The three worlds of the welfare capitalism revisited

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Abstract

We introduce a new way to model the Bismarckian social insurance system, stressing its corporatist dimension. Comparing the Beveridgean, Bismarckian and Liberal systems according to the majority voting rule, we show that for a given distribution of risks inside the society, the Liberal system wins if the inequality of income is low, and the Beveridgean one wins if the inequality of income is high. Using a utilitarian criterion, the Beveridgean system always dominates, and the Bismarckian system is preferred to the Liberal one when agents are sufficiently heterogeneous in risk inside each group.

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1 Introduction

This paper compares the three main systems of welfare capitalism – Beveridgean, Bismarckian, Liberal – as analyzed by Esping-Andersen (1990) from both a positive and normative perspective. To do so, we introduce a new way to model a Bismarckian type of social insurance to account for the fact that Bismarckian systems are organized around groups of agents. We aim to focus on the redistributive design of these different regimes and compare the preferred systems under both perspectives.

The background for our considerations is the following: In many countries with a Bismarckian system, like Germany, Austria, France or Belgium, a variety of social protection funds for illness, occupational injury, family or pension cover specific groups of people. For instance, the set of French social insurance funds refers to professional groups such as railway employees, employees of the public transportation system, seamen, civil servants, workers in the agricultural sector, entrepreneurs, etc. For occupational injury, the German insurance system is similarly organized on a professional group basis: Specific employer's mutual insurance associations cover the commercial, agricultural or the public sector as well as railway workers, firefighters and employees of local authorities etc.[1] There are other examples in Bismarckian countries where the formation of groups are a result of the agents' choice. For instance, in Belgium or Germany, people can choose between a (large) panel of health insurance funds. These funds are organized on the level of geographic coverage, employers, craft guilds, etc.[2]

The recognition of this organizational and strongly corporatist feature of the Bismarckian system goes back to the seminal work of Esping-Andersen (1990): "[C]orporatism was typically built around occupational groups seeking to uphold [...] status distinctions and used these as the organizational nexus for society and economy." (p. 60)[3] To be precise, Esping-Andersen (1990) clustered welfare states along "conservative", "social-democratic" and "liberal" regime types. In line with the established economic literature we remain for the first two systems with the nomenclature of "Bismarckian" and "Beveridgean" systems.

[1] German Social Security Law, Book Nr. VII

[2] In Germany, before the amendment to the Social Security Law in 1996, people had to insure themselves according to the selection criterion of the health insurance funds. Therefore, these funds covered only people who exactly matched with their selection criterion, e.g., they lived in a specific geographic region, they worked for a specific employer or in certain craft guilds etc. Nowadays, people can choose in which fund they want to be insured, cf. German Social Security Law, Book Nr. V. Further source: www.prospeur.org

[3] Cited after Esping-Andersen (1991).

The conventional characterization for all of theses systems is their degree of income redistribution. First, Liberal systems are associated with a really low degree of income redistribution since they mainly encourage private insurance. Second, Beveridgean systems, based on the principles of uniformity of benefits, universality and uniqueness, are associated with a high degree of income redistribution. This is due to proportional tax rates but flat benefits. Finally, Bismarckian systems are associated with a lower degree of income redistribution. Most often, they have been modeled in the literature like a private insurance equivalent, organized by the state, where individuals pay proportional taxes and receive proportional benefits.

The problem with this way of modeling the Bismarckian system is that it ignores the "corporatist" attribute of such systems: If individuals are differentiated by income and by risk, then a pooling of individuals with specific risks inside each fund leads to "intra-group horizontal redistribution" in the Bismarckian system, i.e., it leads to redistribution from low to high risk agents inside each fund.[4] As a consequence, each fund is characterized by its specific average risk. Transferred to the level of individual preferences, this implies that individuals who bear a high risk may benefit from a low average group risk. In a similar vein, the introduction of both income and risk heterogeneity of individuals leads to another kind of horizontal redistribution inside the Beveridgean systems: Here, redistribution is based on individual risk and on the distribution of risks inside the whole society. In our terminology, this type is called "global horizontal redistribution"and it complements the usual vertical redistribution of the Beveridgean system. Again, transferred to the level of individual preference, poor and/or high risk individuals benefit from the Beveridgean system. The Liberal system is characterized by neither horizontal nor vertical redistribution, since it consists of a private insurance mechanism, with a contribution rate that is proportional to individual risk and income.

There are two strands of literature which are related to our model. The first strand has determined both the type and the size of social insurance or social security systems, respectively, and therefore refers to the explicit distinction between Bismarckian and Beveridgean systems, see Casamatta et al. (2000b) (for social insurance) and Pestieau (1999) (for social security). They analyze the optimal size of the system (in terms of the tax rate) with

[4] In our model, individuals can be thought of being differentiated with respect to risk along a horizontal axis and with respect to income along a vertical axis. The notion of "horizontal" refers to the redistribution of risk (i.e., from low risk to high risk people) with two dimensions: within the whole society, or within groups. Accordingly, "vertical" refers to redistribution of income (i.e., from rich to poor people). See also Section 3.2.

the type of system chosen at the constitutional stage.[5] (2004) study both the level of tax rate and the type of system within a probabilistic model of electoral competition. Moreover, Conde-Ruiz and Profeta (2007) provide an OLG model of social security where the size and the type of system is determined simultaneously, yet issue-by-issue.

We complement this first strand of literature in two ways. Firstly, the Bismarckian system is modeled as a corporatist one, which enables us to clearly distinguish it from the Liberal system. Secondly, the choice of the system is determined alternatively according to a positive and a normative criterion, that we are able to compare.

The second strand of literature this paper refers to analyzes the link between income inequality and the level of redistribution inside society. Indeed, in our model, the degree of inequality of income and of risk crucially alters the choice of an agent, which affects the choice of the system for both positive and normative criteria. The link between income inequality and redistribution has first been highlighted in the Meltzer and Richard (1981) general equilibrium model of a labor economy where the share of redistributed income is determined by majority voting.[6] Their main finding is that if mean income rises relative to the income of the median voter, then redistribution increases. In other words, a more unequal income distribution leads to more redistribution. In addition to the standard redistributive mechanism from rich to poor, also insurance motives have been introduced in the analysis of welfare policies. For instance, Moene and Wallerstein (2001) show that the redistributive and the insurance mechanism work in opposite directions in the sense that support for social insurance spending declines with an increase of income inequality.[7] Finally, Kim (2007) extends the analysis of redistribution based on insurance motives by introducing a distribution of risks inside the society, where the level of risk depends on the agent's sector of activity. The main result of this model is that political demand for unemployment insurance is clearly influenced by both the distribution of risks and income.

[5]See also Cremer et al. (2007) for the effect of myopic and non-myopic individuals for social security.

[6]In addition, see Romer (1975) and Roberts (1977) on whose results Meltzer and Richard (1981) build upon.

[7]Moene and Wallerstein (2001) focus on the impact of income inequality on the support of welfare spending when welfare benefits are targeted towards the employed or the unemployed. See also Iversen and Soskice (2001) for a similar model analyzing social policy preferences which depend on different types of skill investments reflecting unemployment risks. Bénabou (2000) analyzes the impact of inequality and redistributive policies that enhance efficiency within a stochastic growth model.

As already indicated, our model provides a complete differentiation of individuals along three dimensions: income, individual risk and group risk. For instance, Casamatta et al. (2000b) introduce heterogeneity of individuals by a one dimensional differentiation with three discrete levels of income but the same probability of receiving income or relying on social benefits. Casamatta et al. (2000a) and Conde-Ruiz and Profeta (2007) differentiate along two dimensions, namely age (working young vs. retired old) and the level of income (continuous in Casamatta et al. (2000a), discrete in Conde-Ruiz and Profeta (2007)). A related double differentiation of individual along income and probability to become sick is found in Gouveia (1997) who analyzes the outcome of majority voting over the public provision of a private good (in particular, health care).

We concentrate on the case of insurance systems covering unemployment, occupational injury or health risks. Individuals earn a wage income in the good state of the world and receive an insurance benefit in the bad state of the world. Furthermore, they are members of a group which is characterized by an income distribution and a group-specific risk distribution. This implies that groups can be ranked according to the average risk of its members. We incorporate into our analysis a Liberal insurance system reflecting an actuarial fair private insurance, a Beveridgean system involving redistribution for the whole society and a Bismarckian system comprising redistribution between high and low risk individuals within a group. In a two stage model, first, the system of insurance is decided and second, the level of the tax rate determined. The choice of the tax rate and the choice of the system are determined according to a positive criterion, then compared to a normative one.

