# Fighting Collaborative Tax Evasion

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**Abstract**

We build a model where a buyer negotiates a price discount with a seller in exchange for not asking the receipt and paying in cash, allowing tax evasion. Sellers and buyers are heterogeneous with respect to their tax morale and to their cost of managing payment instruments different from cash. We study how a tax rebate for the buyer and a tax on cash withdrawals affect tax evasion and government revenue. We found that an appropriate mix of these two instruments can reduce tax evasion while, at the same time, raising additional revenue. We discuss the implementation problems of the tax on cash withdrawals and we suggest how to overcome them.

**JEL**: O17, H21

**Keywords**: collaborative tax evasion; cash tax

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# 1 Introduction

The first law and economic model of tax evasion by Allingham and Sandmo (1972) explains evasion as a result of a cost benefit analysis by perfectly rational sellers of goods and services. They choose to evade if the expected cost of the sanction, given the auditing probability, is lower than the tax payments. A great deal of economic literature followed their pioneering work (Sirinvasan 1973, Yitzhaki 1974, Baldry 1979, Marrelli 1984, Reinganum and Wilde 1985, Usher 1986, Marrelli and Martina 1988, Andreoni 1992, among others), adding many elements to their baseline framework. What is in general missing in the literature, besides few exceptions, is the role of the buyer: asking for a receipt of the transaction makes tax evasion more difficult, while paying cash and not asking for a receipt facilitates evasion, since there is no proof of the transaction. Since buyers have the power to facilitate tax evasion, it is plausible that many sellers will try to induce a cooperative behavior from them, for instance offering a price discount. When the two parties reach an agreement, the so called "Collaborative tax evasion" takes place.

Our goal is to identify the optimal policies to curb this collaborative tax evasion while, at the same time, raising additional tax revenue. Besides the tax rate, we consider two policy instruments: a tax deduction for the buyer that keep the receipts of the transactions and a tax on cash withdrawals (TCW henceforth).

The tax deduction, or tax credit, is a standard instrument, implemented in many tax codes around the world and sometimes very creatively: in Taiwan, China, Puerto Rico and in the city of Sao Paulo, for instance, the receipt can be used to claim lottery tickets (Marchese 2009, Fabbri 2013). The purpose of tax deductions is to reduce evasion by rewarding honest taxpayers rather than punishing dishonest ones and many studies suggested that this strategy can be effective[1] (Among others, Alm, McClelland, and Schulze 1992 and Berhan and Jerkins 2005.) The TCW, conversely, is a rather new instrument. Its purpose is obviously to curb the incentives of the buyer to cooperate with the seller[2]: if evasion can be perpetrated only in case of cash transactions, then making the use of cash more costly should induce less cooperation. We are aware of only

[1]Frey (1997) also points out that implementing punishing schemes might actually crowd out the intrinsic motivation to pay taxes and can also result in more tax evasion. This result is consistent with experimental evidence that suggests a drop in cooperation when fines for deviations are introduced (Scholtz and Lubell 2001).

[2]There exist also some literature against the effectiveness of policy instruments that reduce the incentives to cooperate on tax evasion, like Santoro (2006).

two countries that implemented[3] this tax, Pakistan in 2001 and India from 2005 to 2009 (the so called Banking Cash Transaction Tax or BCCT). In both cases, however, the official reason for the introduction of the tax was not a reduction of tax evasion nor the collection of additional tax revenue. Instead the tax was supposed to provide more information for the tax enforcing authorities to better guide the audits[4]. Here we provide a throughout analysis of the effects of the TCW and of its interactions with the tax deductions. Differently from India and Pakistan, however, we focus on the role of these instruments as tools to both reduce tax evasion and increase government revenue.

We build a model where price taking sellers (small vendors, family businesses or independent professionals) choose how much of their aggregate tax liability to report and where customers decide whether to facilitate tax evasion, accepting a price discount for not asking a receipt of the transaction. If the seller chooses to evade, he enters in a bargaining round with the customer. If a deal is reached, then the customer receives a discount and the transaction is completed without receipt. In this case the transaction must be completed in cash, since credit or debit cards, bank transfers, checks and other payment instruments leave a paper trial. If there is no deal, there is no discount for the customer and the seller issues the receipt. In this case the buyer is not constrained to use cash. Sellers and buyers are heterogeneous with respect to their honesty, or tax morale, and with respect to their cost of managing payment instruments different from cash. In the baseline model, we assume that the identity of buyers and sellers is public information and we use the Nash bargaining solution. Prior to the evasion decision by the seller and before the bargaining game, a legislator, whose objectives are to fight tax evasion and to maximize revenue, commits to a tax rate, to a tax rebate and to a TCW rate.

Although the model is highly stylized along many dimensions, it is still not possible to completely solve it analytically. Therefore we proceed as follows. First, we derive analytical results for the effects of the policy instruments on tax evasion for which we have an explicit solution. For the effects of the policy instruments on government revenue and for the optimal policy, we resort to a numerical solution. We start calibrating the model to a fictitious "prototype economy ",

[3] Recently a team of Italian journalists lead by Milena Gabanelli proposed the introduction of the cash tax in Italy through a TV program called "Report".

