Market-Dependent Preferences, Positive and Negative Network Effects and Welfare

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Abstract: This article specifies the conditions on the utility function and on the cost function for which congestion or production overcapacity emerge in a market when information is perfect and prices are flexible. Consumers are of two types: some appreciate a large production market, some don't. A typology underlines the shape of utility functions and cost functions which generate an optimal production overcapacity or an optimal congestion at the pure strategy Nash equilibrium. We show that the sign of the consumption externalities is endogenous. The result is independent of the nature (positive or negative) of the consumption externalities that affect the consumers' preferences. A welfare analysis is performed. We underline the conditions under which the social planner chooses whether to clear the market or not. Application to a network economy is developed. Extensions to the following three cases are provided: 1) unsold stocks of goods with or without warehouse inventories, 2) quality of goods and 3) software markets.[1]

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I. Introduction

In this theoretical paper on network goods, we demonstrate, on the one hand, that positive consumption externalities do not necessarily lead to market congestion. On the other hand, negative consumption externalities do not necessarily entail market production overcapacity. Therefore, the shapes of both utility and cost functions are central in our theory.

In this paper, we consider consumers who are sensitive to the market size. As usual in economic literature, consumption externalities are considered to be positive or negative if for instance the consumer's utility increases or decreases with the market size. We highlight two types of consumers according to the sign of these function values: those who buy more on large network markets and those who buy more on small network markets. We point out that when the market size varies, their utility also varies, but not necessarily in the same direction. When considering every impact of positive or negative consumption externalities, four possible cases may arise.

1. **Positive consumption externalities with production overcapacity:** the more Internet users there are, the more Internet providers increase Internet connection speed. Consumers thus experience positive consumption externalities. This case has been well studied by the Network literature.

2. **Negative consumption externalities with production overcapacity:** Cyber security problems may occur considering the fact that some networks require the user to reveal personal or even confidential data to sign in. Indeed, the more Internet users there are, the higher the risks of hacking become. This affect negatively the user's utility.

3. **Positive consumption externalities with congestion:** The Covid-19 pandemic led to lock-downs all over the world. The Internet Network almost became the unique substitute for the outside world, increasing people interactions

and cooperation if we consider the educational field. On line classes developed sometimes creating congestion despite greater positive consumption externalities.

4. **Negative consumption externalities with congestion**: Consider a road network. When the number of drivers increases, the traffic becomes less and less fluid which causes congestion and air pollution. In general, motorists are averse to traffic jams. Furthermore, environmental concerns go against positive consumption externalities.

We depart from the literature by making no assumptions about the sign of the consumption externalities which are arbitrary functions from the modeling point of view. A negative (positive) consumption externality thus corresponds to a decreasing (increasing) marginal utility with respect to the demanded quantity which is a decreasing (an increasing) function of the market size. In other words, this means that the inverse demand is a decreasing (increasing) function of the market size. In this setting, it turns out that the sign of the externality is endogenous. We point out that it is relevant to consider its sign after calculating the pure strategy Nash equilibrium. Indeed, this sign can change depending on the neighborhood in which the equilibrium is reached.

In the remaining of the paper, we say that consumption externalities are positive or negative in equilibrium if the marginal utility function increases or decreases in the neighborhood of the equilibrium. Depending on the shape of the α-concave utility function, either the market is balanced or not.

The additive separable shape of the utility function chosen by Katz and Shapiro (1985)[21] helps understand why in equilibrium consumers' expectations are fulfilled. Similarly, in Amir and Lazzati (2011)[3], the existence of a symmetric equilibrium is due to some properties of the inverse demand function which are direct consequences of the shape of the consumer's utility function. Moreover, Amir, Evstigneev and Gama (2021)[4] investigate the viability problem with an argumentation in the direction of technological progress and exogenous entry of firms. They obtain the surprising result that 'monopoly leads to the highest prospects for viability' p 1207.

We consider a monopoly and we study the conditions on the utility and the cost functions for which consumers' expectations are fulfilled or not. As mentioned in Amir, Evstigneev and Gama (2021) [4], "the comparison of consumer surplus requires a more restrictive condition on demand. [...] A theoretical investigation at hand appears needed as a way to gauge the well-foundedness of the conventional view, which was based on a number of observed cases and stylized facts but little formal analysis". In the single network industry case, our paper explores the role of both the consumption externalities' sign and the cost function's shape on the optimal solutions of the market.

After the general model, we present an example with quadratic utility function and an interaction cost function for the monopoly. Depending on the values of the parameters, either the consumer's expectations are fulfilled or not. Moreover, whatever the consumer's type, it is shown that the social planner chooses the same network capacity as the market would have chosen, but provides the consumers with more consumption in the case where their expectations are fulfilled. Depending on the values of parameters, the social planner may choose either production overcapacity or congestion in the single network industry.

Extension in the direction of the emergence of an excess of supply of goods on non-network market is provided. Replacing the size of the network by the display of goods allows us to understand the emergence of permanent unsold stocks of goods. Traditional models explain temporary excess of supply by considering demand a random variable, (Prescott (1975)[24], Bryant (1980)[7], Lucas and Woodford (1993)[23], Eden (1990)[16] and Dana (1993)[12]), (Rey and Tirole (1986)[25], Deneckere *et al*. (1996)[14]), (Khan and Thomas (2007)[20]), Herbert E. Scarf (1960)[27], Schutte (1984)[28]. However, a permanent unsold stocks of goods cannot be considered a random variable. For that reason, the literature introduces fixed prices under certainty to generate excess of supply, Kawasaki, McMillan and Zimmermann (1983)[19], Carlton (1986)[8], Mathewson and Winter (1987), Shaffer (1991)[29]. At the macroeconomic level, fixed prices result in excess of supply on the goods market and excess of supply on the labor market, resulting in short term unemployment, Benassy (1975)[6], and Grandmont (1977)[18]. Another argument is unobserved consumer's preferences, Lazear (1986)[22]. The main problem with those approaches is that in the long run it is hard to assume that prices will not be adjusted. Consequently, there is no convincing theory of permanent unsold stock of goods, i.e., a persistent gap between sales and warehouse inventories, as observed on many markets, Wouter J. Den Haan (2013)[33] and Dudin, Nazarov and Yakupov (2015, p 264)[15]. Our general setting provides a theoretical framework for explaining such observed facts without fixed price or random variables.

The paper is organized as follows. Section II. is devoted to the model. Section III. presents the solution of the model. Section IV. exposes an example applied to Network goods. Section V. discusses and extends the results before Section VI. concludes.

II. The model

Prior to presenting the consumer's behavior, we must first define the consumer's production-dependent preferences and then provide some useful definitions. We start by defining the set of available goods, the demand vector q and the supply vector Q. The latter represents the size of the market. We then study the properties of individual preferences that depend on production and show that under the usual Debreu assumptions, there is a utility function that reflects these preferences. Using this utility function, which depends on both quantities demanded and quantities produced, we can define two types of consumers: Those who value a large market size and those who prefer a small market size. Then we define the strategic game played by the consumer and a representative monopoly. Finally, we present a somewhat generalized Inada-like condition. We need this condition to prove the existence of a solution of the game in pure strategies with perfect information and flexible prices characterized either by production overcapacity or by congestion.

A. The consumer's market production dependent preferences

Define by $Z = \mathbb{R}^n_+$ the set of the n goods produced by a monopoly. Denote $q = (q_1, ..., q_n) \in \mathbb{R}^n_+$ the vector of quantities of goods a consumer wants to buy, and $Q = (Q_1, ..., Q_n) \in \mathbb{R}^n_+$ the monopoly production of these n goods. In this subsection, we consider that a consumer k who buys $q_i, i = 1, \ldots, n$ units of goods is sensitive to the quantity Q_i of goods produced on market i. For that reason, we start by defining the properties of his Q-dependent preferences. The consumer k's preferences on X are denoted $\succeq_k$: $(q', Q') \succeq_k (q'', Q'')$, $(q', Q') \in \mathbb{R}^{2n}_+$, $(q'', Q'') \in \mathbb{R}^{2n}_+$ means that (q', Q') is at least as good as (q'', Q'') for consumer k. We will use the following assumptions and definitions. We assume:

Completness C: for any two consumption points $(q', Q') \in \mathbb{R}^{2n}_+, (q'', Q'') \in \mathbb{R}^{2n}_+$, either $(q', Q') \succeq_k (q'', Q'')$ or $(q'', Q'') \succeq_k (q', Q')$.

Reflexivity R: for all $(q, Q) \in \mathbb{R}^{2n}_+, (q, Q) \succeq_k (q, Q)$.

Transitivity T: $(q', Q') \in \mathbb{R}^{2n}_+$, $(q'', Q'') \in \mathbb{R}^{2n}_+$, $(q''', Q''') \in \mathbb{R}^{2n}_+$:

$$(q', Q') \succeq_k (q'', Q'') \succeq_k (q''', Q''') \Rightarrow (q', Q') \succeq_k (q''', Q''').$$

Continuity $Cont$: $\forall (q'', Q'') \in \mathbb{R}^{2n}_+, \{(q', Q') \in \mathbb{R}^{2n}_+ : (q', Q') \succeq_k (q'', Q'')\}$ and $\{(q', Q') \in \mathbb{R}^{2n}_+ : (q'', Q'') \succeq_k (q', Q')\}$ are closed sets.