In the following we show that by majority voting, the Liberal system wins if the inequality of income is low, and the Beveridgean system wins if the inequality of income is high. Employing a utilitarian criterion, the Beveridgean system dominates both Bismarckian and Liberal systems. However, if we compare the Bismarckian and Liberal systems, the Bismarckian one might dominate if agents are very heterogenous in risks inside each group.

This paper is organized as follows: Section 2 introduces the model. In Section 3 we analyze the pairwise preferences of individuals and determine the type of welfare system chosen by majority voting. In section 4 we analyze the outcome of a utilitarian social planner and compare the results of both criteria. We conclude in Section 5.

2 The model

The society is divided into groups which are denoted by $k = 1, \ldots, M$ and there are N_k members per Group k.[8] There are N agents in the society with $N = \sum_{k=1}^{M} N_k$. An agent i of Group k has an income w_i and a risk p_i to lose this income. A high level of p_i implies that agent i is risky in terms of bad health or unemployment, for instance. Each group k is characterized by a specific distribution function of risk f_k. To concentrate our analysis on the heterogeneity of the distribution of risk, we suppose that the distribution function of income g is similar in each group. Moreover, for the sake of the readability of our results, we assume that the distribution of incomes and risks are independent. Therefore, groups are heterogeneous with respect to the risks but homogeneous with respect to the income distribution. We describe the distribution of income and risk in the following in more detail.

2.1 Distributions of income and risk

The distribution of income for each group is represented by the probability density function g defined on $[w_{inf}; w_{sup}]$ with average income $\overline{w} = \int wg\,(w)\,dw$. The function g is positively skewed such that median income w_m is lower than average income $\overline{w}$. Income levels can then be ranked as $0 \leq w_{inf} \leq w_m \leq \overline{w} \leq w_{sup}$.

The distribution of risk depends on the group k and implies a group-specific risk probability density function f_k defined on $[p_{inf}; p_{sup}]$. This function f_k is positively skewed, as well, and produces a particular intra-group average risk $\overline{p}_k$, where $\overline{p}_k = \frac{1}{N_k} \int pf_k(p)dp$ and $N_k = \int f_k(p)dp$ with $f_k \geq 0$. Let f be the risk probability density function of the whole society, i.e., $f = \sum_{k=1}^{M} f_k$. For M groups, we normalize $N = 1 = \int f$. The average risk in the whole society is $\overline{p} = \int pf(p)dp$.

We assume that the intra-group average risks are ranked as

$$\overline{p}_1 < \overline{p}_2 < \overline{p}_3 < \ldots < \overline{p}_M \tag{1}$$

In addition, we postulate that $p_{m,k} = p_m$ for every k, i.e., the median risk of each group $p_{m,k}$ corresponds to the median risk in society p_m, even if the distribution of risk inside each group is different.

How can we justify these two assumptions? It is clear that there is a majority of low-risk people in each group. It is reasonable to assume

[8] These groups could be professional groups (e.g., the service sector, the agricultural sector, the industrial sector etc.) or other types of groups.

that the groups are mainly differentiated by the distribution of their high-risk people. This implies that the groups have different average risks $\overline{p}_k$ (i.e., $\overline{p}_1 < ... < \overline{p}_M$), but approximatively similar median risks $p_{m,k}$ (i.e., $p_{m,k} = p_m$ for every k).

Finally, based on the positive skewness of function f_k, we postulate in the following of the paper

$$\forall k,\, p_m < \overline{p}_k$$

which implies $p_m < \overline{p}$.

The ranking of risks can then be summarized as $0 \leq p_{inf} \leq p_m \leq \overline{p} \leq p_{sup} \leq 1$. In particular, notice that we have median risk p_m is lower than average risk $\overline{p}$. In the following, we will present an empirical justification for the relationship between median risk and average risk.

2.2 Empirical evidence

It is a well-known and stylized fact for income distributions in many developed countries that these distributions exhibit positive skewness, see, e.g., Neal and Rosen (2000).[9] However, the positive skewness of risk distribution that we employed here can only be approximated by empirical evidence since individual risk is not observable, of course. To establish the positive skewness of the risk distribution we can refer to the same line of argument as before: We need to have a majority of low-risk and a minority of high-risk people in each group. As in our model risk refers to the probability of having to rely on (social) insurance benefits due to unemployment, occupational injury or illness we provide for each of these risk factors an empirically observable proxy in the following:

For unemployment we compare median and average duration of unemployment using data from OECD countries for 2000-2010.[10]

We find that the proportion of countries where median duration of unemployment is clearly smaller than average duration is substantial for the whole time period, see Figure 1. However, the proportion of countries where the relationship between median and average duration remains unclear (as both location parameters fall in the same range) is not negligible. But since the average duration of unemployment is a careful calculation of our own when OECD data was not available, average duration will de facto in many

[9]Early contributions to the literature analyzing functional forms of earnings capacities are Staele (1942), Miller (1955), or Harrison (1981).

[10]For reasons of comparability across all OECD countries we chose unemployment rates of male work force.

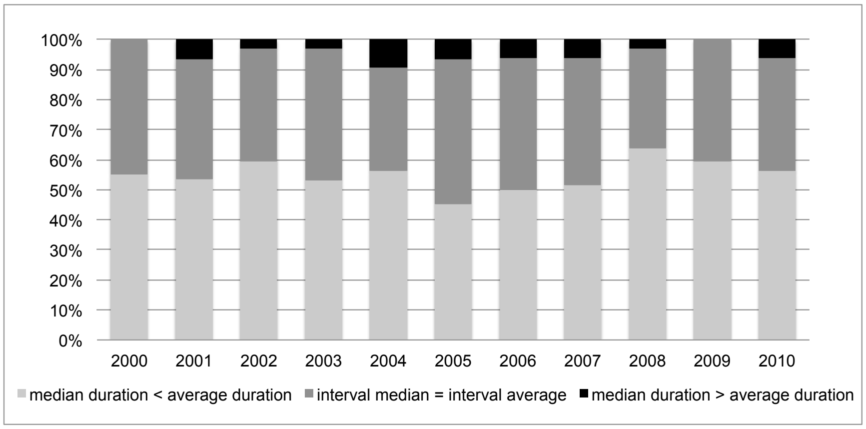

Figure 1: Median vs. average duration of unemployment in OECD countries, proportions of countries relative to all OECD countries, 2000-2010, male work force.

Source: OECD Labour Market Statistics, own calculations.

cases be higher than median duration. Overall, less than 5% of total observations exhibit a reverse relationship with average risk higher than median risk.[11]

For illness our basic hypothesis is that people being affected by chronic health problems or disability bear a higher risk of having to rely on insurance payments. Since these people constitute a minority in society, average risk will be lower than median risk. Indeed, data from OECD (2010b) shows that the self-assessed prevalence of chronic health problems or disability is lower than 15% on OECD average for the whole working age population. Even for age group 50-64, the proportion of people with self-assessed chronic health problems or disability is lower than 25% on average and only for few countries a little higher than 30%. Given a minority of people bearing a high risk due to chronic health problems and disabilities, the majority of people has quite a low risk.

Finally, for occupational injury the risk of accidents is clearly depending on the employment sector. Data from the European Labour Force Survey in 2007 shows that even for highly risky employment sectors, such as construc-

[11] Data from OECD (2010a). Estimation of median and average duration of unemployment and calculations of average duration of unemployment are available from the authors upon request.

tion, manufacturing or agriculture, the percentage of workers who report one or more work accident per year is less than 5.5% (construction, men).[12] In summary, the distribution of incidences of occupational injury is such a majority of people is not affected, while on average the risk is higher due to few people being really affected.

2.3 The three systems

The agent i earns with probability $(1-p_i)$ an income w_i which is subject to a payroll tax t, such that $(1-t)w_i$ is his net of tax income. With probability p_i the agent receives a social insurance benefit b_i which is in case of a Beveridgean system (BE) identical for all agents $b_i = b^{BE}$. In case of a Bismarckian system (BI), the social benefit b_i is proportional to the individual income but the coefficient of proportionality is identical for all agents inside his group k, i.e., $b_i = b_k^{BI}(w_i) = c_k \cdot w_i$. Finally, in case of a "Liberal" system (L), benefits that an agent receives in the bad state of the world are actuarially computed, based on both his risk p_i and the wage w_i that he would receive in the good state of the world, i.e., $b_i = b^L(p_i, w_i)$. None redistribution occurs in this last system.