[4] Indeed, the tax in India was abolished in 2009 exactly claiming its scarce contribution once more sophisticated information collection technologies were implemented.

choosing both the value of the parameters and the calibration targets in a rather arbitrary way. We choose this approach for two reasons: first because we want to study the effect of the policy instruments in general, to highlight the principles that should guide the anti evasion policy for a large set of countries. For the same reason, we also make an effort to study the robustness of all our results to a wide range of alternative parameterizations and calibrations. Second, because it is difficult to identify the exact value of some of the key economic variables that we need in order to credibly calibrate the model to isolate a specific policy for a country.

We show that a small tax deduction can be effective at both reducing tax evasion and increasing government revenue and that the deduction rate must be higher the higher the prevailing tax evasion rate in the country and the higher the tax rate. We also show that the TCW can actually increase evasion in economies or sectors of the economy where the use of cash is high. The reason is that, for individuals with high costs of using alternative paying instruments, taxing cash raises the threat value in the bargaining process, inducing more rather than less evasion. Obviously the higher the TCW, the smaller the percentage of individuals that will use cash. We show that the first effect prevails for a small rate of the TCW, while the second for high rates. Therefore we conclude that taxing cash is effective at reducing evasion and increasing revenue only if the rate is high enough, and the TCW rate must be higher the larger the mass of individuals with high costs of using alternative paying instruments.

We then isolate the optimal policies to maximize the government revenue while, at the same time, keeping evasion below a low threshold value. Overall, the main result of the analysis is that, with an appropriate mix of cash taxes and tax rebates, it is possible to eliminate tax evasion and, at the same time, raising a substantial amount of additional tax revenue. We also provide an example of the optimal policy for a real world country, Italy.

The main problem with our optimal policy, however, is that the TCW is not easy to implement. On the one hand, the TCW can foster the emergence of a parallel cash economy (Gordon 1995, Morse et al. 2005), significantly reducing its incidence: firms and consumers can use whatever cash they have for the transactions, bypassing the banking system. Besides the reduced incidence of the tax, this will have important side effects for the entire economy, as the financial intermediation improves the functioning of the payment system fostering investments and growth. In addition, there is the possibility of a bank run at the moment of the announcement of the tax and before its introduction. Moreover, since banks typically charge a fee for credit card use to both sellers and buyers, this can translate in a loss of profits for the sellers and in a loss of purchasing power for the buyers. Finally, the TCW should be implemented in all the countries that issue the same currency and, to avoid arbitrage, the rate should be the equal or, at least, not very different. We propose a throughout discussion of these issues, suggesting how to overcome the implementation problems.

While the symmetric information assumption between seller and buyer might be sensible for repeated interactions, for spot transactions it is more reasonable to assume asymmetric information. We explore also this possibility by modeling the bargaining process as a double auction à la Chatterje and Samuelson (1983). Overall, the results are very similar.

The rest of the paper is organized as follows. Section 2 briefly summarizes the related economic literature. Section 3 describes the model and the analytical results. Section 4.1 illustrates the model calibration for the prototype economy. Section 4.2 reports the comparative static results for the prototype economy and its robustness. Section 4.3 discusses the optimal policy for the prototype economy and its robustness. In Section 5 we discuss the implementation problems of the TCW and propose how to overcome them. Section 6 summarizes the (very similar) results obtained under the assumption of asymmetric information. Section 7 offers some concluding remarks. Appendix A specializes the results of Section 4.3 for Italy and Appendix B provides the analytical solution of a simplified version of the model under complete and incomplete information.

## 2 Related literature

The paper follows quite abundant economic literature on tax evasion. We do not include a complete review of the works on tax evasion since there already are very good and exhausting ones in the literature (Andreoni, Erard and Feinstein 1998, Slemrod and Yitzhaki 2002, Cowell 2004, Marchese 2004, Sandmo 2005, Slemrod 2007 and Franzoni 2008). We limit ourselves to a brief account of the few works that specifically tackled collaborative tax evasion.

The first work on collaborative tax evasion is Gordon (1990), that suggests that the under-the-counter cash sales at a discount price, on which the seller evades taxes, serve also as a means of price discrimination. Namely the receipt of the transactions is the basis on which all the after sale services are based, including the possibility of returning a defective item and the possibility of suing the seller. Customers who are not interested in this insurance, for various reasons, are willing to pay a lower price. The second work is Boadway, Marceau and Mongrain (2002), that model evasion as a collusion between two taxpayers, the buyer and the seller. They assume that cooperative tax evasion efforts can reduce the detection probability more than individual efforts, which gives an incentive to cooperation, and analyze the possibility of punishment schemes for the non cooperating parties. They show that tax evasion might increase after an increase in sanctions because it leads to a larger reduction in the deviation payoff. The third work is Chang and Lai (2004), that models collaborative tax evasion as a bargaining game between a seller and a buyer. The authors are interested in the role of social norms (Akerlof 1980) in shaping the incentives of the agents: evading taxes induces psychological costs, associated to the feelings of guilt, shame etc., but these costs are higher if there is a higher social sanction of evasion, that is, the lower the tax evasion rate. They also show that, contrary to the standard tax evasion model, if the economy is in a bad equilibrium with widespread evasion, a more severe fine or enforcement can actually induce more tax evasion: a higher expected fines raises the gains from trade and, given the low social sanction, evasion increases.