Strong monotonicity in q (SM_q): $(q', Q) \in \mathbb{R}^{2n}_+, (q'', Q) \in \mathbb{R}^{2n}_+, q' \geq q''$ (a coordinatewise inequality) and there is at least one i, so that ${q'}_i > {q''}_i \Rightarrow (q', Q) \succ_k (q'', Q)$.

We will use the following theorem:

THEOREM **1** *(Debreu [13]) Suppose $Y \in R^n$. If the preference relation $\preceq$ on Y is $C, T, Cont$, then there exists a continuous utility representation $U : Y \to R$.*

COROLLARY **1** *If preferences are Q^k-dependent, there exists a continuous Q^k-dependent utility function $U : \mathbb{R}^{2n}_+ \to R$ that represents these preferences.*

Proof. Using assumptions $C, R, T, Cont$, this is a direct consequence of the Theorem of Debreu[13]). $\square$

There are nice examples of such preferences. Indeed, on the energy markets, consumers may substitute gaz for oil, oil for coal etc. Consequently, their preferences are market dependent, see page 3 Abada, Gabriel, Briat and Massol (2013)[1].

DEFINITION **1** *We call market-sensitive consumer any consumer who has Q^k-dependent preferences.*

It is useful to distinguish the two following types of consumers, according to the value of $k = +, -$.

DEFINITION **2** *A Q^+-consumer is a consumer whose utility increases with the quantity of goods produced on the market.*

DEFINITION **3** *A Q^--consumer is a consumer whose utility decreases with the quantity of goods produced on the market.*

Q is a parameter for the market-sensitive consumer and it represents an additional instrument for the monopoly which chooses both the price and the quantity to be produced. For this reason we call it a modified monopoly. We will build a one-period model considering only one good, one market-sensitive consumer and a modified monopoly.

A..1 The static strategic game

The static strategic game in which a monopoly plays with only one type of consumer at the time, consists in:

1. A set of players $N = \{C, M\}$, where C is a market-sensitive consumer, M for the modified monopoly.

2. A market-sensitive consumer has a set of actions : $\mathcal{A} = \{q\}$, where $q \in \mathbb{R}^+$ is the demanded quantity, and for the monopoly $\mathcal{A}^M = \{p_m, Q_m\}$, where $p_m \in \mathbb{R}^+$ is the monopoly price given the demand function, and $Q_m \in \mathbb{R}^+$ is the market production of good.

3. A Q-dependent preference relation is defined on the set of consequences $\mathcal{C} = \mathcal{A} \times \mathcal{A}^M = \{(p_m, q, Q_m)\}$. The consumer's preferences relation is represented by his surplus and the monopoly's preference relation by its profit.

Define $X = \mathbb{R}^+ \times \mathbb{R}^+$. $U(q, Q_m)$ represents the preferences of a market-sensitive consumer. It satisfies the following assumption.

ASSUMPTION **1** *:*

- $U(q, Q_m)$ *is defined on* X*;*
- $U_q(\cdot, Q_m)$ *is* (-1)*-concave in* q *for all* $Q_m \geq 0$*;*
- $U_q(q, \cdot)$ *is concave in* Q_m *for all* $q \geq 0$*;* $U_Q(q, Q) \neq$ *constant* ,
- $U(q, Q_m)$ *has continuous partial derivatives up to third order in* X*, finite on the points such that* $q > 0$*,* $Q_m > 0$*;*
- $U_q(q, Q_m) > 0$*,* $U_{qq}(q, Q_m) < 0$*,* $U_{Q_m Q_m}(q, Q_m) \leq 0$ *for all* $(q, Q_m) \in X$*, they may take on infinite values on points such that* $q = 0$ *or* $Q_m = 0$*;*

Denote p the after-tax market price of the good: $p = p_m + \tau$, where τ is a tax. The consumer's preferences are represented by $V(q, p, Q_m) = U(q, Q_m) - pq$, where V — the surplus function — and $U(q, Q_m)$ — the utility function. From assumption 1 it follows $U(\cdot, Q_m)$ and $V(\cdot, Q_m)$ are two one-to-one $\mathcal{C}^2$ concave functions in q. A rational market-sensitive consumer maximizes his surplus function with respect to q, given Q_m and p. In this section, the government receives τq and redistributes it as a lump sum tax in proportion γ to the consumer $\gamma \tau q = \gamma T$, the rest $(1-\gamma)T$ is attributed to the monopoly. A rational market-sensitive consumer solves Problem $\mathcal{P}$:

$$\mathcal{P} := \max_q \; U(q, Q_m) - pq + \gamma T. \tag{1}$$

A market-sensitive consumer considers T a constant.

B. The modified monopoly

The modified monopoly operates under certainty and sells its product to the market-sensitive consumer. The total cost function is $C(q_m, Q_m)$.

ASSUMPTION **2** *:*

- $C : \mathbb{R}^+ \times \mathbb{R}^+ \to [0, \infty)$;
- C *is continuously differentiable on* $(0, \infty) \times (0, \infty)$;
- $C(\cdot, Q_m)$ *is strictly increasing and convex for every* $Q_m \geq 0$;
- $C(q_m, \cdot)$ *is strictly increasing and convex for every* $q \geq 0$;
- $C_q(0, .) = 0$.

The monopoly chooses both the price p_m and the production Q_m by solving Problem $\mathcal{P}_{M1}$:

$$\mathcal{P}_{M1} : \begin{cases} \max\limits_{p_m, Q_m} \Pi_1(p_m, Q_m) = p_m q(p_m, Q_m) - C(q_m, Q_m) + (1 - \gamma)T, & (2) \\ \text{subject to: } (q_m, Q_m) \in X. & (3) \end{cases}$$

C. The welfare economy

This Subsection determines the welfare maximizing tax τ_w that leads the economy to the welfare maximizing consumption q_w^k and the welfare maximizing production Q_w^k. The social welfare is defined as $W^k(q_w^k, Q_w^k) = V^k(q_w^k, p_w^k, Q_w^k) + \Pi(q_w^k, Q_w^k)$. The social planner solves the following program $\mathcal{P}_{sp}$

$$\mathcal{P}_{sp} := \max_{q_w^k, Q_w^k} \; W^k(q_w^k, Q_w^k) \; \text{ subject to: } (q_w^k, Qw^k) \in X.$$

III. Solutions of the model

We analyse separately the market economy solution and the welfare solution.

A. The market economy

As the market economy solution is computed without any social planner's intervention, we consider $\tau = T = 0$ so that $p = p_m$. Under certain conditions on the shape of the utility function, either an interior solution or a corner solution may emerge at the pure strategy Nash equilibrium, $\forall k = +, -$[2] .

A..1 The consumer's solution

Assume that the marginal utility U_q is a one-to-one function. The first order condition is

$$p_m \quad = \quad U_q(q, Q_m) \tag{4}$$

Since the consumption marginal utility $U_q(q, Q_m) > 0$ we have $p_m(q, Q_m) > 0$. Consequently, along the consumer's optimum $p = U_q(q, Q_m)$, p is a decreasing function of q. Therefore we have $p_q < 0$. Note that the price is production dependent $p_{Q_m} = U_{qQ_m}$. Examples are provided in Section IV.

A..2 The Modified monopoly's solutions

We now turn to present the solution of $\mathcal{P}_{M1}$. Replace the solution of the market-sensitive consumer (4) into the profit function of the monopoly. We suppose the consumer has enough budget to cover their expenditures. Since in perfect information the monopoly takes the consumer's solution $p = U_{q_m}(q_m, Q_m)$ as given, choosing p_m is equivalent to choose $q_m = q$ and Q_m.

Problem $\mathcal{P}_{M1}$ becomes

$$\max_{q_m, Q_m} \Pi_1(q_m, Q_m) = U_{q_m}(q_m, Q_m)q_m - C(q_m, Q_m) \tag{5}$$

$$\text{subject to: } (q_m, Q_m) \in X. \tag{6}$$

ASSUMPTION **3** *:*

[2]This can be interpreted as an equilibrium or an over-production (excess of supply of good).

- $\lim_{q_m \to 0^+} U_{q_m}(q_m, Q_m) q_m = 0$;
- *There exists a point* $(\overline{q}_m, \overline{Q}_m) \in int\, X$[3] *so that*
  1. $U_q(\overline{q}_m, \overline{Q}_m)\overline{q}_m - C(\overline{q}_m, \overline{Q}_m) > U_q(q_m, 0)q_m - C(q_m, 0)$ *for all* $q_m \geq 0$;
  2. $U_q(\overline{q}_m, \overline{Q}_m)\overline{q}_m - C(\overline{q}_m, \overline{Q}_m) > 0$;
- Π_1 *is bounded on* X;

The firm does not have any revenue if they do not sell any quantity (point 1 of assumption 3). From assumption 3 it follows the solution is on $int\ X$. We do not consider the cases where the market size or the sold quantity are unbounded (point 3 of assumption 3) and solutions at points so that $q_m = 0$ or $Q_m = 0$ (point 2 of assumption 3).

Since the solution is an interior point it must satisfy the first order condition for a local extremum: $\Pi_{1_{q_m}} = 0$ and $\Pi_{1_{Q_m}} = 0$:

$$S_1 \begin{cases} U_{q_m q_m}(q_m, Q_m) q_m + U_{q_m}(q_m, Q_m) - C_q(q_m, Q_m) = 0, & (7) \\ U_{q_m Q_m}(q_m, Q_m) q_m - C_{Q_m}(q_m, Q_m) = 0. & (8) \end{cases}$$

According to Assumption 1 and 2, the profit is quasi-concave in q_m (see Corollary 2 in Appendix A) and concave in Q_m. Problem S_1 may have multiple solutions, but for sure one of them is a maximum because the function is bounded, according to Assumption 3. The first relation captures the monopoly's maximizing-profit behavior. From this relation, the monopoly extracts the demanded quantity that maximizes its profit, given the market size Q_m. Knowing this maximal quantity, the second relation indicates that the marginal revenue of an extra unit of market size should be equal to the marginal cost of this extra unit of market size.

