule, so his vote can never change the decision. Thus the situation where no partisan votes is stable.

Interestingly, a high minimum quorum relative to the total voting rights of unconditionals discourages voting and favours the rejection of resolutions. Non voting equilibrium $\mathcal{A}$ no longer exists whenever $(\alpha_U^F + \alpha_U^A) \geq Q - \alpha_1^F$. However, it is interesting to note that for intermediary minimum quorum voting equilibrium against may exist as the following example shows.

**Example 3** *$Q = 25\%$, $\alpha_U^F = 15\%$, $\alpha_U^A = 7\%$, $\alpha_1^F = 1\%$, $\alpha_1^A = 4\%$, $\alpha_2^A = 3\%$, $\alpha_3^A = 2\%$, $\alpha_4^A = 1\%$,...*

The coalition $\left\{\alpha_1^A, \alpha_2^A, \alpha_3^A\right\}$ does support a voting equilibrium $\mathcal{A}$.

• Equilibria $\mathcal{F}$

In equilibrium, just enough partisans of the resolution vote so that the resolution is adopted. The existence of at least an equilibrium where the resolution passes depends on the shareholding structure of $\mathcal{P}^F$ and the minimum quorum. When the minimum quorum is sufficiently high ($Q > Max(Q^F, 2\alpha_U^A)$), there always exists a voting equilibrium $\mathcal{F}$. When the minimum quorum is intermediate ($\alpha_U^A + \alpha_U^F < Q < 2\alpha_U^A$), there exists a voting equilibrium $\mathcal{F}$ if and only if $Q^F \leq 2\alpha_U^A$.

**Example 4** *Suppose the minimum quorum is* $Q = 25\%$ *and consider the following shareholding structure:*

$$\begin{aligned} \alpha_U^A &= \alpha_U^F = 6\% \\ \mathcal{P}^A &= \{15\%, ...\} \\ \mathcal{P}^F &= \{10\%, 8\%, .....\} \end{aligned}$$

From Proposition 2, a non voting equilibrium $A$ exists since $Q^A = 22\% < Q$ and there also exists a voting equilibrium $F$ since $Q^F \leq 16\% < Q$. The resolution may be rejected (equilibrium $A$) or adopted (equilibrium $F$). The dominant shareholder who is against may thus be countered.

### Coalitions and the role of the quorum rule

With a high minimum quorum, the presence of unconditionals affects the results only marginally and it is the presence of the voting rule which really matters. Actually, when there are no unconditionals, the results are similar to those stated in Proposition 2; the only difference is that no equilibrium coalition of voters against may exist. Obviously, with no unconditional for, there is no more incentive to vote against a resolution since the same result (against) obtains at no cost without voting.

**Corollary 2** *Suppose there are no unconditional shareholders but the minimum quorum rule applies. In that case,*

(1) Equilibria $\mathcal{F}$ exist if and only if $Q > \alpha_m^F$,

(2) There exists a non voting equilibrium $\mathcal{A}$ if and only if $Q > \alpha_1^F$.

Therefore the minimum quorum requirement by itself increases the incentive to form voting coalitions in favor of the resolution, which never exist in equilibrium without a minimum quorum. Moreover, with a high enough minimum quorum the situation where no partisan votes becomes an equilibrium.These two results are in contradiction with Ritzberger (2005).

**Example 5** *Consider the following shareholding structure:*

$$\begin{aligned} \mathcal{P}^A &= \{14\%, ...\} \\ \mathcal{P}^F &= \{9\%, 8\%, 7\%, 4\%, 3\%, .....\} \end{aligned}$$

With no quorum requirement, no equilibrium exists. With a minimum quorum $Q = 8.5\%$, there exists a unique equilibrium where the coalition $V^F = \{8\%, 7\%\}$ votes. With a minimum quorum $Q = 25\%$, both equilibria exist: the coalitions $\{9\%, 8\%, 7\%, 4\%\}$ and $\{9\%, 8\%, 7\%, 3\%\}$ support a voting equilibrium $\mathcal{F}$, and there also exists an equilibrium $\mathcal{A}$ where no shareholder votes.