Hence, under the Liberal system, the budget constraint for each agent i is given by $(1-p_i)\, tw_i = p_i b^L(p_i, w_i)$ which immediately implies

$$b^L(p_i, w_i) = \frac{1-p_i}{p_i} tw_i$$

Under the Bismarckian system, the budget constraint in Group k is
$\frac{1}{N_k}\int\int((1-p)tw)\, f_k(p)g(w)\, dpdw = \frac{1}{N_k}\int\int pb_k^{BI}(w)\, f_k(p)g(w)\, dpdw$
and since $b_k^{BI}(w_i) = c_k \cdot w_i$ it implies
$\frac{1}{N_k}\int\int((1-p)tw)\, f_k(p)g(w)\, dpdw = \frac{1}{N_k}\int\int pc_k w f_k(p)g(w)\, dpdw$
thus $c_k = \frac{1-\overline{p}_k}{\overline{p}_k}t$, and finally

$$b_k^{BI}(w_i) = \frac{1-\overline{p}_k}{\overline{p}_k} tw_i$$

Lastly, under the Beveridgean system, the social insurance budget constraint satisfies the identity $\int\int((1-p)tw)f(p)g(w)\, dpdw = \int\int pb^{BE}f(p)g(w)\, dpdw$, which implies

$$b^{BE} = \frac{1-\overline{p}}{\overline{p}} t\overline{w}$$

with $\overline{p} = \int pf(p)dp$ since $N = 1$.

[12] cf. Eurostat (2009)

The welfare function under the Beveridgean system, for an individual i of risk p_i if the tax rate is t, is now:

$$W^{BE}(t,p_i,w_i) = (1-p_i)U((1-t)w_i) + p_i U\left(\frac{1-\overline{p}}{\overline{p}}t\overline{w}\right)$$

Analogously, the group-specific welfare function for the Bismarckian system for a member i of Group k, is

$$W_k^{BI}(t,p_i,w_i) = (1-p_i)U((1-t)w_i) + p_i U\left(\frac{1-\overline{p}_k}{\overline{p}_k}tw_i\right)$$

and the welfare function of an agent i under the Liberal system is:

$$W^{L}(t,p_{i,}w_i) = (1-p_i)U((1-t)w_i) + p_i U\left(\frac{1-p_i}{p_i}tw_i\right)$$

We aim to determine the preferred system according to two alternative criteria, one positive, majority voting, and one normative, utilitarian criterion. In both cases, the timing of decisions is as follows: In a first stage, the welfare system is chosen. In a second stage, the level of the tax rate is chosen, according to the studied criterion. We will solve these games by backward induction.

For the sake of simplicity we specify the utility function to be $U(x) = \ln x$.

3 Majority voting

3.1 Choice of the tax rate

Maximizing the level of the welfare of a given agent i with respect to the tax rate t_i yields the same preferred tax rate under the three systems:

$$t_i^* = p_i \tag{2}$$

The preferred tax rate does not depend on the income. Moreover, since agents are differentiated by their risk p_i, their preferences are single peaked with respect to the tax rate. As a result, according to the majority rule, the tax rates that are chosen in both the Beveridgean and Bismarckian systems are those preferred by the median voter, i.e.:

$$t_{BE}^* = t_m^* = p_m$$
$$t_{BI}^* = t_{m,k}^* = p_{m,k}$$

Since all groups have approximately similar median risks $p_{m,k}$ (i.e., $p_{m,k} = p_m$ for any k), the tax rate chosen by majority voting corresponds to the choice of the society's median agent and is the same in both the Beveridgean and Bismarckian systems

$$t^*_{BE} = t^*_{BI} = t^*_m = p_m$$

In the Liberal system the choice of the tax rate is made independently by each agent and corresponds to his personal level of risk[13]

$$t^*_L = t^*_i = p_i$$

Incorporating the chosen tax rates in the welfare functions gives:

$$W^{BE}(t^*_m, p_i, w_i) = (1 - p_i)\ln((1 - p_m)\, w_i) + p_i \ln\left(\frac{1 - \overline{p}}{\overline{p}} p_m \overline{w}\right) \tag{3}$$

$$W^{BI}_k(t^*_m, p_i, w_i) = (1 - p_i)\ln((1 - p_m)\, w_i) + p_i \ln\left(\frac{1 - \overline{p}_k}{\overline{p}_k} p_m w_i\right) \tag{4}$$

$$\begin{aligned} W^L(t^*_i, p_i, w_i) &= (1 - p_i)\ln((1 - p_i)\, w_i) + p_i \ln\left(\frac{1 - p_i}{p_i} p_i w_i\right) \\ &= \ln\left((1 - p_i) w_i\right) \end{aligned} \tag{5}$$

3.2 Individual preferences on the system

Before determining the system that would be chosen by majority voting, we need to study individual preferences for the systems by each agent using the tax rates we have just determined. We focus on a pairwise comparison of the three systems to have a complete ranking of the systems for each agent.

3.2.1 Bismarck or Beveridge?

We start by comparing the Beveridgean and Bismarckian systems with the tax rates obtained by majority voting.

Proposition 1.

Agent i of Group k prefers a Beveridgean system to a Bismarckian one iff $w_i < r_k \overline{w}$, where $r_k = \frac{1-\overline{p}}{\overline{p}} \frac{\overline{p}_k}{1-\overline{p}_k}$ is an increasing function of $\overline{p}_k$, and thus of k.

This agent prefers the Bismarckian system iff $w_i > r_k \overline{w}$

[13] For the sake of simplicity we refer to the term "tax rate" also in case of a Liberal system. "Contribution rate" would be a more precise terminology.

Proof. From Equations (3) and (4)

$$\begin{array}{rcl} W^{BE}(t_m^*, p_i, w_i) & > & W_k^{BI}(t_m^*, p_i, w_i) \\ & \Longleftrightarrow & \\ p_i \ln\left(\dfrac{1-\overline{p}}{\overline{p}} p_m \overline{w}\right) & > & p_i \ln\left(\dfrac{1-\overline{p}_k}{\overline{p}_k} p_m w_i\right) \end{array}$$

which is equivalent to

$$\frac{1-\overline{p}}{\overline{p}}\overline{w} > \frac{1-\overline{p}_k}{\overline{p}_k} w_i$$

i.e., equivalent to $w_i < r_k \overline{w}$, where r_k is clearly an increasing function of $\overline{p}_k$, and $\overline{p}_k$ is an increasing function of k according to Inequality (1). □

Note that the coefficient r_k is a measure of average risk in group k, $\overline{p}_k$, relative to the average risk of society, $\overline{p}$. If the average risk in Group k coincides with the society's average risk, then r_k is equal to 1. If the average risk in group k is lower (higher) than the society's average risk, then r_k is strictly smaller (larger) than 1. Since r_k is an increasing function of k, we can write:

$$r_1 < r_2 < \ldots < r_j < 1 < r_{j+1} < \ldots < r_M$$

With the usual way of modeling the Bismarckian system, there is only one group, i.e., $M = 1$. In this case, $\overline{p}_1 = \overline{p}$ so that $r_1 = 1$. It immediately implies that an agent of income w_i prefers a Beveridgean system if $w_i < \overline{w}$ and a Bismarckian if $w_i > \overline{w}$. It is a particular case of our Proposition 1.

With the more realistic way of modeling the Bismarckian system that we adopt here, the Bismarckian system is particularly interesting for agents who belong to low-risk groups, i.e., to Group k with k low. Agents who bear a high risk benefit from a group with a low mean risk because of the intra-group horizontal redistribution.

Both the Beveridgean and the Bismarckian systems imply horizontal redistribution (i.e., from low risks to high risks agents), but the only system with vertical redistribution (i.e., from rich to poor agents) is the Beveridgean one. Then, poor agents prefer Beveridge to Bismarck.