Note that relation (8) can be rewritten as $p_{Q_m} q_m - C_{Q_m}(q_m, Q_m) = 0$. It is important to note here that the term $U_{q_m Q_m}(q_m, Q_m)$ can change it sign in the neighborhood of an equilibrium. Consider the following utility function $U(q, Q) := (-AqQ^2 + BQ + C - q)q$. We have $U_{qQ} = B - 4AqQ$ which can be positive for

[3] $int\, X$ denotes the interior points of the set X.

the market equilibrium but negative for the social planner's solution, since the values of the solution change, see Appendix B for an example with graphic.
Assume that the modified monopoly faces a Q^+ consumer, the higher the production, the higher their willingness to pay and $p_{Q_m} > 0$. Suppose the modified monopoly faces a Q^--consumer. If the firm wants to decrease production overcapacity it must also lower the price. Then again $p_{Q_m} > 0$. Consequently, whatever the type of consumer the monopoly faces, relation (8) may have a solution so that $q^\star \neq Q^\star$.

Examples presented in this paper illustrate that it is possible for consumption externalities to be negative while the first derivative of the price with respect to the market size is positive. Let us denote the elasticity of the cost to the market size $\varepsilon_{C/Q} := C_{Q_m} \times (Q_m/C)$ and the elasticity of the price to the market size $\varepsilon_{p/Q} := p_{Q_m} \times (Q_m/p_m)$.

PROPOSITION **1** *The problem of the modified monopoly has a solution if* $p_{Q_m}(q_m, Q_m) \geq 0$ *and this solution secures a positive profit if* $\varepsilon_{C/Q} \geq \varepsilon_{p/Q}$.

Proof. To prove Proposition 1 we solve the system (7), (8). From (7) we have

$$q^\star_m = \frac{C_{q_m}(q_m, Q_m) - U_{q_m}(q_m, Q_m)}{U_{q_m q_m}(q_m, Q_m)}. \tag{9}$$

From (8) we have

$$q^\star_m = \frac{C_{Q_m}(q_m, Q_m)}{U_{q_m Q_m}(q_m, Q_m)}. \tag{10}$$

Using $p_m = U_{q_m}(q_m, Q_m)$ and $p_{Q_m} = U_{q_m Q_m}$ rewrite the two previous relations to get the following equality

$$\frac{C_{q_m}(q_m, Q_m) - p(q_m, Q_m)}{U_{q_m q_m}(q_m, Q_m)} = \frac{C_{Q_m}(q_m, Q_m)}{p_{Q_m}(q_m, Q_m)}$$

$$\iff \frac{C_{q_m}(q_m, Q_m) - p_m(q_m, Q_m)}{C_{Q_m}(q_m, Q_m)} = \frac{U_{q_m q_m}(q_m, Q_m)}{p_{Q_m}(q_m, Q_m)}.$$

On account of the concavity of the utility function, we have $U_{q_m q_m} \leq 0$. Furthermore, $C_{q_m}(q_m, Q_m) - p(q_m, Q_m) \leq 0$ as the opposite of the market power of the firm in Lerner's sense. Therefore, from the previous equality we deduce the condition that $p_{Q_m}(q_m, Q_m) \geq 0$ for the equality to hold. The solution should also secure a positive profit, i.e., $q_m^\star \geq \frac{C(q_m,Q_m)}{p_m(q_m,Q_m)}$. Using relation (10), we have

$$\frac{C_{Q_m}(q_m, Q_m)}{U_{q_m Q_m}(q_m, Q_m)} \geq \frac{C(q_m, Q_m)}{p(q_m, Q_m)} \iff \frac{C_{Q_m}(q_m, Q_m)}{C(q_m, Q_m)} \geq \frac{p_{Q_m}(q_m, Q_m)}{p_m(q_m, Q_m)}$$

Multiply the last inequality by Q_m and obtain the condition $\varepsilon_{C/Q} \geq \varepsilon_{p/Q}$.[4] □

B. The emergence of a production overcapacity

B..1 Utility of the type $U(q_m, Q_m) = f_1(q_m) + g_1(Q_m)$

This type of utility function is commonly used in network literature to ensure a market equilibrium, Katz and Shapiro (1985)[21]. It leads to a Q_m-independent price. Indeed, equation (8) has no solution and there is no over-production of goods. The maximization problem of the modified monopoly in this case is equivalent to the maximization problem of the monopoly in the classical micro-economic settings.

B..2 General type of utility

The following two theorems give sufficient conditions for emergence of production overcapacity, congestion or clear market if we observe Q_m^* (q_m^*).

THEOREM **2** *Under assumptions 1-3 suppose the observed market size is Q_m^*.*

1. *If $\Pi_{1_{q_m}}(Q_m^*, Q_m^*) = 0$, then the optimal quantity sold is $q_m^* = Q_m^*$ and the market clears.*

2. *If $\Pi_{1_{q_m}}(Q_m^*, Q_m^*) < 0$, then the optimal quantity sold is $q_m^* < Q_m^*$ and there is an overcapacity in the market.*

3. *If $\Pi_{1_{q_m}}(Q_m^*, Q_m^*) > 0$, then the optimal quantity sold is $q_m^* > Q_m^*$ and there is a congestion in the market.*

[4] Similar conditions on elasticities often come up in oligopoly models, see e.g. Cosandier, Garcia and Knauff (2018).

Proof. Since $U_{q_m}(\cdot, Q_m)$ is (-1)-concave for all Q_m, Π_1 is quasi-concave in q_m for all $Q_m \geq 0$ (see Corollary 2 in Appendix A) and Π_1 is bounded (Assumption 3) there are two possibilities:

1. there exists a_m such that Π_1 is increasing in q_m in $[0; a_m]$ and decreasing in q_m in $(a_m; \infty)$ at $Q_m = Q_m^*$;
2. Π_1 is decreasing in q_m at $Q_m = Q_m^*$.

Depending on the sign of $\Pi_{1_{q_m}}(Q_m^*, Q_m^*)$ the monopoly sells more, equal or less than the observed market size Q_m^*. □

THEOREM **3** *Under assumptions 1-3 suppose the observed demand for the good is q_m^*.*

1. *If $\Pi_{1_{Q_m}}(q_m^*, q_m^*) = 0$, then the optimal market size is $Q_m^* = q_m^*$ and the market clears.*
2. *If $\Pi_{1_{Q_m}}(q_m^*, q_m^*) < 0$, then the optimal market size is $Q_m^* < q_m^*$ and there is a congestion in the market.*
3. *If $\Pi_{1_{Q_m}}(q_m^*, q_m^*) > 0$, then the optimal market size is $Q_m^* > q_m^*$ and there is an overcapacity in the market.*

Proof. From assumption 1 ($U_q(q_m, \cdot)$ is concave in Q_m) and assumption 2 ($C(q_m, \cdot)$ is convex in Q_m) it follows $\Pi_1(q_m, \cdot)$ is concave and therefore quasi-concave in Q_m and the proof follows that of Theorem 2. □

C. The welfare economy

The social planner chooses the welfare maximizing tax / subsidy so that the consumption is q_w. As in Katz and Shapiro (1984) welfare is written as a function of the firm's individual levels of output, i.e., the market size $Q_w := Q_m$, according to Section C. The market price is $p_w = p_m + \tau$.

C..1 The consumer's solutions

The first order condition of the market-sensitive consumer is modified as follows:

$$p_m + \tau \quad = \quad U_{q_w}(q_w, Q_w) \tag{11}$$

As explained in Subsection A.1, by the same argument, the economic interpretation of the marginal condition above is that $q_p < 0$ and $p_{Q_m} = U_{qQ}$. Moreover, the demanded quantity by the consumer is a decreasing function of the tax τ, $q_\tau < 0$.

C..2 The modified monopoly solution

Replacing the solution of the market-sensitive consumer (11) into the profit function of the monopoly, Problem $\mathcal{P}_{M1}$ becomes

$$\max_{q_w, Q_w} \Pi_1(q_w, Q_w) = (U_{q_w}(q_w, Q_w) - \tau)q_w - C(q_w, Q_w) + (1-\gamma)T \tag{12}$$

$$\text{subject to: } (q_w, Q_w) \in X. \tag{13}$$

The first order condition for a local extremum is $\Pi_{1_{q_w}} = 0$ and $\Pi_{1_{Q_w}} = 0$.

$$S_1(\tau)\begin{cases} U_{q_w q_w}(q_w, Q_w)q_w + U_{q_w}(q_w, Q_w) - C_{q_w}(q_w, Q_w) - \tau = 0, & (14) \\ U_{q_w Q_w}(q_w, Q_w)q_w - C_{Q_w}(q_w, Q_w) = 0. & (15) \end{cases}$$

Since both monopoly and social planner's solve their respective problem by maximizing with respect to both outputs (q_w and Q_w), there is, *a priori*, no reason to expect the outcome to have fulfilled expectations, i.e, to obtain $Q_w = q_w$ as part of the solution. If $\tau > 0$ then $q_w < q_m$ and reciprocally. In relation (15) it is important to note that the value of the first derivative with respect to the market size of the marginal utility is not equivalent to the value of the consumption externalities in equilibrium, i.e., evaluated at the optimal values of q_w and Q_w. The solution of the system 14-15 is $q_w^* = q_w(\tau)$ and $Q_w^* = Q_w(\tau)$.