# 5 Conclusion

In this paper, we investigate the respective roles of the quorum rule and the presence of unconditional shareholders (or equivalently mandatory voting for some shareholders) in annual meetings. In a framework where information is perfect, shareholders differ in size and opinion, and voting is costly, we find that both the quorum rule and the presence of unconditional shareholders increase the incentive to vote for other shareholders who vote strategically. We show the existence of equilibria where coalitions of shareholders for (resp. against) the resolution win even though the largest shareholder is against (resp. for). For some shareholding structures, equilibria leading to a different outcome (adoption or rejection of a resolution) may coexist. Our results therefore differ substantively from Ritzberger's (2005) according to which a unique equilibrium exists if and only if largest (or dominant) shareholder is for a resolution in which case only one shareholder votes.

The concentration of the shareholding structure plays a key role in our analysis. In particular, equilibrium coalitions against a resolution necessarily involve sufficiently large shareholders, namely at least larger than the largest strategic shareholder in favor of the resolution. We therefore predict that a resolution may only be rejected when opponents hold large stakes. This is not always the case for equilibria in favor of a resolution. Indeed, when the minimum quorum is sufficiently high, a coalition gathering only small shareholders in favor of the resolution may vote in equilibrium. Widespread ownership is consistent with the adoption of resolutions but not with rejection. Voting equilibria in favor or against a resolution are thus fundamentally different. From our model, it should be easier to oppose the management by sponsoring and vote a resolution directed against him rather than vote against a management sponsored resolution. This somewhat highlights the importance of federating small shareholders in associations, through proxy voting, as an effective corporate control mechanism.

Even though the presence of unconditional shareholders and a minimum quorum rule both facilitate the existence of voting equilibria, these two parameters do not play the same role. The existence of unconditional shareholders is not equivalent to a decrease of the minimum quorum rule. Rather than the total -for and against together- share held by unconditional shareholders, it is the difference between the respective shares the unconditional against and for the resolution that plays a key role for the existence and the type of the equilibria. Finally, it is worth noting that the introduction of voting constraints such as a quorum rule actually enriches the results in terms of number and nature of equilibria. It would be interesting in a future research to test empirically our theoretical findings, in particular whether voting equilibria against a resolution are more likely in the presence of large shareholders against a resolution.

# 6 Appendix

## Proof of proposition 1 (large unconditionals)

Suppose $\alpha_U^F + \alpha_U^A \geq Q$, in other words unconditionals are large enough to reach the minimum quorum.

**Case 1 -** $\alpha_U^F > \alpha_U^A$

***Existence of equilibrium $\mathcal{F}$.*** (F1) and (F2) always hold in case 1. Since the majority in favor of $F$ is obtained without any partisan's participation, no partisan of $F$ votes and (F4a) is never verified (condition (F4b) is irrelevant here). Thus the existence of an equilibrium $\mathcal{F}$ is guaranteed if and only if $\alpha_U^F > \alpha_U^A + \alpha_1^A$ (F3). Note that only the unconditionals vote in equilibrium $\mathcal{F}$ when it exists.

***Existence of equilibrium $\mathcal{A}$.*** If it exists, it can only be an equilibrium where a subset of partisans of $A$ vote since the minimum quorum is reached. Also (A3b) always holds in that case. The necessary and sufficient conditions for an equilibrium simplify to:

$$\alpha_U^F + \alpha_1^F - \underset{V^A}{Min}(\alpha_j^A) \leq \sum_{V^A} \alpha_i^A + \alpha_U^A - \underset{V^A}{Min}(\alpha_j^A) < \alpha_U^F$$