An individual i prefers a Beveridgean system if his income w_i is such that $w_i < r_k \overline{w}$, as shown in Figure 1. In each group, there may be a proportion of agents who prefer the Beveridgean system and another that prefer the Bismarckian one. The proportion of agents who prefer the Beveridgean system is increasing with the average risk of the group. As a consequence, according to the ranking of the $\overline{p}_k$, the proportion of agents who prefer a

Beveridgean system is the lowest in Group 1 and the highest in Group M. Note that the individual choice of the system only depends on the group the individual belongs to, and on his individual income w_i, but not on his individual risk p_i. Finally, every agent of Group k prefers a Beveridgean system if

$$w_{sup} < r_k \overline{w}$$

which is more likely to be true for high k (that is for a high average risk), whereas every agent prefers a Bismarckian system if

$$w_{inf} > r_k \overline{w}$$

which is more likely to be true for low k (that is for a low average risk).

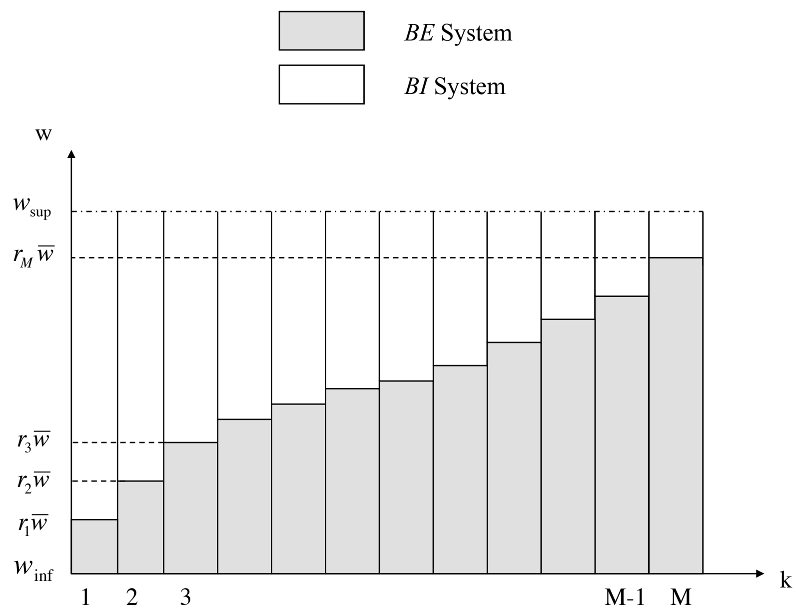

Figure 2: Individual preferences: BE or BI?

3.2.2 Bismarck or Liberal?

Now we compare the Bismarckian and Liberal systems. An agent i of Group k prefers a Bismarckian system if

$$W_k^{BI}(t_m^*, p_i, w_i) > W^L(t_i^*, p_i, w_i)$$

which leads to the following proposition:

Proposition 2.

Agent i of Group k prefers a Bismarckian system to a Liberal one iff $p_i > \widehat{p}_k$, where $\widehat{p}_k$ only depends on the group of the agent, and is an increasing function of k.

This agent prefers the Liberal system iff $p_i < \widehat{p}_k$

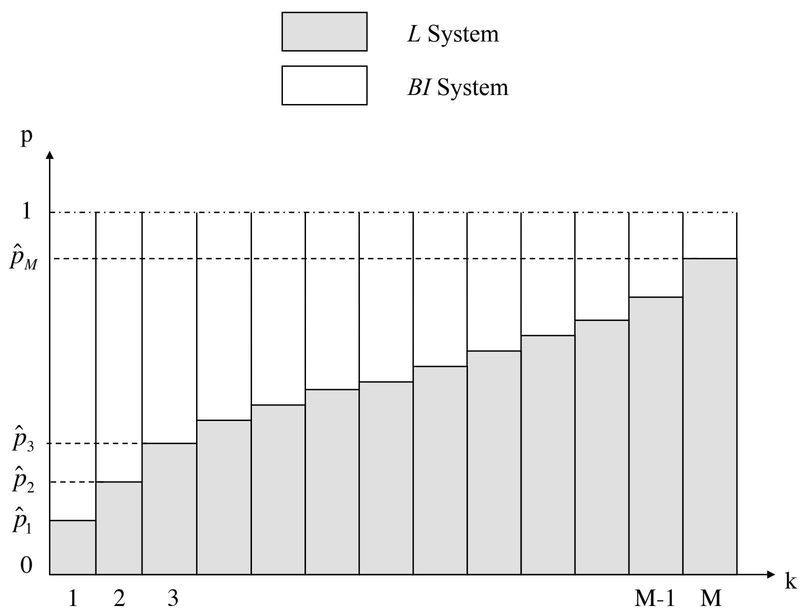

Figure 3: Individual preferences: L or BI?

Proof. See Appendix A. □

Note that the choice between L and BI does not depend on the income earned by the agent in the good state of the world, because in both systems there is no vertical redistribution.

A Bismarckian system implies intra-group horizontal redistribution (i.e., between low risk to high risk agents) in opposition to the Liberal system. As a result, the high risk people prefer the Bismarckian system to the Liberal one. For a given agent of risk p_i, the Bismarckian system is more interesting if the other agents of the group are low risk. If k is low (i.e., $\overline{p}_k$ low), Group k is a very low risk group. It is then interesting to have a Bismarckian system for an agent of this group, as it appears in Figure 2.

3.2.3 Beveridge or Liberal?

Now we compare the Beveridgean and Liberal systems. An agent i prefers the Beveridgean system iff

$$W^{BE}(t_m^*, p_i, w_i) > W^L(t_i^*, p_i, w_i)$$

For given p_m, $\overline{p}$ and $\overline{w}$, we define

$$H(p_i, w_i) = W^{BE}(t_m^*, p_i, w_i) - W^L(t_i^*, p_i, w_i) \tag{6}$$

which depends both on the distribution of risks and on the income of the agent relatively to the average income.

The study of Function H leads to the following proposition:

Proposition 3.

Agent i prefers a Beveridgean system to a Liberal one iff $w_i < \widehat{w}(p_i)$ *where* $\widehat{w}(p_i)$ *is an increasing function of* p_i, *with* $\widehat{w}(0) = 0$ *and* $\widehat{w}(1) = +\infty$

This agent prefers the Liberal system iff $w_i > \widehat{w}(p_i)$

Proof. See Appendix B. □

Figure 4 presents the partition of the population between those who prefer a Liberal system and those who prefer a Beveridgean system. The preference depends both on the income and the risk supported by the agent. The curve representing the function $\widehat{w}$ characterizes the boundary between both regimes. Therefore, a combination of income and risk on the boundary makes the agent indifferent between both regimes.

An agent i of income w_i and risk p_i prefers a Beveridgean system against a Liberal one iff $H(p_i, w_i) > 0$, where H (defined in (6)) is an increasing function of p_i, and a decreasing function of w_i. Agents with a sufficiently high income and relatively low risk prefer the Liberal system. Agents with a sufficiently low income but relatively high risk prefer the Beveridgean system because they benefit from vertical redistribution.

A Beveridgean system implies both global horizontal and vertical (i.e., from rich to poor agents) redistribution. Both high risk and/or poor agents have an incentive to choose a Beveridgean system to benefit from redistribution. Conversely, low risk and/or rich agents benefit more from supporting their preferred tax rate. In addition, these agents do not benefit from redistribution. Therefore, they are in favor of a Liberal system.

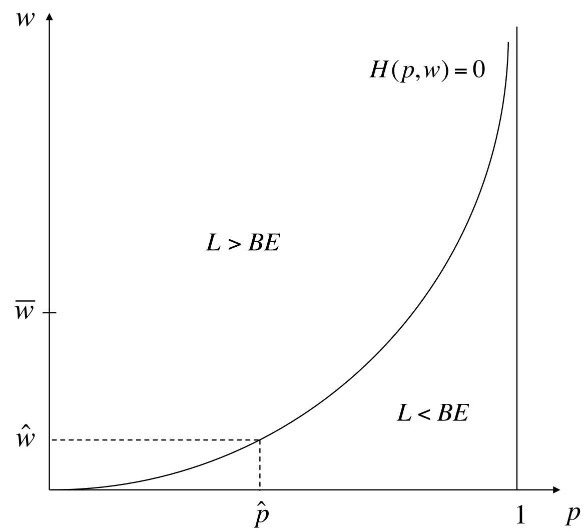

Figure 4: Individual preferences: L or BE?