Differently from these previous works on collaborative tax evasion, in this paper we take a normative perspective and we focus on different policy instruments than enforcement and fines.

The paper is also related to the literature on the optimal inflation tax (Friedman 1969; Phelps 1973; Chamley 1985; Woodford 1990, among others). Like the inflation tax, the TCW reduces the purchasing power of the consumers. Unlike the inflation tax, however, it is selective, in the sense that it reduces the purchasing power only for the goods that are paid in cash. Thus its incidence is lower the bigger the possibility of using the electronic currencies and the lower the cost associated to credit and debit card use for the customers. An additional point is that there is a further cost from the introduction of the TCW, which we do not consider in the paper: the reduction of the seignorage revenue. Although this source of revenue is only of limited importance for low inflation, advanced economies, it can be important for developing economies which are not able to manage a tax collection system or that have high rates of irregular activity. Nicolini (1998) and Koreshkova (2006) already discussed the role of the inflation tax as a way to raise revenue from tax evaders or businesses that operate in the underground sector. Differently from their work, in this paper we consider how to raise revenue from tax evaders by reducing the tax evasion rate.

## 3 The Model

We propose a game-theoretic approach to tax compliance where buyers, sellers and the government interact strategically. Specifically, the economy is composed by a unit measure of price taking, risk neutral, sellers, a unit measure of risk neutral customers and by the government. We model a single transaction between a seller and a buyer about a given good or service. The government implements a tax and enforces the law with random audits resulting in the payment of a fine for those who are caught evading. The seller can evade taxes by not issuing the receipt failing to report the transactions to the tax authority. Since we are interested in collaborative tax evasion, we assume that, in case there is a receipt (non cooperative buyer), it is impossible for the seller to evade taxes. We also assume that the buyer has the right to ask for a receipt and that the seller cannot refuse to provide one. In this setting, which is indeed similar to what happens in many real world situations (doctors, contractors, plumbers, shops etc.), a negotiation between the seller and the buyer is likely: the seller may offer a price discount to the buyer in exchange for not providing the receipt, allowing tax evasion. This setting has been called in the literature collaborative tax evasion to stress that the buyer's cooperation is essential to evade taxes. If the buyer cooperates we assume that the good is paid in cash since electronic payments leave a trace that will impair the possibility of evading taxes.

The sellers are heterogeneous with respect to their tax morale: honest sellers will always issue the receipt, while less honest sellers will always bargain with the buyers. The buyers are heterogenous along two dimensions. The first is tax morale, as for the sellers. Honest buyers will always ask for a receipt, preventing the buyers from evading, while less honest buyers will always bargain. We consider tax morale in our model since, as noted by Gordon (1989), Andreoni, Erard and Feinstein (1998) and Feld and Frey (2002), it is almost impossible to account for the observed compliance rate just with expected penalties and risk aversion.[5]

[5] According to Gordon (1990), there is also another reason, beyond honesty, for which the buyers need a receipt: to have a formal guarantee on the product, so that, for instance, they can return a defective item. Following this argument, more risk averse consumer should ask for a receipt more often. In what follows, we will ignore this feature, mainly because, as narrative evidence suggests, the sellers typically offer the same kind of customer service even when the fail to provide the receipt (in fact this is part of the argument they use to convince the buyers to go

The second is the cost associated to managing payment instruments different from cash. Some individuals, like the elderly or the less financially educated, might find it cumbersome to use a credit card, while others might find it easy.

The government, whose goal is to maximize the revenue from taxation minus the cost imposed on honest people who bear the cost of managing payment instruments different from cash, announces and commits to a policy $\mathcal{P} = \{t, \tau, \vartheta\}$. A policy is made up of a tax rate $t$ and two instruments to fight tax evasion: a tax on cash withdrawals $\vartheta$ and a tax deductions for the buyer that keep the receipts of the transactions $\tau$, both of which are designed to reduce the buyers and sellers incentives to cooperate. After having observed the policy $\mathcal{P}$ and the other relevant parameters of the economy such as the probability of auditing, the fine, etc. one buyer and one seller are randomly matched and play the following game. First, the seller decides the amount to evade (possibly zero) and the buyer the preferred means of payment cash or electronic payment; then we utilize the Nash bargaining solution to solve the negotiation between buyer and seller over the price discount.