Let $\Delta^A$ represent the set of coalitions of partisans against that cannot be challenged, i.e. $\Delta^A = \{V^A \subset \mathcal{P}^A \mid \sum_{V^A} \alpha_i^A + \alpha_U^A \geq \alpha_1^F + \alpha_U^F\}$ ($\Longleftrightarrow$ (A2)) and define:

$$\begin{aligned} V_m^A &= Arg \underset{V^A \in \Delta^A}{Min} \left\{ \sum_{V^A} \alpha_i^A - \underset{V^A}{Min}(\alpha_j^A) \right\} \\ \text{and } \alpha_m^A &= \sum_{V_m^A} \alpha_i^A - \underset{V_m^A}{Min}(\alpha_j^A) \end{aligned}$$

An equilibrium voting coalition against exists when $\alpha_U^F > \alpha_U^A$ if and only if $\alpha_m^A < \alpha_U^F - \alpha_U^A$. Indeed, if $\alpha_m^A < \alpha_U^F - \alpha_U^A$, $\alpha_m^A$ verifies the necessary and sufficient conditions and the corresponding coalition $V_m^A$ is therefore an equilibrium voting coalition (it may not be unique). If conversely $\alpha_m^A \geq \alpha_U^F - \alpha_U^A$, the second inequality can never be verified and no voting equilibrium exists.

**Case 2 -** $\alpha_U^A \geq \alpha_U^F$

Case 2, is symmetric to case 1.

***Existence of equilibrium*** $\mathcal{A}$**.** (A1) always holds in case 2. Since the majority in favor of $A$ is obtained without any partisan's participation, no partisan of $A$ votes (conditions (A3a) and (A3b) are irrelevant here). Thus the existence of an equilibrium $\mathcal{A}$ is guaranteed if and only if $\alpha_U^A \geq \alpha_U^F + \alpha_1^F$ (A2). Note that only the unconditionals vote in equilibrium $\mathcal{A}$ when it exists.

***Existence of equilibrium*** $\mathcal{F}$**.** Some partisans of $F$ must vote for $F$ to pass in equilibrium. With large unconditionals, (F2) is always verified and (F4b) can never hold and is therefore irrelevant. The necessary and sufficient conditions simplify to:

$$\alpha_1^A + \alpha_U^A - \underset{V^F}{Min}(\alpha_i^F) < \sum_{V^F} \alpha_i^F + \alpha_U^F - \underset{V^F}{Min}(\alpha_i^F) \leq \alpha_U^A$$

By the same reasoning as before, an equilibrium with a voting coalition for exists when $\alpha_U^A \geq \alpha_U^F$ if and only if $\alpha_m^F \leq \alpha_U^A - \alpha_U^F$ where $\alpha_m^F$ represents the minimum, among all coalitions for that cannot be challenged, of total partisan votes for less the share of the smallest partisan of the coalition.

**Remarks**

- The necessary and sufficient conditions for the existence of a voting equilibrium $\mathcal{A}$ (resp. $\mathcal{F}$) in case 1 where $\alpha_U^F > \alpha_U^A$ (resp. in case 2 where $\alpha_U^A \geq \alpha_U^F$) imply $\underset{V^A}{Min}(\alpha_i^A) > \alpha_1^F$ (resp. $\underset{V^F}{Min}(\alpha_i^F) > \alpha_1^A$). Thus the smallest partisan of the voting coalition must be larger than the largest opposed partisan in equilibrium.

- If $\alpha_U^F > \alpha_U^A$ (resp. $\alpha_U^A \geq \alpha_U^F$) and $\alpha_1^A + \alpha_U^A \geq \alpha_1^F + \alpha_U^F$ (resp. $\alpha_1^F + \alpha_U^F > \alpha_1^A + \alpha_U^A$), then the situation where $\alpha_1^A$ (resp. $\alpha_1^F$ ) votes for and all other partisans do not vote is an equilibrium $\mathcal{A}$ (resp. $\mathcal{F}$). There may exist other voting equilibria $\mathcal{A}$ (resp. $\mathcal{F}$). No equilibrium $\mathcal{F}$ (resp. $\mathcal{A}$) exists in that case.

## Proof of proposition 2 (small unconditionals)

Suppose $\alpha_U^F + \alpha_U^A < Q$, or unconditionals alone are not large enough to reach the minimum quorum.