C..3 The social planner's solution

When replacing the surplus function by its expression in terms of utility and profit function by their respective expression, everything cancels out except the consumer's utility and the cost function of the modified monopoly. The problem is:

$$\max_{q_w,Q_w} W(q_w, P_w) = U(q_w, Q_w) - C(q_w, Q_w) \tag{16}$$

$$\text{subject} \quad \text{to: } (q_w, Q_w) \in X. \tag{17}$$

To exclude degenerate solutions in the boundary of X (i.e. a point $(q_w, Q_w) \in X$ so that $q_w = 0$ or $Q_w = 0$) and unboundedness of the welfare function we make the following assumption.

ASSUMPTION **4** *:*

- *There exists a point* $(\overline{q}_w, \overline{Q}_w) \in int\, X$ *so that*
  1. $U(\overline{q}_w, \overline{Q}_w) - C(\overline{q}_w, \overline{Q}_w) > U_w(q_w, 0) - C(q_w, 0)$ *for all* $q_w \geq 0$*;*
  2. $U(\overline{q}_w, \overline{Q}_w) - C(\overline{q}_w, \overline{Q}_w) > U_w(0, Q_w) - C(0, Q_w)$ *for all* $Q_w \geq 0$*;*
- W *is bounded on* X*;*

From point 1 of assumption 4 it follows the solution of the social planner is an interior point. Again, we do not consider the cases when the solution is at points so that $q_w = 0$ or $Q_w = 0$. The choice of the social planner (q_w^*, Q_w^*) is a solution of the system

$$\begin{cases} U_{q_w}(q_w, Q_w) - C_{q_w}(q_w, Q_w) = 0, & (18) \\ U_{Q_w}(q_w, Q_w) - C_{Q_w}(q_w, Q_w) = 0. & (19) \end{cases}$$

To find the optimal value of τ solve either

$$q_w^* = q_w(\tau), \text{ or } Q_w^* = Q_w(\tau). \tag{20}$$

Having in mind that the utility function $U(q_w, Q_w)$ is concave in q_w and in Q_w (assumption 1) and the cost function $C(q_w, Q_w)$ is convex in q_w and Q_w (assumption 2) analogously to Theorems 2 and 3 are the following two theorems.

THEOREM **4** *Under assumptions 1-4 suppose the observed market size is* Q_w^**.*

1. *If* $W_{q_m}(Q_m^*, Q_m^*) = 0$*, then the social planner clears the market.*

2. *If* $W_{q_m}(Q_m^*, Q_m^*) < 0$*, then the social planner chooses production overcapacity.*

3. *If* $W_{q_m}(Q_m^*, Q_m^*) > 0$*, then the social planner chooses congestion.*

THEOREM **5** *Under assumptions 1-4 suppose the observed demand for the good is* q_w^**.*

1. *If* $\Pi_{1_{Q_m}}(q_m^*, q_m^*) = 0$*, then the social planner clears the market.*

2. *If* $W_{Q_m}(q_m^*, q_m^*) < 0$*, then the social planner chooses congestion.*

3. *If* $W_{Q_m}(q_m^*, q_m^*) > 0$*, then the social planner chooses production overcapacity.*

The proof is similar to the proof of Theorems 2 and 3 presented in section B..2. In the next section theorems are illustrated by examples.

IV. Economic Applications: Network goods

We now turn to present an economic application of our general modeling of preferences, dealing with network goods, with either market equilibrium, or production overcapacity or congestion in pure strategy Nash equilibrium. Suppose for instance that a Q^k-consumer buys $q \in \mathbb{R}^+$ units of debit to a network provider while the network supplies a maximum of $Q_m \in \mathbb{R}^+$ units which represents the maximum size of the network. Moreover, suppose that there are $i = 1, \ldots, N$ users at the same time (the model is static). As in Katz and Shapiro (1985)[21], let us define the size of the network as $Q = \sum_{i=1}^{N} q_i$. Note that $q \in \left[0, \sum_{i \neq j}^{N-1} q_j - q_i\right]$. If a Q^+-consumer is connected to a small network, then their utility increases as the size of the network increases before the maximum size Q_m of the network is reached. If too many users are connected at the same time, then their utility start to decrease. The latter situation encapsulates the possibility of congestion.

A. The consumer

The following example illustrates the functioning of the general model.[5] A rational consumer maximizes their surplus

$$\max_{q_m} \left(-Aq_mQ_m^2 + BQ_m + C - (q_m - D)\right) q_m - p_m q_m.$$

The solution is $q_m = \frac{BQ_m+C-(p_m-D)}{2AQ_m^2+2}$, from what we extract the price

$$p_m = -2Aq_mQ_m^2 + BQ_m + C - (q_m - D) - q_m. \tag{21}$$

B. The modified monopoly

The rational modified monopoly maximizes its profits given the Q^k-consumer demand $\Pi(q_m, Q_m) := p_m q_m - q_m Q_m$. Replace p_m by the consumer's solution and solve the following programme

$$\max_{q_m, Q_m} (-2Aq_mQ_m^2 + BQ_m + C - q_m - (q_m - D))q_m - q_mQ_m.$$

A maximum in $int\, X$ must satisfy first order conditions (FOC):

$$\Pi_{q_m} = 0 \iff C - (q_m - D) + (B-1)Q_m - 4Aq_mQ_m^2 - 3q_m = 0, \tag{22}$$

$$\Pi_{Q_m} = 0 \iff q_m(B - 4Aq_mQ_m - 1) = 0. \tag{23}$$

There are two solutions. The first one is the pure network solution, but direct check shows it is not a maximum point for the values we have chosen for A, B, C, D to fit our model.

$$q_{m_0} = 0, \qquad Q_{m_0} = \frac{C+D}{1-B}. \tag{24}$$

[5]The choice of the shape of the consumption externalities and the shape of the cost function of the monopoly is crucial. Indeed, if we consider affine consumption externalities $aQ + b$ with a separable quadratic cost function, it is not possible to generate the very natural case of congestion with negative consumption externalities. Therefore, one can think of using quadratic consumption externalities of the following form: $aQ^2 + bQ + c$, but with linear, bi-linear or quadratic cost functions, solutions are not easily tractable.

The second one is

$$q_m^\star = \frac{C+D}{4}, \qquad Q_m^\star = \frac{B-1}{A(C+D)}. \tag{25}$$

Note that the difference $Q_m^\star - q_m^\star = \frac{B-1}{A(C+D)} - \frac{C+D}{4}$ can be either positive or negative, characterizing production overcapacity or congestion. The externality $EXT := B - 4AqQ$ can either be positive or negative. Table 1 hereafter simulates all the possible economic cases.

After introducing a tax/subsidy in the previous model, the after tax price becomes $p+\tau$. It leads to the following relation $p = C+D-2q+BQ-2AQ^2-\tau$. Rewrite the modified monopoly problem and solve the first order condition to have

$$q_{m_0} = 0, \qquad Q_{m_0} = \frac{C+D-\tau}{1-B}. \tag{26}$$

or

$$q_m^\star = \frac{C+D-\tau}{4}, \qquad Q_m^\star = \frac{B-1}{A(C+D-\tau)}. \tag{27}$$

If the social planner only manipulates the market size, then it is easy to see that $\tau = 0$. The social planner keeps the market size solution. If he chooses to tax/subsidy consumption, i.e., to solve $q_m(\tau) = q_w$ then the tax/subsidy would be $\tau = -(C+D)$.

C. The social planner

The social planner maximizes the welfare function. He solves the following programme:

$$\max_{q_w, Q_w} (-Aq_w Q_m^2 + BQ_w + C - (q_w - D))q_w - q_w Q_w.$$

Note that the condition $A > 0$ implies that the welfare function is concave in Q_m and in q_m. Again points such as q_w are not maximum points for the values we have chosen for A, B, C, D. A direct check shows that the solution is

$$q_w = \frac{C+D}{2}, \ Q_w = \frac{-1+B}{A(C+D)}.$$

The price is obtained by replacing the welfare maximizing quantities into relation (21). We obtain

$$p_w = \frac{(B-1)B}{A(C+D)} - \frac{(B-1)^2}{A(C+D)}$$

It is worth noticing that the social planner chooses the same network size as the market, lower-script w_0. Therefore, only one instrument is needed to decentralize the market equilibrium. However, he chooses twice the consumption as the market. Therefore, it is possible that he chooses congestion while the market has chosen production overcapacity . The reverse is not possible.

D. Continuing with the example

D..1 No fulfilled expectations equilibrium

Our example has been chosen in order to illustrate that, as in Katz and Shapiro (1985)[21], the consumer's expectation on the size of the network Q_e and the expected price p_e to pay to access the network may differ from the modified monopoly optimal solution. Indeed, if the consumer also chooses to use a network which size optimizes the value of the expected consumption externalities EXT_e, the solution is $Q_e = \frac{B}{2Aq_m}$. Replacing q_m into the previous expression and obtain $Q_e = \frac{2B}{A(C+D)}$. Solving the first-order equation $Q_e = Q_m$ for B leads to $B = -1$. The condition for the concavity of the utility function with respect to q is $-2 - 2AQ^2 < 0$ which involves $A > 0$. For $q_m > 0$ we must have $C + D$. Consequently, for $B = -1$ we always have $Q_m < 0$. There is no fulfilled expectations equilibrium.