In the next sections we describe in detail the case where in the bargaining round buyers and sellers have perfect information about each other type. This setting provides an analytical solution for the equilibrium tax evasion and for the government revenue that allows us to highlight the main trade offs of the Government policies.

## 3.1 Sellers and Buyers

The utility of the seller in case of tax evasion ($e$), which requires cooperation from the buyer, is the following:

$$\begin{aligned} v_s^e &= (1-\pi)\left[p(1-t)+te-d-\upsilon\right]+\pi[p(1-t)-d-fte-\upsilon] \\ &= p(1-t)+te\left[1-\pi\left(1+f\right)\right]-d-\upsilon \end{aligned}$$

where $p$ is the price of the good (taken as given by the seller), $t$ is the tax rate, $d$ is the discount bargained with the buyer, $\pi$ is the audit probability, $f$ is the fine and $\upsilon$ is the individual cost of

without receipt).

tax evasion representing differences in innate honesty between sellers. Thus in case of no audit, with probability $1-\pi$, the seller earns the evaded amount $te$ minus the discount, while in case of audit, he is forced to pay the full amount of taxes plus a fine which is computed on the amount evaded $fte$. We follow Yitzhaki (1974) setting a penalty on the evaded tax rather than on the evaded amount as in Allingham and Sandmo (1972). In addition, we simplify the model assuming that the random audit, and the resulting penalty, are contemporaneous with the evasion decision while, in the real world, penalties are assessed later (Andreoni 1992). The distribution of the cost of tax evasion, which is higher the higher honesty, is characterized by the pdf $g_\upsilon$ and by the cdf $G_\upsilon$. If the buyer and the seller do not reach a deal in the bargaining round[6] or if the seller does not engage in tax evasion at all, the utility is simply equal to $v_s^0 = p(1-t)$. Comparing $v_s^0$ with $v_s^e$ we notice that the cost of cheating is $d+\upsilon$ while the benefit is the evaded amount minus the expected sanction. To make the analysis interesting we assume that $1-\pi(1+f)>0$ so that a trade off exist. This assumption implies a limit to the audit probability $\pi$ and to the fine $f$.

The utility of a buyer in case of cooperation is

$$\begin{aligned} v_b^e &= (1-\pi)[u-(p-d)(1+\vartheta)] + \pi[u-(p-d)(1+\vartheta)] - s \\ &= u-(p-d)(1+\vartheta) - s \qquad (1) \end{aligned}$$

where $u$ is the utility from purchasing the good, $\vartheta$ is the tax on currency withdrawals and $s$ is the cost of tax evasion or tax morale, which is characterized by the pdf $g_s$ and by the cdf $G_S$. Since to evade the tax the transaction must be paid in cash, the buyer has to pay $\vartheta$ on the negotiated effective amount of the transaction $p-d$.

If the buyer does not cooperate, he must still decide whether to use cash for the transaction or an electronic means of payment (Card). In the former case the utility for the buyer is

$$v_b^0(cash) = u - p(1+\vartheta-\tau) \qquad (2)$$

Instead if he uses a card the utility becomes

[6] Even if there are gains from collaborative tax evasion, bargaining might fail in the incomplete information case studied in the appendix.

$$v_b^0(card) = u - p(1-\tau) - c \tag{3}$$

where $\tau$ is the tax rebate and $c \in [0, p]$ is the cost associated to payment with traceable instruments such as cheques or credit cards. As it will become clear, the behavior of those individuals who benefit from the use of cards (negative $c$) is identical to the one of those who have $c = 0$. We denote by $g_c$ the pdf and by $G_c$ the cdf of $c$. We also assume that the distribution of $c$ is independent from the distribution of $s$. Then, if the buyer decides not to cooperate with the seller and asks for a receipt, he receives a tax rebate on the full amount of the transaction $p$ and being free to choose among different payment instruments, he will choose the one with the lower cost. More formally, cash will be preferred to card if and only if $c \geq p\vartheta$. Then, from now on we denote the utility of a buyer who does not cooperate with

$$v_b^0 = u - p(1-\tau) - \min\{p\vartheta, c\}.$$

The tax rebate and the TCW affect the buyer's incentive to cooperate rather than the terms of the gamble faced by the tax evader. Moreover, both instruments are independent from the probability that the illicit transaction is discovered. This implies that, while risk aversion is crucial in the analysis of the equilibrium effects of fines and enforcement, it is just marginal here.