**Existence of equilibrium $\mathcal{F}$.**

If it exists, it can only be an equilibrium where a subset of partisans of $F$ vote. As no partisan of $A$ ever votes in equilibrium $\mathcal{F}$ (P1), if no partisan of $F$ votes either, the quorum cannot be reached, so for never passes.

Let $\Delta^F$ represents the set of coalitions of partisans for that cannot be challenged, i.e. $\Delta^F = \{V^F \subset \mathcal{P}^F \mid \sum_{V^F} \alpha_i^F + \alpha_U^F > \alpha_1^A + \alpha_U^A\}$ and:

$$V_m^F = Arg \underset{V^F \in \Delta^F}{Min} \left\{ \sum_{V^F} \alpha_i^F - \underset{V^F}{Min}(\alpha_j^F) \right\}$$

***Case 1* - $Q \leq 2\alpha_U^A$** The necessary and sufficient conditions simplify to:

$$\alpha_U^A + \alpha_1^A - \underset{V^F}{Min}(\alpha_j^F) < \sum_{V^F} \alpha_i^F + \alpha_U^F - \underset{V^F}{Min}(\alpha_j^F) \leq \alpha_U^A$$

$V_m^F$ verifies the first inequality since by definition it belongs to $\Delta^F$. It also verifies the second inequality if $\alpha_m^F = \sum_{V_m^F} \alpha_i^F - \underset{V_m^F}{Min}(\alpha_j^F) \leq \alpha_U^A - \alpha_U^F$. Moreover the second inequality can never hold if $\alpha_m^F > \alpha_U^A - \alpha_U^F$ since $\alpha_m^F$ minimizes the value of $\sum_{V^F} \alpha_i^F - \underset{V^F}{Min}(\alpha_j^F)$ in $\Delta^F$. Note that the above conditions imply $\underset{V^F}{Min}(\alpha_j^F) > \alpha_1^A$. This case with relatively high $\alpha_U^A$ is actually similar to the large unconditionals case with $\alpha_U^A \geq \alpha_U^F$ since $Q$ plays no role in case 1. $V_m^F$ is an equilibrium coalition, but others may exist.

***Case 2* - $2\alpha_U^A < Q \leq 2\alpha_U^A + \alpha_1^A$** The necessary and sufficient conditions simplify to:

$$\alpha_U^A + \alpha_1^A - \underset{V^F}{Min}(\alpha_j^F) < \sum_{V^F} \alpha_i^F + \alpha_U^F - \underset{V^F}{Min}(\alpha_j^F) < Q - \alpha_U^A$$

As in case 1, the first inequality is verified for $V_m^F$ since it belongs to $\Delta^F$. It also verifies the second inequality if

$$Q_m^F \equiv \sum_{V_m^F} \alpha_i^F + \alpha_U^F + \alpha_U^A - \underset{V_m^F}{Min}(\alpha_j^F) < Q$$

Moreover, if $Q_m^F \geq Q$, the second inequality can never be verified for any non contestable coalition.

Thus there exists an equilibrium voting coalition for if and only if $Q_m^F < Q$. In other words, the effective quorum for less the smallest share among voting partisans is below the minimum quorum which ensures that all shareholders are pivotal. $V_m^F$ is an equilibrium coalition, but others may exist.

***Case 3* - $2\alpha_U^A + \alpha_1^A < Q$** The necessary and sufficient conditions in that case are:

$$\begin{aligned} Q - \alpha_U^A - \underset{V^F}{Min}(\alpha_j^F) &\leq \sum_{V^F} \alpha_i^F + \alpha_U^F - \underset{V^F}{Min}(\alpha_j^F) < Q - \alpha_U^A \\ \text{or } 0 &\leq \left\{ \sum_{V^F} \alpha_i^F + \alpha_U^F + \alpha_U^A \right\} - Q < \underset{V^F}{Min}(\alpha_j^F) \end{aligned}$$