3.2.4 Summary of individual preferences

Overall, there are three types of redistributive mechanisms which essentially determine individual preferences. They are summarized in Table 1.

Redistributive mechanism	**effective in**	**not effective in**
Vertical redistribution	BE	BI and L
Global horizontal redistribution	BE	BI and L
Intra-group horizontal redistribution	BI	BE and L

Table 1: Summary of redistributive mechanisms

Figure 5 gives an overview of the partition functions for individual preferences for a given Group k. First, the Beveridgean system is clearly preferred by agents who are characterized by a combination of very low income and very high risk. However, Beveridge is also preferred by poor agents who support a relatively small risk if the "income effect" of a high vertical redistribution dominates.

Second, the Liberal system is preferred by agents with a low risk, from the "quite rich" to the very rich agents because low risk agents are against horizontal and rich agents are against vertical redistribution. However, a Liberal system is also preferred by poor agents who have a really low risk:

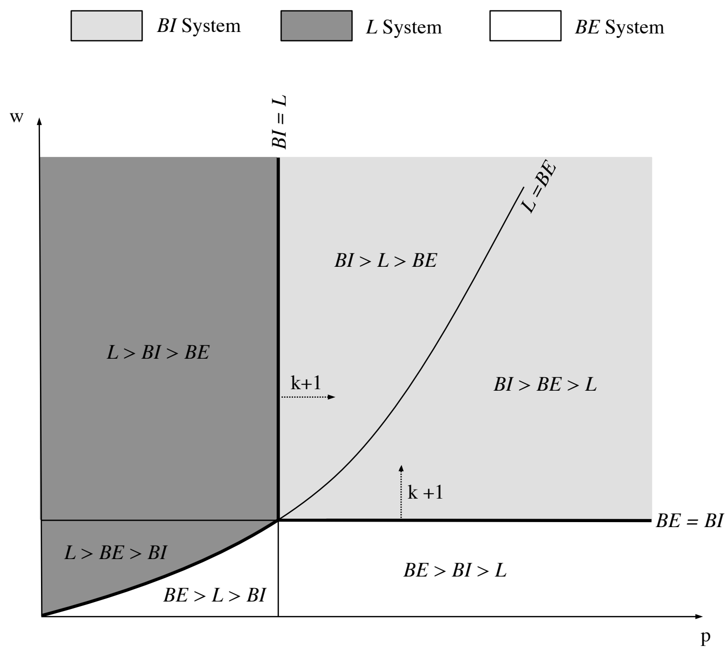

Figure 5: Overview of partition functions for individual preferences

If this "low risk effect" dominates the income effect from vertical redistribution inside the Beveridgean system, then also these agents prefer Liberal to Beveridge. The additional advantage of a Liberal system is that the tax rate is not chosen by a decision-maker, but is the one preferred by the agent.

Third, agents are in favor of a Bismarckian system if they are sufficiently rich and have a level of risk beyond a certain threshold since the Bismarckian system features intra-group horizontal but no vertical redistribution.

The impact of a higher group risk on the partition space of individual preferences can be seen by considering preferences of Group $k+1$. Compared to Group k, the indifference curves of agents in this group will move for $BE = BI$ upwards and for $BI = L$ to the right. This is indicated in Figure 5. As a consequence, the space where the Bismarckian system is preferred becomes smaller because a Bismarckian system is more favorable for lower group risk.

3.3 Choice of the system under majority voting

Before studying the choice of a utilitarian planner we first focus on a simple positive decision rule, majority voting. In order to avoid a Condorcet paradox, we restrict our analysis to pairwise comparisons of the choice of the systems. Note that there is no unanimity inside a group about the preferred system.

3.3.1 Bismarck or Beveridge?

In the following we define as the "poor" those agents whose income is lower than the average income ($w_i < \overline{w}$) and as the "rich" those agents who get an income higher than average ($w_i > \overline{w}$). We study the impact of a "mean preserving spread" (referred as MPS hereafter).[14] It means that rich people become richer, poor people become poorer, but the average income stays unchanged.
Recall that the indicator of risk of group k relative to society's risk, r_k, is ranked as follows: $r_1 < ... < r_j < 1 < r_{j+1} < ... < r_M$

Proposition 4.

(i) If the inequality of income is low, i.e. here, if $r_j\overline{w} < w_{\inf} < w_{\sup} < r_{j+1}\overline{w}$*, then in Groups* 1, 2, ..., j*, there is unanimity in favor of the Bismarckian system, and in Groups* $j+1$, ..., M*, there is unanimity in favor of the Beveridgean system.*

(ii) A Mean Preserving Spread (MPS) of the distribution of incomes implies a lower political support for the Bismarckian system in Groups 1, ..., j*, and a lower political support for the Beveridgean system in Groups* $j+1$, ..., M*.*

(iii) With a sufficiently large MPS, there is a majority in favor of the Beveridgean system.

Proof. See Appendix C. □

The decisive factor which determines the type of system is income inequality.

An interpretation of this Proposition is as follows:

(i) If the inequality of income is low, the impact of vertical redistribution can be neglected. In this case, comparing BI and BE systems means that two different types of horizontal redistribution are compared: Intra-group horizontal redistribution in the BI system and global horizontal redistribution in the BE system. The intra-group horizontal redistribution is more favorable for groups $1, ..., j$ because for them $\overline{p}_k < \overline{p}$ holds true. Therefore, each member in these groups prefers a BI system.

[14]Note that a MPS is equivalent to second-order stochastic dominance, this is well defined in Mas-Colell et al. (1995), chapter 6.D.

The reverse is true for groups $j+1, ..., M$: they benefit more from global horizontal redistribution since $\overline{p} < \overline{p}_k$. Consequently, all agents in these groups prefer a BE system.

(ii) Let us consider Groups $1, ..., j$ following a MPS. The rich agents of these groups will still prefer the BI system. The same is true for the "rather" poor people, because they benefit from their low intra-group risk $\overline{p}_k$ in the BI system. The poorest people become even poorer with the MPS, so that they finally prefer BE because it allows vertical redistribution.

For agents who belong to Groups $j+1, ..., M$, again, the reverse is true. With a MPS, the poor agents of these groups will still prefer BE, and also the "rather" rich people. The richest agents become even richer with the MPS, so that they finally prefer BI because it does not imply vertical redistribution.

(iii) In case of a very large inequality of income (i.e., very large MPS), the effect of vertical redistribution dominates horizontal redistribution: people with an income w_i lower than $\overline{w}$ almost all prefer BE. Then, there is a majority for BE since $w_m < \overline{w}$.

This effect is in line with the result of Meltzer and Richard (1981) which states that when the share of redistributed income is determined by majority voting, a more unequal income distribution leads to more redistribution.[15]

3.3.2 Bismarck or Liberal?

We now focus on the choice between a Bismarckian and a Liberal system. The inequality of income has no impact on the political support of the Liberal system against the Bismarckian one, because in both systems the social benefit is proportional to the income, i.e., there is no vertical redistribution.

Recall we have assumed that the median voter is the same in each group, i.e., $p_{m,k} = p_m$ for every k.

Proposition 5.

According to the majority voting criterion a Liberal system is adopted against a Bismarckian one.

[15]Roberts (1977) has shown that the median voter's preferences are decisive if voting rights are universal and the election rule is majority voting. Furthermore, it is well-known that in modern societies, the distribution of income is right skewed so that the average income exceeds median income. For instance, Brown (1977) states that the distribution of income in modern societies is well approximated by a lognormal distribution. See also Neal and Rosen (2000) for a recent update.

Proof. We can set

$$h_k(p_m) = W_k^{BI}(t_m^*, p_m, w_i) - W^L(t_i^*, p_m, w_i) = p_m \ln\left(\frac{1-\overline{p}_k}{\overline{p}_k}\frac{p_m}{1-p_m}\right)$$

Since we have assumed that $p_{m,k} = p_m$ for every k, and $p_m < \overline{p}_k$ for every k, then $h_k(p_m) < 0$.

From Proposition 1 and its proof, we are then able to state that for all agents i with $p_i \leqslant p_m$, that represent at least 50% of the voters, the Liberal system is preferred. □

The main advantage of the Liberal system is that each agent can choose his individually preferred tax rate. The main advantage of the Bismarckian system is intra-group horizontal redistribution; however, this advantage only applies to a minority of people. As a result, the Liberal system is preferred by a majority of agents.