——————— insert Table 1 here. ————————

	Consumption externalities			
	Overcapacity	Congestion	Overcapacity	Congestion
	$A = 8$, $B = 4$	$A = 8$, $B = 4$	$A = 1$, $B = 2$	$A = 8$, $B = 4$
	$C = -3$, $D = 4$	$C = -2.5$, $D = 4$	$C = 2$, $D = -1$	$C = 1$, $D = 4$
p_m	1.43	1.375	2	3.53
q_m	0.25	0.375	0.25	1.25
Q_m	0.375	0.25	1	0.075
$EXT(A, B, q_m, Q_m)$	> 0	> 0	> 0	> 0
$V(q_m, p_m, Q_m)$	0.13	0.21	0.125	1.63
Π	0.26	0.42	0.25	3.26
W	0.49	0.63	0.375	4.89
	The social planner			
	Congestion	Congestion	Overcapacity	Congestion
p_w	0.375	0.25	1	0.075
q_w	0.5	0.75	0.5	2.5
Q_w	0.375	0.25	1	0.075
$EXT(A, B, q_w, Q_w)$	< 0	< 0	$= 0$	< 0
$V(q_w, p_w, Q_w)$	0.53	0.84	0.5	6.53
Π	0	0	0.	0
W	0.53	0.84	0.5	6.53
	The Katz and Shapiro case			
	Overcapacity	Overcapacity	Overcapacity	Congestion
p_e	0.5	0.75	0.5	2.5
q_e	0.25	0.375	0.25	1.25
Q_e	1	0.66	4	0.2
$EXT(A, B, q_w, Q_w)$	< 0	< 0	< 0	< 0
$V(q_w, p_e, Q_e)$	0.32	0.40	0.68	1.82

Table 1: All possible economic cases

Note that the consumer's surplus evaluated at the welfare maximizing variables equals the welfare since in this example we have $p_w q_w = q_w Q_w = \frac{B-1}{2A}$.

V. General discussion and extensions

A. General discussion

How can we account for these results? How to understand that a Q^--consumer's behavior is compatible with a pure strategy Nash equilibrium with an optimal over-production of goods? To answer these questions, it is important to economically interpret the marginal condition of system S_1. The first relation captures the monopoly's maximizing profit behavior. The second relation indicates that the marginal revenue of an extra unit of production should be equal to the marginal cost of this extra unit of production. Moreover, $U_{qQ}(q, Q) > 0$ is a necessary condition for equation 8 in system S_1 to have a solution and it means that the price is increasing in Q, even if the utility is decreasing in Q, which characterizes a Q^--consumer. In that case, the firm produces more than the maximal quantity $q^\star < Q^\star$. If $U_{qQ}(q, Q) < 0$ there will be no excess supply of goods.

B. Extensions

B..1 Unsold stocks of goods

As discussed in the introduction, there are particular markets for which unsold stocks of goods are permanent. This section analyzes the emergence of permanent unsold stocks of goods in a very simple modeling of the consumer's and firm's behaviors, thus extending the main results of the management and economic literature. Let be Q the display of one type of good and q the demand for it.

1. DEFINITION **4** *Given $Q' > Q''$, a picky consumer is a Q^+-consumer such as $(q, Q') \succ_P (q, Q'')$.*

2. DEFINITION 5 *Given $Q' > Q''$, a consumer who faces Fear Of Missing Out (namely a FOMO consumer) is a Q^--consumer such as $(q, Q'') \succ_F (q, Q')$.*

B..1.1 The consumer

Assume that the utility function has the following shape

$$U(q_m, Q_m) := A(q_m - q_m^2/4) + BQ_m + Cq_mQ_m,\ A > 0,\ C > 0,\ B - \text{arbitrary}.$$

The consumer is

- picky at q_m if $B + Cq_m > 0$;
- without any preference for display at q_m if $B + Cq_m = 0$;
- fomo at q_m if $B + Cq_m < 0$.

Consumer maximizes its surplus $S(q_m, Q_m, p_m) := U(q_m, Q_m) - p_mq_m$ and solves

$$\max_q \ (A(q_m - q_m^2/4) + BQ_m + Cq_mQ_m - p_mq_m.$$

The solution is $p_m = A - Aq_m/2 + CQ_m$.

B..1.2 The modified monopoly

The rational modified monopoly maximizes its profits given the consumer's demand $\Pi(q_m, Q_m) := p_mq_m - q_m^2/2 - Q_m^2/2$ and solves the following programme

$$\max_{q_m, Q_m} \ (A - Aq_m/2 + CQ_m)q_m - q_m^2/2 - Q_m^2/2.$$

The system of the First-Order Condition (FOC) is the following

$$\Pi_{q_m} = 0 \iff A - Aq_m + CQ_m - q_m = 0, \quad (28)$$

$$\Pi_{Q_m} = 0 \iff Cq_m - Q_m = 0 \quad (29)$$

The profit function is concave as long as $\Delta_m := A + 1 - C^2 > 0$. The solution is:

$$q_m^* = \frac{A}{A + 1 - C^2},$$

$$Q_m^* = Cq^* = \frac{CA}{A + 1 - C^2}.$$

- If $0 < C < 1$ then $q_m^* > Q_m^*$ and there is a congestion in the market;
- if $C = 1$ then $q_m^* = Q_m^*$ and the market clears;
- if $C > 1$ then $q_m^* < Q_m^*$ and there is an overcapacity in the market;

Welfare considerations

A tax/subsidy τ is now introduced on the market. It is an instrument for the social planner to lead the economy to the maximum of welfare.

B..1.3 The consumer's behavior and Welfare

A rational consumer maximizes its surplus and solves

$$\max_{q_w, Q_w} \; A(q_w - q_w^2/4) + BQ_w + Cq_wQ_w - (p_w + \tau)q_w.$$

The solution is $p_w = A - Aq_w/2 + CQ_w - \tau$.

B..1.4 The modified monopoly's behavior and Welfare

The rational modified monopoly maximizes its profits given the consumer's demand $\Pi(q_w, Q_w, \tau) := p_w(\tau)q_w - q_w^2/2 - Q_w^2/2$ and solves the following programme

$$\max_{q_w, Q_w} \; (A - Aq_w/2 + CQ_w - \tau)q_w - q_w^2/2 - Q_w^2/2$$

The solution is

$$q_w^\star = \frac{A - \tau}{A + 1 - C^2}, \tag{30}$$

$$Q_w^\star = \frac{C(A - \tau)}{A + 1 - C^2}. \tag{31}$$

B..1.5 The Social planner and the welfare maximizing tax/subsidy

The social planner chooses the welfare maximizing demand q_w and the welfare maximizing size of the market Q_w. He solves the following problem

$$\max_{q_w, Q_w} A(q_w - q_w^2/4) + BQ_w + Cq_wQ_w - q_w^2/2 - Q_w^2/2$$

First order conditions are:

$$\Pi_{q_w}(q_w, Q_w) = A - Aq_w/2 + CQ_w - q_w = 0 \tag{32}$$

$$\Pi_{Q_w}(q_w, Q_w) = B + Cq_w - Q_w = 0 \tag{33}$$

The welfare function is concave as long as $\Delta_w := A/2 + 1 - C^2 > 0$. The solution is

$$q_w^* = \frac{A + BC}{A/2 + 1 - C^2}, \ (A + BC > 0),$$

$$Q_w^* = B + Cq^* = B + \frac{C(A + BC)}{A/2 + 1 - C^2}.$$

The consumer is

- Picky at q_w^* if $B + Cq_w^* > 0$, i.e. $Q_w^* > 0$. As we mentioned before monopoly clears the market if $C = 1$. The social planner will choose the same if additionally $B = 0$. If $B > 0$, $C = 1$ the monopoly will generate overcapacity and if $B < 0$, $C = 1$ there will be a congestion.

- Without any preference for display at q_w^* if $B + Cq_w^* = 0$ and the social planner will set $Q_w^* = 0$.

- FOMO at q_w^* if $B + Cq_w^* < 0$, then $Q_w^* < 0$. The social planner will solve the problem at the points $Q_w = 0$. In this case assumption 4 is violated.

Note that the social planner chooses the same network size, so much so that the welfare maximizing level of the consumption externalities is exactly the same as the market one. Contrary to Katz and Shapiro (1985), the social planner chooses the same Network size, but not the demanded quantity. Therefore, the tax/subsidy is obtained by solving the following first-order equation: $q_m(\tau) = q_w$, making the market solution equal to the welfare maximizing one. This leads to the following tax/subsidy

$$\tau_w = \frac{-A^2/2 - AC + AC^2 + BC(C^2 - A - 1)}{A/2 + 1 - C}. \tag{34}$$

Table 2 presents all relevant cases.