Let $\Omega^F$ represent the set of coalitions of partisans for that reach the minimum quorum, i.e. $\Omega^F = \{V^F \subset \mathcal{P}^F \mid \sum_{V^F} \alpha_i^F + \alpha_U^F + \alpha_U^A \geq Q\}$. As $2\alpha_U^A + \alpha_1^A < Q$, no $V^F \in \Omega^F$ can be challenged ($\Omega^F \subset \Delta^F$). Define:

$$V_Q^F = Arg \underset{V^F \in \Omega^F}{Min} \left\{ \sum_{V^F} \alpha_i^F \right\}$$

It is then easy to see that $V_Q^F$ is an equilibrium voting coalition for. Indeed, by definition $V_Q^F$ reaches the minimum quorum since it belongs to $\Omega^F$. It also verifies the second inequality which ensures that any shareholder belonging to $V_Q^F$ is pivotal. Indeed, suppose this is not the case (i.e. $\left\{\sum_{V_Q^F} \alpha_i^F + \alpha_U^F + \alpha_U^A\right\} - Q \geq \underset{V_Q^F}{Min}(\alpha_j^F))$. This means that the coalition $V_Q^F \diagdown \underset{V_Q^F}{Min}(\alpha_j^F)$ belongs to $\Omega^F$. This contradicts the fact that $V_Q^F$ has the smallest size in this set.

**Remarks:** - When there exists a coalition $V^F \mid \sum_{V^F} \alpha_i^F + \alpha_U^F + \alpha_U^A = Q$ (this is the case with small shareholders, each one having one vote), the voting coalition may match exactly the minimum quorum $Q$ in equilibrium.

- If $\alpha_1^F + \alpha_U^F > \alpha_1^A + \alpha_U^A$ and $\alpha_U^F + \alpha_U^A + \alpha_1^F \geq Q$, there exists a voting equilibrium where only the largest shareholder for votes.

**Existence of equilibrium $\mathcal{A}$.**

The necessary and sufficient conditions (A2), (A3a) and (A3b) for a voting equilibrium against to exist are:

$$\sum_{V^A} \alpha_i^A + \alpha_U^A \geq \alpha_U^F + \alpha_1^F$$

$$\sum_{V^A} \alpha_i^A + \alpha_U^A - \underset{V^A}{Min}(\alpha_j^A) < \alpha_U^F$$

$$\sum_{V^A} \alpha_i^A + \alpha_U^F + \alpha_U^A - \underset{V^A}{Max}(\alpha_j^A) \geq Q$$

As mentioned in the text, these conditions imply together that

$$\underset{V^A}{Min}(\alpha_j^A) > \alpha_1^F$$

$$\alpha_U^F > \alpha_U^A$$

$$2\alpha_U^F > Q$$

So in the case where $Q \geq 2\alpha_U^F$, no voting equilibrium exists. However voting equilibria against may exist if $\alpha_U^F + \alpha_U^A < Q < 2\alpha_U^F$.

They are characterized as follows. Let us consider $\Omega^A = \{V^A \subset \mathcal{P}^A \mid \sum\limits_{V^A} \alpha_i^A + \alpha_U^F + \alpha_U^A - Max(\alpha_j^A) \geq Q$ and $\sum_{V^A} \alpha_i^A + \alpha_U^A \geq \alpha_U^F + \alpha_1^F\}$.

Let $V_M^A = Arg \underset{V^A \in \Omega^A}{Min} \left\{ \sum_{V^A} \alpha_i^A - \underset{V^A}{Min}(\alpha_j^A) \right\}$. Thus if $\alpha_M^A = \sum_{V_m^A} \alpha_i^A - \underset{V_m^A}{Min}(\alpha_j^A) \geq \alpha_U^F$, no voting equilibria exists. When $\alpha_M^A < \alpha_U^F$, $V_M^A$ is a voting coalition equilibrium against.

When $\alpha_U^F + \alpha_U^A < Q$, a non voting equilibrium against exists if and only if (A'2) holds i.e.

$$\begin{aligned} \alpha_U^F + \alpha_U^A + \alpha_1^F &< Q \\ \text{or } \alpha_U^F + \alpha_1^F &\leq \alpha_U^A \end{aligned}$$

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