3.3.3 Beveridge or Liberal?

Let us study now the majority choice between Beveridgean and Liberal systems.

According to Proposition 3, we know that an agent of income w_i and risk p_i prefers the Beveridgean system against the Liberal one iff $w_i < \widehat{w}(p_i)$. The function $\widehat{w}$ depends on p_m, $\overline{p}$, and $\overline{w}$. In the following proposition, we show that the political support for BE (against L) depends on the level of inequality of income in the society.

Proposition 6.

(i) If the inequality of income is low, here more precisely, if $\frac{w_{\inf}}{\overline{w}} > \eta$, where η only depends on the distribution of risks, and $0 < \eta < 1$, then a majority of agents prefers a Liberal system to a Beveridgean one.

(ii) A Mean Preserving Spread (MPS) of the distribution of income implies a higher political support for the Beveridgean system.

(iii) With a sufficiently large MPS, there is a majority in favor of the Beveridgean system.

Proof. See Appendix D. □

Again, there is a precise interpretation of this Proposition.

(i) If the inequality of income is low, vertical redistribution does not matter. In this case, BE vs. L means horizontal redistribution vs. no redistribution at all. Horizontal redistribution is in favor of poor agents which are a minority. In turn, there is a majority for the L system.

(ii) The higher inequality of income, the stronger vertical redistribution. Poor agents are in favor of vertical redistribution. Since they constitute more than 50% of the population, the support for BE increases.

(iii) With a sufficiently large inequality of income, the effect of vertical redistribution dominates, so poor agents will be in favor of BE.

3.3.4 Condorcet winner?

Taking into account the results of Propositions 4, 5 and 6 we can now try to find out if one of these welfare systems is a Condorcet winner. It is clear that the Bismarckian system can never be the Condorcet winner since this system is dominated by the Liberal one. The Liberal system is the Condorcet winner if there is a low inequality of income. The Beveridgean system is the Condorcet winner if there is a high inequality of income. For intermediate levels of income inequality there might be no Condorcet winner.

4 Utilitarian criterion

In this section, we focus on a normative criterion, analyzing the choice of a utilitarian social planner.

4.1 Choice of the tax rate

The Liberal system is characterized by total liberty of choice for any individual. Similarly to the majority voting analysis, the individually preferred tax rate is applied, so that

$$t_i^L = p_i$$

Under the Beveridgean system, the utilitarian planner determines the common optimal tax rate, t_u^{BE}, by maximizing the average of the individual welfares. Since

$$\begin{aligned} U_{BE} &= \iint W^{BE}(t_u^{BE}, p, w) f(p) g(w) dp dw \\ &= \iint (1-p)\ln((1-t_u^{BE})w) + p \ln\left(\frac{1-\overline{p}}{\overline{p}} t_u^{BE} \overline{w}\right) f(p) g(w) dp dw \\ &= (1-\overline{p})\ln(1-t_u^{BE}) + \overline{p}\ln\left(\frac{1-\overline{p}}{\overline{p}} t_u^{BE}\right) + (1-\overline{p})\overline{\ln w} + \overline{p}\ln \overline{w} \end{aligned}$$

then $\max_{t_u^{BE}} U_{BE}$ implies $t_u^{BE} = \overline{p}$.

Under the Bismarckian system, the utilitarian planner determines the optimal tax rate of Group k, $t_{u,k}^{BI}$, by maximizing the average of the individual welfares of Group k. Since

$$
\begin{aligned}
U_{BI,k} &= \iint W_k^{BI}(t_{u,k}^{BI}, p, w) f_k(p) g(w) dpdw \\
&= \iint (1-p) \ln \left(\left(1 - t_{u,k}^{BI}\right) w \right) + p \ln \left(\frac{1-\overline{p}_k}{\overline{p}_k} t_{u,k}^{BI} w \right) f(p) g(w) dpdw \\
&= (1-\overline{p}) \ln \left(1 - t_{u,k}^{BI}\right) + \sum_{k=1}^{M} \overline{p}_k \ln \left(\frac{(1-\overline{p}_k)}{\overline{p}_k} \right) N_k + \overline{p} \ln t_{u,k}^{BI} + \overline{\ln w}
\end{aligned}
$$

and maximizing $U_{BI,k}$ yields an optimal tax rate independent on the group, i.e., $t_{u,k}^{BI} = t_u^{BI} = \overline{p}$.

4.2 Choice of the system

Let us first compare the Beveridgean system with the Bismarckian system.

The Beveridgean system yields the following social welfare:

$$
\begin{aligned}
U_{BE}^* &= (1-\overline{p}) \ln(1 - t_u^{BE}) + \overline{p} \ln \left(\frac{1-\overline{p}}{\overline{p}} t_u^{BE} \right) + (1-\overline{p}) \overline{\ln w} + \overline{p} \ln \overline{w} \\
&= (1-\overline{p}) \ln(1-\overline{p}) + \overline{p} \ln \left(\frac{1-\overline{p}}{\overline{p}} \overline{p} \right) + (1-\overline{p}) \overline{\ln w} + \overline{p} \ln \overline{w} \\
&= \ln(1-\overline{p}) + (1-\overline{p}) \overline{\ln w} + \overline{p} \ln \overline{w} \qquad (7)
\end{aligned}
$$

where $\overline{\ln w}$ stands for the mean of $\ln w_i$.

The Bismarckian system produces the social welfare:

$$
\begin{aligned}
U_{BI}^* &= (1-\overline{p}) \ln \left(1 - t_u^{BI}\right) + \sum_{k=1}^{M} \overline{p}_k \ln \left(\frac{1-\overline{p}_k}{\overline{p}_k} \right) N_k + \overline{p} \ln t_u^{BI} + \overline{\ln w} \\
&= (1-\overline{p}) \ln(1-\overline{p}) + \sum_{k=1}^{M} \overline{p}_k \left(\ln \left(\frac{1-\overline{p}_k}{\overline{p}_k} \right) \right) N_k + \overline{p} \ln \overline{p} + \overline{\ln w} \qquad (8)
\end{aligned}
$$

The Liberal system gives the social welfare:

$$
\begin{aligned}
U_L &= \iint W^L(t^*; p; w) f(p) g(w) dpdw \\
&= \iint \ln ((1-p) w) f(p) g(w) dpdw \\
&= \overline{\ln w} + \overline{\ln (1-p)} \qquad (9)
\end{aligned}
$$

The comparison of the different welfare functions leads to the following propositions.

Proposition 7.
A utilitarian planner always adopts a Beveridgean system against a Bismarckian system.

Proof. Comparing (7) and (8) gives

$$U_{BE}^{*} \geq U_{BI}^{*} \Leftrightarrow \overline{p} \ln \left(\frac{1-\overline{p}}{\overline{p}} \right) + \overline{p} \ln \overline{w} \geq \sum_{k=1}^{M} \overline{p}_k \left(\ln \left(\frac{1-\overline{p}_k}{\overline{p}_k} \right) \right) N_k + \overline{p} \overline{\ln w}$$

which is true due to the Jensen inequality since functions $g(x) = x \ln \left(\frac{1-x}{x} \right)$ and $h(w) = \ln w$ are both concave functions. □

Under the utilitarian criterion, the Beveridgean system is always preferred even if the distribution of risk is strongly asymmetric in favor of low risk agents (say, 85% of agents have a risk lower than the average risk).

The concavity of the ln function implies that the high risk lose too much when a Bismarckian system is adopted compared to the loss of the low risk when a Beveridgean system is implemented.

Proposition 8.
When comparing Bismarckian and Liberal systems, for a given global distribution of risk f, the utilitarian planner adopts the Liberal system when agents are sufficiently homogeneous in risks inside each group, and the Bismarckian one when agents are sufficiently heterogeneous in risks inside each group.