We model the negotiation between the seller and the buyer over the price discount through Nash bargaining. The solution for the bargaining under perfect information is defined as

$$d^* = \arg\max_d \left(v_s^e - v_s^0\right)^\beta \left(v_b^e - v_b^0\right)^{1-\beta}$$
$$s.t. \quad v_s^e \geq v_s^0 \,,\; v_b^e \geq v_b^0$$

where $\beta$ is the bargaining power of the seller. The solution for the optimal discount is

$$d^*(\upsilon, s, c) = \beta \frac{p(\tau+\vartheta) + s - \min\{p\vartheta, c\}}{1+\vartheta} + (1-\beta)\left[et(1-\pi(1+f)) - \upsilon\right] \tag{4}$$

for all $\upsilon$ such that $v_s^e \geq v_s^0$, i.e,

$$\upsilon \leq et(1-\pi(1+f)) - d^*(\upsilon, s, c) \tag{5}$$

and for all the couples $s$ and $c$ such that $\upsilon_b^e = \upsilon_b^0$, i.e.,

$$s \leq d^*(\upsilon, s, c)(1+\vartheta) - p(\tau+\vartheta) + \min\{p\vartheta, c\}. \tag{6}$$

The optimal discount is zero in case conditions (5) and (6) do not hold. If the seller finds an agrement with the buyer his utility $\upsilon_s^e = p(1-t) + te\left[1-\pi\left(1+f\right)\right] - d^*(\upsilon, s, c) - \upsilon$ is increasing in the amount evaded since the positive direct effect of an increase in the amount evaded $t\left[1-\pi\left(1+f\right)\right]$ is larger than the indirect negative effect due to the increase in the discount that the seller must concede to the buyer $(1-\beta)\, t\left[1-\pi\left(1+f\right)\right]$, so that $e = p$.[7]

By plugging the optimal discount (4) into (6) we find

$$s \leq (1+\vartheta)\left\{et[1-\pi\left(1+f\right)] - \upsilon - \frac{p(\tau+\vartheta) - \min\{p\vartheta, c\}}{1+\vartheta}\right\}. \tag{7}$$

## 3.2 Tax evasion

We then use condition (7) to compute the equilibrium level of tax evasion. First, consider the buyers for which $c < p\vartheta$ to obtain a threshold value $\tilde{s}_1(\upsilon, c)$ such that all the buyers of type $c < p\vartheta$ with an innate honesty lower than $\tilde{s}_1(\upsilon, c)$ will cooperate. Next, define with a $\tilde{\upsilon}_1$ such that $\tilde{s}_1(\tilde{\upsilon}_1, c) = 0$, the seller honesty level that makes no buyer eager to cooperate. Doing the same for $c > p\vartheta$ we obtain a second threshold $\tilde{s}_2(\upsilon)$ which does not depend on $c$ and such that all the buyers of type $c > p\vartheta$ with an innate honesty lower than $\tilde{s}_2(\upsilon)$ will cooperate. Finally, we define a $\tilde{\upsilon}_2$ such that $\tilde{s}_2(\tilde{\upsilon}_2) = 0$ and we get the following expression for total tax evasion $E$ :[8]

$$E = \int_0^{p\vartheta}\left[\int_0^{\tilde{\upsilon}_1}\int_0^{\tilde{s}_1(\upsilon,c)} g_s\, ds\, g_\upsilon\, d\upsilon\right] g_c dc + [1-G(p\vartheta)]\int_0^{\tilde{\upsilon}_2}\int_0^{\tilde{s}_2(\upsilon)} g_s\, ds\, g_\upsilon\, d\upsilon. \tag{8}$$

Since fighting tax evasion is an important intermediate objective and a good in itself, we are interested in the effect of the government policy $\mathcal{P} = \{t, \tau, \vartheta\}$ on the total amount of tax evasion $E$. Of course the simplest way to have zero tax evasion is to have no taxation at all. However, a

[7] Schweizer (1983) and Cremer and Gahvari (1993, 1999), assume that the revenue concealed from the tax authorities by the seller entails an extra cost thus obtaining an interior solution for the amount evaded. We do not include such a cost since we lack the necessary data to assess the increase in expenditure needed to conceal more revenue.

[8] By plugging the expression (4) into (5) instead of (6) we find exactly the same results.

State to function needs a minimum amount of resources and, as shown by the following derivative, the amount of tax evasion is increasing in the tax $t$. This is in contrast to the basic Allingham and Sandmo model where an increase in the tax rate would reduce evasion. But perfectly in accordance with most empirical literature which has found that a higher tax rate generally leads to less compliance (Clotfelter, 1983; Crane and Nourzad, 1992; Alm, 2012):

$$\begin{aligned}\frac{\partial E}{\partial t} &= \int_0^{p\vartheta}\left[\int_0^{\tilde{\upsilon}_1} g(\tilde{s}_1(\upsilon,c))p[1-\pi\,(1+f)](1+\vartheta)g_\upsilon\, d\upsilon\right]g_c\, dc + \\ &\quad [1-G(p\vartheta)]\left[\int_0^{\tilde{\upsilon}_2} g(\tilde{s}_2(\upsilon))p[1-\pi\,(1+f)]g_\upsilon\, d\upsilon\right].\end{aligned}$$