——- insert table 2 here ————

Table 2: All possible cases for the Unsold Stock of goods example

Positive consumption externalities	
Production overcapacity	
$A = 9, B = -1.6, C = 1.2,$	$A = 2, B = 2, C = 1.2,$
$p^\star = 5.782$ $q^\star = 1.051$ $Q^\star = 1.261$ $EXT(B, C, q^\star) > 0$	$p^\star = 2.564$ $q^\star = 1.282$ $Q^\star = 1.538$ $EXT(B, C, q^\star) > 0$
The welfare maximizing results	
$p_w = 1.743$ $q_w = 1.743$ $Q_w = 0.492$ $EXT(B, C, q_w) > 0$	$p_w = 5.714$ $q_w = 7.857$ $Q_w = 61.734$ $EXT(B, C, q_w) > 0$
Congestion	
$A = 3, B = -1, C = 0.9,$	$A = 2, B = 1, C = 0.1,$
$p^\star = 3.291$ $q^\star = 0.94$ $Q^\star = 0.846$ $EXT(B, C, q^\star) > 0$	$p^\star = 1.337$ $q^\star = 0.668$ $Q^\star = 0.066$ $EXT(B, C, q^\star) > 0$
The welfare maximizing results	
$p_w = 1.242$ $q_w = 1.243$ $Q_w = 0.118$ $EXT(B, C, q_w) > 0$	$p_w = 1.055$ $q_w = 1.055$ $Q_w = 1.222$ $EXT(B, C, q_w) > 0$

Consider the last set of results. Note that the social planner turns production overcapacity with positive externality into congestion with positive externality. Similarly, market congestion turns to welfare maximizing production overcapacity.

B..2 Inventories

More can be said on the basis of such a modeling. Indeed, it is frequently observed that the demand can be a function of both the monopoly's display and warehouse inventory. To extend our results in that direction, it is possible to consider another class of utility functions that incorporate the inventory I_m. To do so, consider the utility function

$$U^k(q_m, Q_m, I_m)$$

and redo the complete exercise and obtain the consumer's solution $U_{q_m}(q_m, Q_m, I_m) = p$. Using the previous solution of the market-sensitive consumer into the profit function of the monopoly, Problem $\mathcal{P}_{M1}$ becomes $\mathcal{P}_{M1(I)}$

$$\max_{q_m, Q_m, I_m} \Pi_1(q_m, Q_m, I_m) = U_{q_m}(q_m, Q_m, I_m)q_m - C(q_m,Q_m, I_m), \quad (35)$$

$$\text{subject to: } 0 \leq q_m \leq I_m,\ 0 \leq Q_m \leq I_m. \quad (36)$$

The monopoly's marginal condition is: $\Pi_{1_{q_m}} = 0, \Pi_{1_{Q_m}} = 0$ and $\Pi_{1_{I_m}} = 0$

$$S_1(I) \begin{cases} U^k_{q_m q_m}(q_m, Q_m, I_m)q_m + U^k_{q_m}(q_m, Q_m, I_m)) - C_{q_m}(q_m, Q_m, I_m) = 0 = 0, & (37) \\ U^k_{q_m Q_m}(q_m, Q_m, I_m)q_m - C_{Q_m}(q_m, Q_m, I_m) = 0, & (38) \\ U^k_{q_m I_m}(q_m, Q_m, I_m)q_m - C_{I_m}(q_m, Q_m, I_m) = 0. & (39) \end{cases}$$

More pure strategy Nash equilibria can arise. Note that this encompasses all the cases where the display is higher than the warehouse inventory. The unsold stocks of goods emerge in these equilibria: ${q_m}^\star = {Q_m}^\star < {I_m}^\star,\ \ {q_m}^\star < {Q_m}^\star < {I_m}^\star,\ \ {q_m}^\star < {Q_m}^\star = {I_m}^\star$.

B..3 Quality and luxury goods

Our model can also be extended in the direction of the quality of goods. Indeed, denote q the useful quality needed by the consumer and Q the overall quality of a given product. The excess of quality $Q - q$ explains the existence of luxury goods. As Veblen (1889)[32] mentions in his book, page 82 there are consumers who are luxury sensitive, so are our Q^+-consumers. Finally, a consumer who buys a software with Q options usually uses just q of them.

Another possible interpretation of our setting considers Q as a measure of general product quality for the output q, as in the classic paper by Spence (1975)[31] or in the environmental sense as in Amir, Gama and Maret (2019)[2].

VI. Conclusion

In this paper, there are two types of consumers. Consumers who are market production sensitive, i.e. who appreciate the size of the market while buying a type of good. For example the higher the display, the higher the demand, or the higher the size of a network, the higher the utility. Other consumers appreciate tiny markets while buying and the lesser the production the higher the demand. For example, luxury goods are more appreciated than others for their scarcity which makes them valuable.

The firm is a modified monopoly that strategically chooses both the price and the market size. In a static environment with perfect information, no uncertainty and flexible prices, the paper shows in which conditions a production overcapacity emerges, regardless of the consumer's type. It is shown that this depends on the shape of the utility and cost functions.

If a social planner chooses the welfare maximizing tax / subsidy in the economy, he chooses not to clear markets. Some example illustrate the general results of the model. Extensions of the model are provided, when it is necessary to determine the demanded quantity related to the display and warehouse inventories. At the pure strategy Nash equilibrium, it is shown that similar to the benchmark model, unsold stocks of goods may exist. However, other situations are dealt with. For example, it is possible that an excess of warehouse inventories coexist with a very small display with respect to the demanded quantity. Network goods are explored and the shape of the utility function explains some major results of this literature, Katz and Shapiro (1985)[21], Amir and Lazzati (2011)[3] and Amir, Esvtigneev and Gama (2021)[4].

References

[1] Abada, I., Gabriel, S.A., Briat, V., Massol, O., 2013. A Generalized Nash-Cournot Model for the North-Western European Natural Gas Markets with a Fuel Substitution Demand Function: The GaMMES Model. Networks and Spatial Economics 13 (1), 1-42.

[2] Amir, R., Gama, A., Maret, I., 2019. Environmental quality and monopoly pricing, Resource and Energy Economics 58, 101-109.

[3] Amir, R., Lazzati, N., 2011. Network effects, market structure and industry performance. Journal of Economic Theory 146 (6), 2389-2419.

[4] Amir, R., Evstigneev, I., Gama, A., 2021. Oligopoly with network effects: firm-specific versus single network. Economic Theory 71 (3), 1203-1230

[5] Anderson, S. P., Renault, R., 2003. Efficiency and surplus bounds in Cournot competition. Journal of Economic Theory 113(2), 253-264.

[6] Benassy, J.P., 1975. Neo-Keynesian Disequilibrium Theory in a Monetary Economy. Review of Economic Studies 42, 503-523.

[7] Bryant, J., 1980. Competitive Equilibrium with Price Setting Firms and Stochastic Demand,. International Economic Review 21 (4), 619-626.

[8] Carlton D. W., 1986. The Rigidity of Prices. The American Economic Review 76 (4), 637-658.

[9] Cosandier, C., Garcia, F., Knauff, M., 2018. Price competition with differentiated goods and incomplete product awareness. Economic Theory 66, 681-705.

[10] Corstjens, M., Doyle, P., 1981. A Model for Optimizing Retail Space Allocation. Management Science 27 (7), 822-833.

[11] Corstjens, M., Doyle, P., 1983. A Dynamic Model for Strategically Allocating Retail Space. Journal of the Operational Research Society 34 (10), 943-951.

[12] Dana Jr, J. D., 1993. Equilibrium price dispersion under demand uncertainty: the role of Costly Capacity and Market Structure. RAND Journal of Economics 30 (4), 632-660.

[13] Debreu, G., 1959. The theory of Value. Wiley.

[14] Deneckere, R., Marvel, H. P., Peck, J., 1996. Demand Uncertainty, Inventories, and Resale Price Maintenance. The Quarterly Journal of Economics 111 (3), 885-913.

[15] Dudin, A., Nazarov, A., Moiseev, A., 2016. Information Technologies and Mathematical Modelling: Queueing Theory and Applications: 15th International Scientific Conference, ITMM 2016, Named After AF Terpugov, Katun, Russia, Springer In- ternational Publishing AG Switzerland. Proceedings 638.

[16] Eden, B., 1990. Marginal Cost Pricing When Spot Markets Are Complete. Journal of Political Economy 98, 1293-1306.

[17] Ewerhart, C., 2014. Cournot games with biconcave demand. Games and Economic Behavior 85, 37-47.

[18] Grandmont J.M., 1977). The Logic of the Fix Price Method. Scandinavian Journal of Economics,

[19] Kawasaki, S., McMillan, J., Zimmermann, K. F., 1983. Inventories and Price Inflexibility. Econometrica 51 (3), 599-610.

[20] Khan, A., Thomas, J. K., 2007. Inventories and the Business Cycle: An Equilibrium Analysis of (S, s) Policies Inventories and the Business Cycle: An Equilibrium Analysis of (S, s) Policies. The American Economic Review 97 (4), 1165-1188.

[21] Katz, M. L., Shapiro, C., 1985. Network externalities, competition, and compatibility. The American Economic Review 75 (3), 424-440

[22] Lazear, E. P., 1986. Retail Pricing and Clearance Sales. The American Economic Review 76(1), 14-32.

[23] Lucas Jr, R. E., Woodford, M., 1993. Real effects of monetary shocks in an economy with sequential purchases.

[24] Prescott, E. C., 1975. Efficiency of the Natural Rate. Journal of Political Economy 83, 1229-1236.

[25] Rey P., and Tirole, J., 1986. Contraintes verticales: l'Approche principal-agent. Annales d'économie et de Statistique 1, 175-201.

[26] Rockafellar RT., 1993. Lagrange Multipliers and Optimality. SIAM Review 35 (2), 183-238.

[27] Scarf, H. E., 1960. The Optimality of (S,s) Policies in the Dynamic Inventory Problem. Mathematical Methods in the Social Sciences. ed. K. J. Arrow, S. Karlin, and H. Scarf, 196-202.

[28] Schutte, D. P., 1984. Optimal Inventories and Equilibrium Price Behavior. Journal of Economic Theory 33: 46-58.