Proof. Comparing (8) and (9) yields

$$\begin{aligned} U_{BI}^{*} - U_{L}^{*} &= (1-\overline{p}) \ln(1-\overline{p}) + \sum_{k=1}^{M} \overline{p}_k \ln \left(\frac{1-\overline{p}_k}{\overline{p}_k} \right) N_k + \overline{p} \ln \overline{p} + \overline{\ln w} - \overline{\ln w} - \overline{\ln(1-p)} \\ &= \ln(1-\overline{p}) - \overline{\ln(1-p)} - \left[\overline{p} \ln \left(\frac{1-\overline{p}}{\overline{p}} \right) - \sum_{k=1}^{M} \overline{p}_k \ln \left(\frac{1-\overline{p}_k}{\overline{p}_k} \right) N_k \right] \\ &= \left[\ln(1-\overline{p}) - \overline{\ln(1-p)} \right] - \left[\sum_{k=1}^{M} N_k \left(\overline{p} \ln \left(\frac{1-\overline{p}}{\overline{p}} \right) - \overline{p}_k \ln \left(\frac{1-\overline{p}_k}{\overline{p}_k} \right) \right) \right] \end{aligned}$$

The first bracket is positive, and is large if the global inequality of risk is high. The second bracket is positive, and large if the average group risks are very widespread. □

For the comparison between BI and L, income is not important because in both systems, there is no vertical redistribution. For a given f, if agents are homogenous in risks inside each group, there is very little horizontal redistribution in BI, so that BI is less interesting. Conversely, if agents are heterogenous in risks inside each group, there is a strong horizontal redistribution so that BI is more interesting than L even if the tax rate in BI is not the individually preferred one.

Proposition 9.

A utilitarian planner always adopts a Beveridgean system against a Liberal system.

Proof. Comparing (7) and (9) gives

$$U_{BE}^{*} - U_{L}^{*} = \left[\ln(1 - \overline{p}) - \overline{\ln(1 - p)}\right] + \overline{p}\left[\ln \overline{w} - \overline{\ln w}\right]$$

where each bracket is positive by concavity of the ln function. □

The BE system allows both vertical and global horizontal redistribution. Again, the concavity of the ln function implies that the high risk and/or poor agents lose too much when a Liberal system is adopted compared to the loss of the low risk and/or rich agents when a BE system is implemented.

4.3 Do the positive results meet the normative recommendations?

In order to compare the results obtained under both criteria, we need to evaluate the impact of every redistribution mechanism (vertical and horizontal) either with our positive or with our normative criterion.

A utilitarian planner is in favor of vertical redistribution since it benefits low income agents more than it hurts high income agents. This argues for BE rather than for BI or L. Similarly, the majority voting rule supports any vertical redistribution because high income agents are a minority in the society. Again, this argues for BE.

On the one hand, a utilitarian planner is also in favor of any horizontal redistribution since it benefits high risks agents more than it hurts low risk agents. This argues for BE or BI rather than L. On the other hand, the majority voting rule does not support any horizontal redistribution because high risk agents constitute a minority in the society. This argues for L.

In addition, the utilitarian planner gives priority to global horizontal redistribution compared to intra-group horizontal redistribution since the first one is a broader type of redistribution. This argues for BE against BI. The intra-group horizontal redistribution is preferred under a majority voting rule if and only if there is a majority of agents in "good groups", i.e., Groups k such that $\overline{p}_k < \overline{p}$. This last case argues for BI.

As a consequence of all these redistribution effects, the utilitarian planner prefers a more redistributive system, i.e., BE. The preferred system under a majority voting rule depends on the relative sizes of income inequality and risk inequality as well as on the proportion of agents belonging to "good groups".

More precisely:

– the lower risk inequality, the higher the political support for L,

– the higher income inequality, the higher the political support for BE,

– the higher the proportion of agents belonging to "good groups", the higher the political support for BI.

When only BI and L are compared, the utilitarian criterion leads to prefer L only if groups of agents are strongly homogeneous, because there is a low horizontal redistribution and a non-individually preferred tax rate under the BI system. This corresponds to the result with majority voting, since with $p_m < \overline{p}$ only a minority of the society benefits from horizontal redistribution.

5 Conclusion

We have studied the three main types of welfare capitalism within a simple economic model which incorporates groups. In particular, we have introduced a more accurate way to model the corporatist Bismarckian system, taking into account the fact that this system allows intra-group horizontal redistribution, as outlined by Esping-Andersen (1990).

For the choice of the welfare system using the majority voting rule, we have shown the influence of the inequality of income, inequality of risk and the group structure. Under a utilitarian criterion, the Beveridgean system is always preferred. However, if we compare the Bismarckian and the Liberal system, the last one can be adopted if individuals are very homogenous inside each group.

This paper offers preliminary results which allow to state that the main results concerning the choice of the welfare system are crucially modified under the new way of modeling the Bismarckian system. This is a first step in a research program that should be encompassed by the development of new studies incorporating this new way of modeling.

Appendix

A Proof of Proposition 2

According to (4) and (5), $W_k^{BI}(t_m^*, p_i, w_i) > W^L(t_i^*, p_i, w_i)$ is equivalent to

$$(1-p_i)\ln(1-p_m) + p_i \ln\left(\frac{1-\overline{p}_k}{\overline{p}_k} p_m\right) > \ln(1-p_i) \tag{10}$$

We set

$$\begin{aligned} h_k(p_i) &= (1-p_i)\ln(1-p_m) + p_i \ln\left(\frac{1-\overline{p}_k}{\overline{p}_k} p_m\right) - \ln(1-p_i) \\ &= \ln(1-p_m) + p_i \ln\left(\frac{1-\overline{p}_k}{\overline{p}_k} \frac{p_m}{1-p_m}\right) - \ln(1-p_i) \end{aligned}$$

We have

$$h_k(0) = \ln(1-p_m) < 0 \text{ and } h_k(1) \to +\infty$$

and

$$\begin{aligned} h_k'(p_i) &= \frac{1}{1-p_i} + \ln\left(\frac{1-\overline{p}_k}{\overline{p}_k} \frac{p_m}{1-p_m}\right); \\ \lim_{p_i \to 1} h_k'(p_i) &= +\infty \text{ and } h_k'(0) = 1 + \ln\left(\frac{1-\overline{p}_k}{\overline{p}_k} \frac{p_m}{1-p_m}\right) \\ h_k''(p_i) &= \frac{1}{(1-p_i)^2} > 0 \end{aligned}$$

According to the properties of function $h_k(p_i)$, we can distinguish two cases:

– If $h_k'(0) > 0$, then $h_k'(p_i) > 0$ on $[0,1]$ and $h_k(p_i)$ is increasing on $[0,1]$. Then there exists a unique $\widehat{p}_k$ such that $h_k(\widehat{p}_k) = 0$. Moreover,

$$h_k(p_i) > 0 \Longleftrightarrow p_i > \widehat{p}_k$$

– If $h_k'(0) < 0$, then $h_k(p_i)$ is first decreasing and then increasing on $[0,1]$. Thus there exists a unique $\widehat{p}_k$ such that $h_k(\widehat{p}_k) = 0$. Moreover,

$$h_k(p_i) > 0 \Longleftrightarrow p_i > \widehat{p}_k$$

In both cases, $\widehat{p}_k$ depends only on $\overline{p}_k$ and p_m.
In addition, $h_k(\widehat{p}_k) = \ln(1-p_m) + \widehat{p}_k \ln\left(\frac{1-\overline{p}_k}{\overline{p}_k}\frac{p_m}{1-p_m}\right) - \ln(1-\widehat{p}_k) = 0$

According to the implicit function theorem, $\frac{\partial \widehat{p}_k}{\partial \overline{p}_k} = -\left(\frac{\partial h_k}{\partial \widehat{p}_k}\right)^{-1} \times \frac{\partial h_k}{\partial \overline{p}_k} > 0$, and $\overline{p}_k$ is an increasing function of k, so that $\widehat{p}_k$ is an increasing function of k.