We now investigate what is the effect of a tax on cash withdrawals $\vartheta$ and a tax deductions for the buyer who keeps the receipts of the transaction $\tau$. Those two instruments should help to reduce the buyers and sellers incentives to cooperate. The intuition is shown to be correct for a tax deduction by the following derivative:

$$\begin{aligned}\frac{\partial E}{\partial \tau} &= \int_0^{p\vartheta}\left[\int_0^{\tilde{\upsilon}_1} -\frac{pg(\tilde{s}_1(\upsilon,c))}{1+\vartheta}g_\upsilon\, d\upsilon\right]g_c\, dc + \\ &\quad [1-G(p\vartheta)]\left[\int_0^{\tilde{\upsilon}_2} -\frac{pg(\tilde{s}_2(\upsilon))}{1+\vartheta}g_\upsilon\, d\upsilon\right].\end{aligned}$$

The amount of tax evasion is indeed decreasing in the buyer's deduction $\tau$. However, as shown by the next derivative, the effect of the second instrument on the total amount of evasion is more complex:

$$\begin{aligned}\frac{\partial E}{\partial \vartheta} &= \int_0^{p\vartheta}\left[\int_0^{\tilde{\upsilon}_1} g(\tilde{s}_1(\upsilon,c))\frac{\partial \tilde{s}_1(\upsilon,c)}{\partial \vartheta}g_\upsilon\, d\upsilon\right]g_c dc + \\ &\quad [1-G(p\vartheta)]\int_0^{\tilde{\upsilon}_2}\left[\int_0^{\tilde{s}_2(\upsilon)} g(\tilde{s}_2(\upsilon))\frac{\partial \tilde{s}_2(\upsilon)}{\partial \vartheta}\right]g_\upsilon\, d\upsilon,\end{aligned}$$

where

$$\frac{\partial \tilde{s}_1(\upsilon,c)}{\partial \vartheta} = pt(1-\pi\,(1+f)) - \upsilon - p < 0,$$

while

$$\frac{\partial \tilde{s}_2(\upsilon)}{\partial \vartheta} = p\tau[1 - \pi(1+f)] - \upsilon > 0.$$

Indeed, the amount of tax evasion can be both increasing or decreasing in the tax on cash withdrawals $\vartheta$. The threshold $\tilde{s}_1(\upsilon, c)$ such that all the buyers of type $c < p\vartheta$ with an innate honesty lower than $\tilde{s}_1(\upsilon, c)$ will cooperate is decreasing in $\vartheta$ while the threshold $\tilde{s}_2(\upsilon)$ for types $c > p\vartheta$ is increasing. The reason is that if the buyer does not cooperate, he must still decide whether to use cash for the transaction or an electronic means of payment and cash will be preferred to card if and only if $c \geq p\vartheta$. Then, for high $c$ ($c \geq p\vartheta$) buyers prefer to use cash even if they do not want to cooperate with the seller to avoid the cost related to electronic payments. This implies that there is no extra cost for cooperation in terms of tax on cash and this makes cooperation relatively more interesting. A buyer who cooperates pays $\vartheta$ over $(p - d)$ (see 1) while not cooperating the cash needed for the transaction over which $\vartheta$ must be paid is the full amount $p$. Conversely, for low $c$ ($c < p\vartheta$) buyers who do not intend to cooperate with the seller will prefer to bear the cost $c$ and use electronic means of payment. Then, an increase in the tax on cash will make cooperating relatively more costly. Finally, since the derivative of tax evasion with respect to a tax on cash withdrawals is positive for low values of $\vartheta$ and negative for high values, an increase in $\vartheta$ is more likely to decrease tax evasion the larger is $\vartheta$. In other words, a tax on cash withdrawals is an effective tool to fight tax evasion only if it is set sufficiently high.[9]

We can summarize the previous analysis in the following proposition:

**Proposition 1.** *A higher tax rate leads to less compliance while a higher tax deduction increases compliance. Finally, a tax on cash withdrawals is an effective tool to fight tax evasion only if set sufficiently high.*

It is interesting to notice that a decrease in $c$ has the same effect of an increase in $\vartheta$. This implies that an alternative policy to reduce tax evasion entails decreasing the cost associated to managing payment instruments different from cash. The main difference between the two alternatives is on the government revenue: the tax on cash gives an extra revenue, while decreasing $c$ is costly

[9]Notice that for $\vartheta = 0$ $\frac{\partial E}{\partial \vartheta} = \int_0^{\tilde{\upsilon}_2} \left[ \int_0^{\tilde{s}_2(\upsilon)} g(\tilde{s}_2(\upsilon)) \frac{\partial \tilde{s}_2(\upsilon)}{\partial \vartheta} \right] g_\upsilon \, d\upsilon > 0$ while for $\vartheta = 1$ $\frac{\partial E}{\partial \vartheta} = \int_0^{p} \left[ \int_0^{\tilde{\upsilon}_1} g(\tilde{s}_1(\upsilon, c)) \frac{\partial \tilde{s}_1(\upsilon, c)}{\partial \vartheta} g_\upsilon \, d\upsilon \right] < 0$.

and, therefore, infeasible for financially constrained governments. However there is a further difference, as the cost $c$ cannot be reduced for all individuals, even in case of very high government expenditures: part of the cost is made up by the fees that the banks charge and it can be easily eliminated by the government with a subsidy; the remaining part is made up of psychological and cognitive costs, which are very hard to reduce.