[29] Shaffer G., 1991. Capturing Strategic Rent: Full-Line Forcing, Brand Discount, Aggregate Debate and Maximum Resale Price Maintenance. The Journal of Industrial Economics 39 (5), 567-575.

[30] Shapiro, A., Dentcheva, D., Ruszczyński, A., 2009. Lectures on stochastic programming: modeling and theory. Society for Industrial and Applied Mathematics.

[31] Spence, A.M., 1975. Monopoly, quality, and regulation. The Bell Journal of Economics, 417-429.

[32] Veblen, T., 1889. The Theory of the Leisure Class, republished in 1973.

[33] Den Haan, W. J., 2013. Inventories and the role of goods-market frictions for business cycles.

A Mathematical background

The concept of generalized concavity finds many applications in economics. It is used to measure the curvature of functions with specific economic meaning - demand functions, price functions, production functions, distribution functions. Generalized concavity finds many applications in Cournot games. Anderson and Renault (2003), [5], use it to derive two efficiency bounds of oligopolistic competition. Ewerhart (2014), [17], applies generalized concavity to obtain unifying conditions for existence and uniqueness of a Cournot-Nash equilibrium. Here we will provide some general definitions and properties of α-concave functions relevant to our modeling.

We consider a function $f : \Omega \to \mathbb{R}^{+\star}$ defined on a convex set $\Omega \subseteqq \mathbb{R}^{+}$

DEFINITION **6** *A positive function defined on a convex set $\Omega \subseteqq \mathbb{R}^{+}$ is said to be $\alpha-concave$, where $\alpha \in [-\infty, +\infty]$ if for all $q, r \in \Omega$ and all $\lambda \in [0, 1]$ the following inequality holds true:*

$$f(\lambda q + (1-\lambda)r) \geq m_\alpha(f(q), f(r), \lambda)$$

where $m_\alpha : \mathbb{R}^{+\star} \times \mathbb{R}^{+\star} \times [0, 1] \to R$ is defined as follows:

$$m_\alpha(a, b, \lambda) = \begin{cases} a^\lambda b^{1-\lambda} & if\ \alpha = 0, \\ \max\{a, b\} & if\ \alpha = \infty, \\ \min\{a, b\} & if\ \alpha = -\infty, \\ (\lambda a^\alpha + (1-\lambda)b^\alpha)^{1/\alpha} & \text{otherwise.} \end{cases}$$

.

In the case where $\alpha = 0$, the function is log-concave, in the case where $\alpha = 1$, the function is simply concave. The concept of generalized convexity can be stated in an analogous manner with inverse inequalities.

The following lemma is very important because it implies that α-concavity entails β-concavity, for all $\beta \leq \alpha$.

LEMMA **1** *The mapping $\alpha \to m_\alpha(a, b, \lambda)$ is non-decreasing and continuous.*

A proof of this lemma can be found in [30].

We are especially interested in -1-concavity (convexity).

DEFINITION **7** *A positive function defined on a convex set $\Omega \subseteqq \mathbb{R}^+$ is said to be -1-concave (convex), if for all $q, r \in \Omega$ and all $\lambda \in [0, 1]$ the following inequality holds true:*

$$f(\lambda q + (1 - \lambda)r) \geq (\leq)(\lambda(f(q))^{-1} + (1 - \lambda)(f(r))^{-1})^{-1}.$$

The inequality is strict if the function is strictly -1-concave (convex). It is clear that a positive function $f(\cdot)$ is -1-concave (convex) if and only if $1/f(\cdot)$ is convex (concave). From Lemma 1 it follows that all positive constant, linear, log-concave and concave functions are -1-concave. The following property characterizes the positive twice continuously differentiable -1-concave (convex) functions.

PROPOSITION **2** *A positive and twice continuously differentiable function is -1-concave (convex), if and only if for all $q \geq 0$: $f(q)f_{qq}(q) - 2(f_q(q))^2 \leq 0(\geq 0)$.*

Proof. $f(q)$ is -1-concave (convex) $\Leftrightarrow 1/f(q)$ is convex (concave) $\Leftrightarrow (1/f_{qq}(q)) \geq 0(\leq 0) \Leftrightarrow f(q)f_{qq}(q) - 2f_q(q)^2 \leq 0(\geq 0)$ for all $q \in \Omega$. □

The inequality is strict if the function is α-concave, $-1 < \alpha < 0$. Now, we will analyse the function $r(q) = f(q)q$. We will use the properties of the function $\frac{1}{aq+b}$ which supports -1-concave (convex) functions (analogous to the linear supports of concave (convex) functions). From now on, in this section, we suppose f is a positive, continuous and decreasing function, finite on $\mathbb{R}^{+\star}$, it may take on $+\infty$ on 0.

THEOREM **6** *Let f be a strictly -1-concave (convex) function, then $r(q) = f(q)q$ has no local minimum (maximum) greater than 0.*

Proof. Suppose there is a local minimum (maximum) $\underline{q} > 0$. As $f(\cdot)$ is positive, strictly -1-concave (convex) and decreasing $\Rightarrow 1/f(\cdot)$ is positive, strictly convex (concave) and increasing. There exists $a > 0$ and b so that $1/f(q) \geq (\leq) aq + b$, with strict inequality if $q \neq \underline{q}$, furthermore $aq + b > 0$ at least at a vicinity of $\underline{q}$, consequently $f(q) < (>) \frac{1}{aq+b}$ at a vicinity of $\underline{q}$, $q \neq \underline{q}$. Since $\underline{q}$ is a local minimum (maximum) of $r(q)$, for all q at a vicinity of $\underline{q}$, $q \neq \underline{q}$:

$$\underline{q}\frac{1}{a\underline{q} + b} = \underline{q}f(\underline{q}) \leq (\geq) qf(q) < (>) q\frac{1}{aq + b}.$$

The last inequality implies $q\frac{1}{aq+b}$ has a strict local extremum at $\underline{q}$ and direct verification shows it is a contradiction. □

THEOREM **7** *Let f be a -1-concave function, let Ω be an interval where $r(q) = f(q)q$ is increasing. If $f(q) > 0$ and $f_q(q) < 0$ for all $q \geq 0$ then $r(q)$ is concave in Ω.*

Proof. It is a direct consequence of the following two inequalities:

$$f_q(q)q + f(q) \geq 0$$

$$f(q)f_{qq}(q) - 2(f_q(q))^2 \leq 0.$$

□

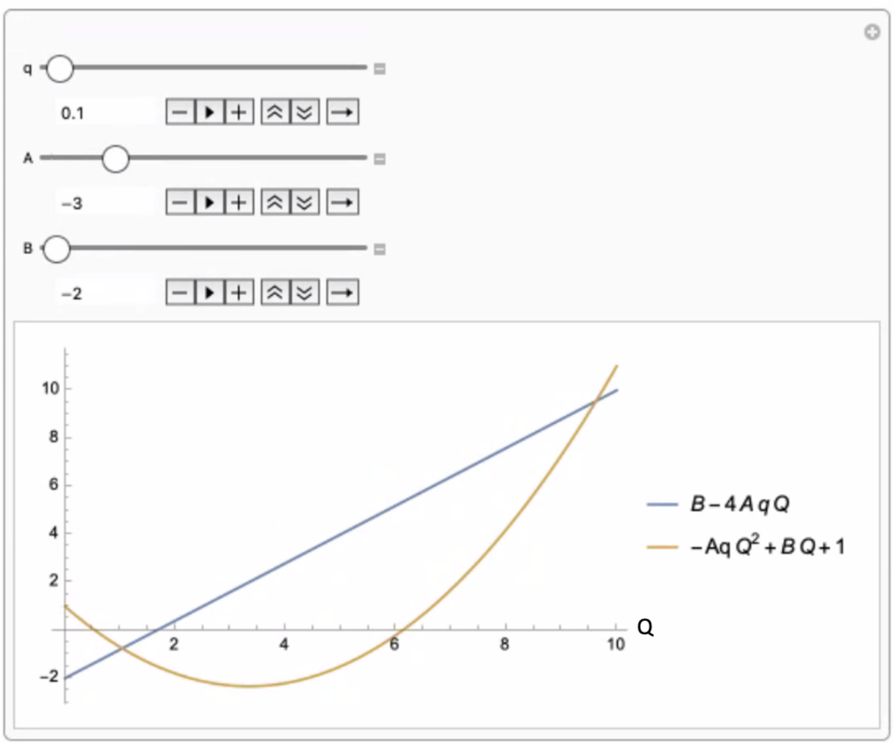
COROLLARY **2** *Let f be a (-1)-concave function, let $c : R_+ \to R_+$ be a convex increasing function then the function $\Pi(q) := f(q)q - c(q)$ is quasiconcave.*

Proof. A direct consequence of Theorem 6 and Theorem 7. □

B Appendix: Examples

A. A simple example where the sign of $U_{q_m Q_m}(q_m, Q_m)$ may differ from that of the consumption externalities

A simple example illustrates that the sign of $U_{q_m Q_m}(q_m, Q_m)$ may differ from that of the consumption externalities. Let us denote the consumption externalities $v(q, Q) := -AqQ^2 + BQ + C$. Consider the following quadratic in q and quadratic in Q utility function $U(q, Q) := q\,(v(q, Q) - q)$. We have $U_q = -q(2AqQ + B)$. On the following graphic, the blue curve represents the crossed derivative U_{qQ}, the yellow curve the value of the externality. The graphic illustrates cases where the blue curve is positive while the yellow one is negative (the blue curve on the following graphic) and $U_qQ = -4AqQ + B$ (the yellow curve on the following graphic).[6]



[6] Recall that the consumer maximizes its surplus and not its utility function.