B Proof of Proposition 3

According to (3), (5) and (6), we have

$$\begin{aligned} H(p_i, w_i) &= W^{BE}(t_m^*, p_i, w_i) - W^L(t_i^*, p_i, w_i) \\ &= (1-p_i)\ln\left[(1-p_m)\,w_i\right] + p_i \ln\left(\frac{1-\overline{p}}{\overline{p}} p_m \overline{w}\right) - \ln\left[(1-p_i)w_i\right] \\ &= \ln(1-p_m) - \ln(1-p_i) + p_i \ln\left(\frac{1-\overline{p}}{\overline{p}} \frac{p_m}{1-p_m} \frac{\overline{w}}{w_i}\right) \end{aligned}$$

Moreover, $H(p_i, w_i) = 0 \Leftrightarrow \ln\left(\frac{1-\overline{p}}{\overline{p}} \frac{p_m}{1-p_m} \frac{\overline{w}}{w_i}\right) = \frac{1}{p_i} \ln\left(\frac{1-p_i}{1-p_m}\right)$

i.e., $H(p_i, w_i) = 0 \Leftrightarrow \frac{1-\overline{p}}{\overline{p}} \frac{p_m}{1-p_m} \frac{\overline{w}}{w_i} = \left(\frac{1-p_i}{1-p_m}\right)^{1/p_i}$

Then, an agent i prefers BE to L iff $H(p_i, w_i) \geq 0$, i.e., iff $w_i \leq \widehat{w}(p_i)$, where $\widehat{w}(p_i) = \frac{1-\overline{p}}{\overline{p}} \frac{p_m}{1-p_m} \overline{w} \left(\frac{1-p_m}{1-p_i}\right)^{1/p_i}$

Let us show that $\widehat{w}(p_i)$ is an increasing function of p_i, with $\widehat{w}(0) = 0$ and $\lim_{p \to 1} \widehat{w}(p) = +\infty$

$\widehat{w}(p_i) = C \times \exp(a(p_i))$, where $C = \frac{1-\overline{p}}{\overline{p}} \frac{p_m}{1-p_m} \overline{w} > 0$ and $a(p_i) = \frac{1}{p_i} \ln\left(\frac{1-p_m}{1-p_i}\right)$

Clearly, we can write that $a(0) = \lim_{p_i \to 0} a(p_i) = -\infty$, and $a(1) = \lim_{p_i \to 1} a(p_i) = +\infty$, thus $\widehat{w}(0) = 0$ and $\widehat{w}(1) = +\infty$

We just have to show that $a(p_i)$ is an increasing function of p_i.

$$\begin{aligned} a(p_i) &= \frac{1}{p_i} \ln(1-p_m) - \frac{1}{p_i} \ln(1-p_i) \\ a'(p_i) &= -\frac{1}{p_i^2} \ln(1-p_m) + \frac{1}{p_i^2} \ln(1-p_i) + \frac{1}{p_i(1-p_i)} \\ p_i^2 a'(p_i) &= -\ln(1-p_m) + \ln(1-p_i) + \frac{p_i}{(1-p_i)} \\ &= \quad -1 - \ln(1-p_m) + \ln(1-p_i) + \frac{1}{(1-p_i)} \end{aligned}$$

Let us show that $p_i^2 a'(p_i) \geq 0$ for any $p_i \in [0;1]$.

We set $b(p_i) = p_i^2 a'(p_i) = -1 - \ln(1-p_m) + \ln(1-p_i) + \frac{1}{(1-p_i)}$ where $b'(p_i) = \frac{-1}{1-p_i} + \frac{1}{(1-p_i)^2} = \frac{p_i}{(1-p_i)^2} > 0$ and $b(0) = -\ln(1-p_m) > 0$

Thus $b(p_i) > 0$ on $p_i \in [0;1]$, so that $p_i^2 a'(p_i) > 0$ and $a(p_i)$ is an increasing function of p_i. $\square$

C Proof of Proposition 4

(i) According to Proposition 1, an agent i of Group k prefers BE to BI iff $w_i < r_k\overline{w}$ where $r_1 < ... < r_j < 1 < r_{j+1} < ... < r_M$.

By assumption, $r_j\overline{w} < w_{\text{inf}}$, then for any agent i of Group k with $k \leq j$, we have $w_i \geq w_{\text{inf}} > r_j\overline{w} \geq r_k\overline{w}$. Thus, there is unanimity in favor of BI in Groups 1, 2, ..., j.

Similarly, by assumption, $w_{\text{sup}} < r_{j+1}\overline{w}$, then for any agent i of Group k with $k \geq j+1$, we have $w_i \leq w_{\text{sup}} < r_{j+1}\overline{w} \leq r_k\overline{w}$. Thus there is unanimity in favor of BE in Groups $j+1$, ..., M.

(ii) Impact of a MPS.

- For $k \leq j$, the agent i prefers BE iff $w_i < r_k\overline{w}$, where $r_k < 1$.

  With a MPS, the proportion of people with $w_i < r_k\overline{w}$ increases, so that the political support for BE increases.

- For $k \geq j+1$, the agent i prefers BI iff $w_i > r_k\overline{w}$, where $r_k > 1$.

  With a MPS, the proportion of people with $w_i > r_k\overline{w}$ increases, so that the political support for BI increases.

(iii) Impact of a sufficiently large MPS.

- For $k \leq j$, with a large MPS, the proportion of people of income $w_i \in [r_k\overline{w}; \overline{w}]$ becomes very small, so that the proportion in favor of BE becomes arbitrarily near that of people of income $w_i < \overline{w}$.

- For $k \geq j+1$, with a large MPS, the proportion of people of income $w_i \in [\overline{w}; r_k\overline{w}]$ becomes very small, so that the proportion in favor of BI becomes arbitrarily near that of people of income $w_i > \overline{w}$.

Finally, whatever the group, if the MPS is sufficiently large, then the proportion of people in favor of BE is arbitrarily close to the proportion of people of income $w_i < \overline{w}$. Since the median income w_m. is lower than $\overline{w}$, we can conclude that with a sufficiently large MPS, there is a majority of people in favor of BE against BI.

D Proof of Proposition 6

(i) For any agent i of risk p_i and income w_i:

$$\begin{aligned} H(p_i, w_i) &= W^{BE}(t_m^*, p_i, w_i) - W^L(t_i^*, p_i, w_i) \\ &= \ln(1-p_m) - \ln(1-p_i) + p_i \ln\left(\frac{1-\overline{p}}{\overline{p}} \frac{p_m}{1-p_m}\right) + p_i \ln\left(\frac{\overline{w}}{w_i}\right) \\ &= \widetilde{h}(p_i) + p_i \ln\left(\frac{\overline{w}}{w_i}\right) \end{aligned}$$

where $\widetilde{h}(p_i) = \ln(1-p_m) - \ln(1-p_i) + p_i \ln\left(\frac{1-\overline{p}}{\overline{p}} \frac{p_m}{1-p_m}\right)$
For an individual of income $\overline{w}$ and risk p_i, $\widetilde{h}(p_i)$ is the difference of welfares under BE and L.
$\widetilde{h}''(p_i) = \frac{1}{(1-p_i)^2} > 0$, and $\widetilde{h}(0) = \ln(1-p_m) < 0$, and $\widetilde{h}(p_m) = p_m \ln\left(\frac{1-\overline{p}}{\overline{p}} \frac{p_m}{1-p_m}\right) < 0$ because $p_m < \overline{p}$.
$\widetilde{h}$ is a convex function with $\widetilde{h}(0) < 0$ and $\widetilde{h}(p_m) < 0$, thus $\widetilde{h}(p_i) < 0$ for all $p_i \leq p_m$. $\widetilde{h}$ is a continuous function, then $\max_{0\leq p\leq p_m} \widetilde{h}(p) < 0$.
Setting $\eta = \exp\left[\max_{0\leq p\leq p_m} \widetilde{h}(p)\right]$, we have then $0 < \eta < 1$. By assumption, $\eta < \frac{w_{\inf}}{\overline{w}}$, thus $\max_{0\leq p\leq p_m} \widetilde{h}(p) < \ln\left(\frac{w_{\inf}}{\overline{w}}\right)$
For every p_i, with $p_i \leq p_m$, we have $H(p_i, w_i) = \widetilde{h}(p_i) + p_i \ln\left(\frac{\overline{w}}{w_i}\right) \leq \max_{0\leq p\leq p_m} \widetilde{h}(p) + \ln\left(\frac{\overline{w}}{w_{\inf}}\right) < 0$
Then, any agent i such that $p_i \leq p_m$ prefers L to BE, i.e., a majority of people are in favor of L.

(ii) A MPS implies that $\ln\left(\frac{\overline{w}}{w}\right)$ increases for a majority of people because $w_m < \overline{w}$, thus it increases the political support for the Beveridgean system.

(iii) With a sufficiently large MPS of the distribution of income, we have $p_i \ln\left(\frac{\overline{w}}{w_i}\right) \geq p_i \ln\left(\frac{\overline{w}}{w_m}\right) > -\widetilde{h}(p)$ for 50% of the population. Then, clearly $H(p_i, w_i) > 0$ for a majority of people. $\square$

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