## 3.3 Net Government Revenue

Next, we are interested in the effect of the government policy $\mathcal{P} = \{t, \tau, \vartheta\}$ on the revenue from taxation minus the socially inefficient cost imposed to those who unwillingly manage payment instruments different from cash. The expression for the government revenue is

$$\int_0^{p\vartheta} \left[ [p\pi t(1+f) + (p - d^*(\upsilon, s, c))\vartheta] \left[ \int_0^{\tilde{\upsilon}_1} \int_0^{\tilde{s}_1(\upsilon,c)} g_s ds g_\upsilon d\upsilon \right] + p(t-\tau) \left[ 1 - \int_0^{\bar{\upsilon}_1} \int_0^{\widehat{s}_1(\upsilon,c)} g_s ds g_\upsilon d\upsilon \right] \right] g_c dc$$
$$+ [1 - G(p\vartheta)] \left\{ [p\pi t(1+f) + (p - d^*(\upsilon, s))\vartheta] \left[ \int_0^{\tilde{\upsilon}_2} \int_0^{\widetilde{s}_2(\upsilon)} g_s ds g_\upsilon d\upsilon \right] + p(t - \tau + \vartheta) \left[ 1 - \int_0^{\tilde{\upsilon}_2} \int_0^{\widetilde{s}_2(\upsilon)} g_s ds g_\upsilon d\upsilon \right] \right\}$$

The first line represents buyers with low cost of managing electronic means of payment ($c < p\vartheta$). Those buyers are either matched with sellers whose innate honesty leads to cooperation to evade taxes (first term) or not (second term). When buyers and sellers collaborate to evade taxes, in case of audit the seller is forced to pay the full amount of taxes plus a fine which is computed on the amount evaded that is $tp\pi\,(1 + f)$. Moreover, since to evade the tax the transaction must be paid in cash, the buyer has to pay $\vartheta$ on the negotiated effective amount of the transaction $p - d$. When instead the matching does not lead to tax evasion the revenue for the government is the tax for the seller net of the rebate for the buyer $p(t - \tau)$.

The second line represents buyers with high cost of managing electronic means of payment ($c > p\vartheta$). In this case when buyers and sellers collaborate to evade taxes (first term) the government cashes in exactly the same amount $p\pi t(1 + f) + (p - d^*)\vartheta$. Conversely, when the matching does not lead to tax evasion (second term) the revenue is $p(t-\tau+\vartheta)$ since the government collects the tax on cash also on those honest buyers who prefer to use cash because of their high cost of using electronic means of payment. For those people the tax represents a pure transfer to the State and should be reimbursed if we want to leave their purchasing power unchanged.

Introducing a tax on cash withdrawals to fight tax evasion imposes a cost on those honest buyers who opt for an electronic mean of payment to avoid the tax $\vartheta$. This cost, which is not a pure transfer, is equal to

$$\int_0^{p\vartheta} c\left[1 - \int_0^{\tilde{v}_1} \int_0^{\widehat{s}_1(v,c)} g_s ds g_v dv\right] g_c dc,$$

and must be accounted for when designing the policy. If a policy forces some individuals to leave cash for a different payment instrument, this is undoubtedly a cost for society and, since $c$ is measured in monetary equivalents, it is possible to subtract it from the government revenue objective. Finally notice that a mass of buyers with negative cost from using electronic means of payment would not affect this expression.

# 4 Numerical Analysis

We now consider an empirically plausible distributions for tax morale and for the cost of managing payment instruments different from cash. Since we do not have an analytical solution, we resort to a numerical solution of a calibrated version of the model.

In the next section we summarize the parametrization and calibration which define what we call the "Prototype Economy". We then proceed by illustrating the results for this fictitious economy. We have two reasons that motivate this approach. First it is difficult to identify, with a reasonable amount of confidence, the exact value of some of the relevant empirical measures, like the current level of tax evasion, that we need to calibrate the model precisely. This forces us to consider a wide range of parameters. Second, we do not want to focus exclusively on a specific country, but rather to provide the widest possible perspective on these issues.

In Section 4.2 we highlight the comparative statics, including the robustness to different, alternative, parameterizations and calibrations.

In Section 4.3 we identify the optimal policy. We consider the net government revenue as a policy objective, preferring it to the gross government revenue and to a measure of welfare (Cowell 1990, Sandmo 2005). The reason why we subtract the cost of credit cards from the total revenue is because, as already stressed, the Government, by imposing a tax on cash $\vartheta$, forces some individuals, who do not cooperate with tax evaders, to bear a cost. We exclude the welfare measure for two reasons: first, it would entail choosing whether to include or not the utility of tax evaders and cooperating buyers; second, in our model, taxation is a waste of resources since there is no public good financed with tax receipts and enjoyed by the agents. In such a context, including tax evaders might lead to strange policy outcomes: indeed facilitating evasion will improve the situation of evaders and force non evaders to evade. Including a public good in the model, on the other hand, would obscure the functioning of the basic mechanisms. Moreover, we lack the necessary data to assess how tax revenue is transformed in the public good.