B. Examples of various utility functions

Examples of utility functions with α concave marginal utilities, $0 < \alpha < 1$, suitable for our model: $u(q) := -e^{-(Aq+B)}, \;\; A > 0, \;\; B \geq 0$; $\;\; u(q) = 1/\rho(Aq+B)^{\rho}, \rho \in (-\infty, 0), \;\; A > 0, \;\; B > 0$; $\; u(q) := \sqrt{ln(q+A)+B}, \;\; A > 0, \;\; B > 0$. The logarithmic utility leads to both -1 concave and convex marginal utility.

Here are some examples of utility functions of a consumer with a preference for displayed goods.

Example 1 - $Q_m{}^{\gamma}u(q_m)$**.** Consider a function of the type $Q_m{}^{\gamma}u(q_m)$. $U^k(q_m, Q_m) = Q_m{}^{\gamma}u(q_m)$, $u(q_m) > 0$ a separable utility function. The consumer is picky in the case of $0 < \gamma < 1$, and FOMO, in case of $-1 < \gamma < 0$, if $\gamma = 0$, then consumer's utility coincides with the utility considered in the classical microeconomic literature. If $u(q_m) < 0$ the consumer's type is opposite. Note that in case $-1 < \gamma < 0$ the function is not defined at the points so that $Q_m = 0$.

Picky consumer

α-concave marginal utility function, $\alpha = -1$, picky consumer.

If the utility function of the picky consumer is $Q_m{}^{0.5}(e - e^{-q_m+1})$, $C_Q(Q_m) = Q_m/4$, $Q^{0.5}(e - e^{-q_m+1})$, then we have $q_m{}^* = 1$, $\gamma Q_m{}^{\gamma-1}u'(q_m{}^*)q_m{}^* = 0.5Q_m{}^{-0.5}$, $Q_m{}^* > q^*$. The unsold stocks of goods exist - Figure 1.

Figure 2 illustrates the case without any unsold stocks of goods: the utility function of the picky consumer is $Q_m{}^{0.5}(e^{-1} - e^{-q_m-1})$, $C_Q(Q_m) = Q_m/4$ and consequently $q_m{}^* = 1$, $\gamma Q_m{}^{\gamma-1}u'(q_m{}^*)q_m{}^* = 0.5Q_m{}^{-0.5}e^{-2}$, $Q_m{}^* < q^*$.

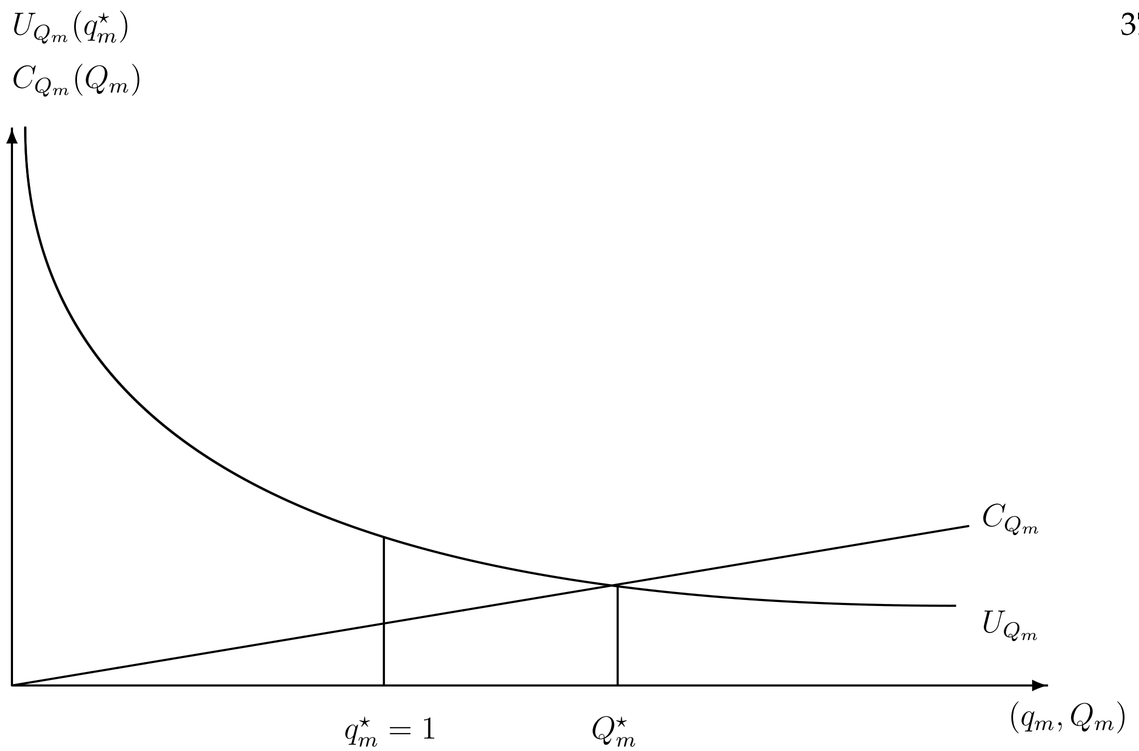


Figure 1: Case with an unsold stock of goods

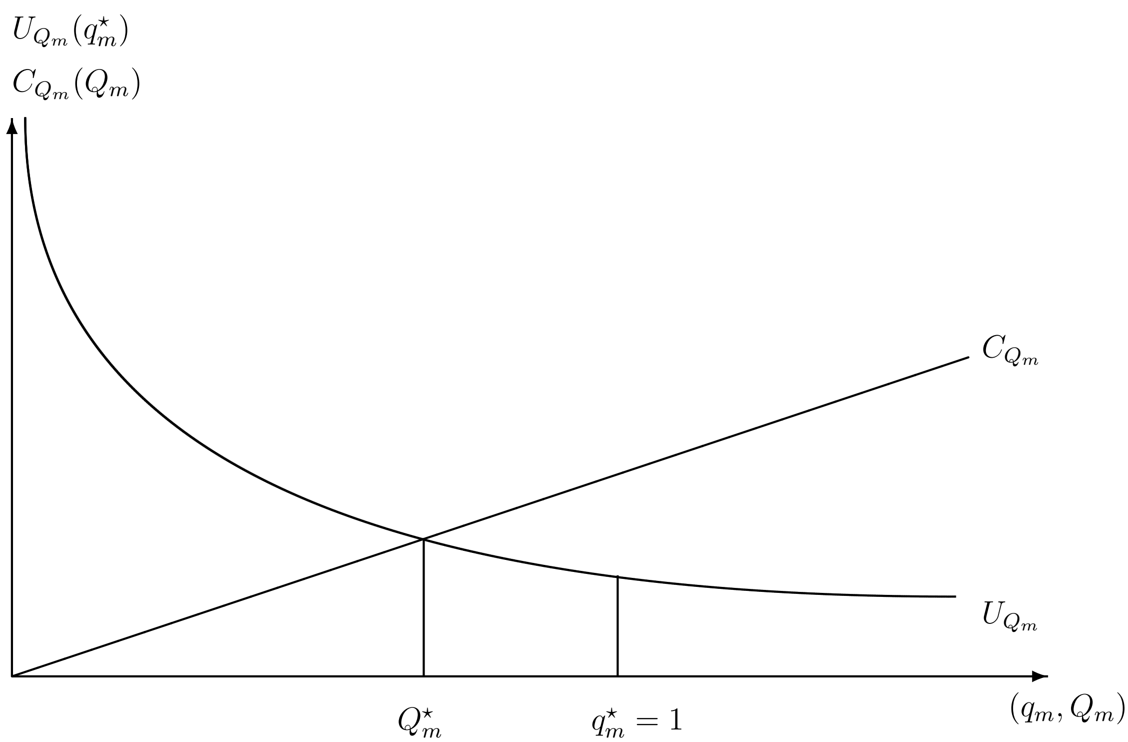


Figure 2: Case without any unsold stock of goods

FOMO consumer

If $-1 < \gamma < 0$, then $\gamma {Q_m}^{\gamma-1} u_{q_m}(q_m)q - C_Q(Q_m) < 0$ and (8) has no solution. There is no possible solution such as $Q_m > q_m$. Here, the marginal revenue with respect to Q_m is negative.

Example 2 - $U(q_m, Q_m) = -\frac{(Q_m+1)^{1/2}}{(q_m+1)}$**.** The consumer is FOMO in this example. The cost function is $C(Q_m) = 1/16Q_m$. The solution is $({q_m}^*, {Q_m}^*) = (1, 3)$, there are unsold stocks of good.

Example 3 - The welfare economy. Consider a FOMO consumer with a utility function $-\frac{(Q_m+1)^{9/10}}{(q_m+1)}$. The monopoly's total cost function is ${Q_m}^2/2$. The market equilibrium is at $({q_m}^*, {Q_m}^*) = (0.443, 0.443)$. The social planner's solution is $({q_w}^*, {Q_w}^*) = (0.091, 0.091)$, $\tau = 0.734$. After the tax the monopoly's solution is the same as the social planner's solution. If $C(Q_m) = {Q_m}^2/4$, the market equilibrium is at $(0.652, 0.652)$, the social planner's solution is $(0.168, 0.168)$, $\tau = 0.625$, but after the tax the monopoly's solution will be $(0.167, 0.217)$ and undesirable unsold stocks of goods will be generated. However, the overall utility at $(0.167, 0.217)$ is greater than the overall utility at $(0.652, 0.652)$.