In Appendix A we also propose an example of the optimal policy for a real world country, Italy, a country where tax evasion is widespread and unanimously considered as one of the biggest national problems.

In all the exercises we assume that the government can freely use all the instruments that we consider and that there are no administrative costs or side effects that must be accounted for. In other words, we abstract from problems associated with the implementation of the policies. While this might be innocuous for the tax rate and the tax rebates, which are typical in almost all countries, it is certainly not so for the tax on cash, which is far more challenging to implement. Since the higher the rate of the tax on cash the higher its implementation problems, we also discuss the optimal policy in case the maximum feasible cash tax rate is 5%.

## 4.1 The Prototype Economy

We start the numerical analysis by choosing a set of parameters and calibration targets that define the baseline, fictitious, prototype economy.

For the tax rate we consider two different choices associated with two different model interpretations. On the one hand, we can interpret the tax rate as the (marginal) tax rate on the firms revenues or profits (which in our simple model coincide). Another possibility is interpreting the tax as a sale tax. Yet another possibility is an interpretation as a value added tax, in which case the price $p$ must be interpreted as the value added of the transaction instead of the transaction price. Our benchmark choices are, respectively, $t = 0.3$ for the income/profit interpretation and $t = 0.1$ for the sale tax/vat interpretation.

For enforcement probability and fine we choose $\pi = 0.01$ and $f = 0.5$ respectively. In the model, we admittedly take a shortcut by assuming a random and constant probability that does not depend on the seller's characteristics and on the evaded amount. It is certainly true that a big firm that evades 90% of its profits faces a higher audit probability than a small, less visible, business that seldom evades a small 10% (Yitzhaki 1987). We also abstract from congestion effects in law enforcement (Galbiati and Zanella 2012), which imply that, for a fixed amount of government resources devoted to enforcement, the individual audit probability decreases the higher the number of individuals that evade. This is the reason why we perform some robustness test on the probability, which can be also interpreted in terms of the size and characteristics of the firm, with higher probabilities corresponding to bigger, more visible, businesses or past evaders. We do not consider the auditing probability as a policy instruments since we do not really have a cost of enforcement in the model and there is no clear way of introducing one[10].

Since there is only one country (Pakistan) with a tax on cash withdrawals, we set $\vartheta = 0$. Next, we normalize the cost of using cash to zero. Provided that the only individuals who do not use cash if $\vartheta = 0$ are the individuals with $c = 0$ (or negative), we need a probability distribution with a mass in 0. Moreover we need to make an assumption about the shape of the distribution: we consider an exponentially declining probability mass for higher cost values representing a society in which most of the individuals have a zero or a very small cost of using payment instruments different from cash, but where a small mass of individuals has a high cost (think about the elderly, for instance). The distribution that we use is the following mixture:

$$g_c(x, \lambda) = \begin{cases} 0 & \text{Prob}\,\lambda \\ \lambda e^{-x\,\lambda} & \text{Prob}\,1-\lambda. \end{cases}$$

In the prototype economy we set $\lambda = 0.2$, or 20% of the population that does not use cash for the transactions. Of course we test the robustness of the model results to alternative assumptions about the cost $c$, in particular to the possibility of a different (higher) value of $\lambda$.

We set $\beta = 0.5$ since we have no particular reason to assign a higher bargaining power to

[10]Reinganum and Wilde (1985) highlight the optimal auditing rule of the tax authority. Slemrod, Blumenthal, and Christian (2001) and Kleven et al. (2011) study the effects of the threat of enforcement on reported income using field experiments.

the buyer or to the seller, but we discuss the robustness of the model results. The values of $p$ and $u$ are just scalings, and do not affect the main findings. We set $p = 10$ and $u = 1.5\,p$. Of course if $u$ is not sufficiently higher than $p$, there is the possibility that an effect of the policy is discouraging the buyer from buying the good, but we rule out this possibility by choosing a high value of $u$. In practice, the number of individuals that uses alternative payment instruments is bigger the bigger is $p$. Ideally we need a model where the cost $c$ is decreasing in $p$, but this is impractical from an empirical perspective, given that we don't have the detailed information needed to parameterize a whole function. Nevertheless, by considering different levels of $\lambda$, we can implicitly take into account this variability: a higher (lower) $\lambda$ is typical of sectors with higher (lower) average transactions values. Therefore the robustness of the comparative static results to different values of lambda must be interpreted as robustness across different sectors of the economy with different transaction values. Notice that, since the price $p$ is just a scaling, we do not change it when we perform this robustness tests (changing it will only deliver different calibrated parameters but exactly the same results).

For the distribution of tax morale, we consider an extremely versatile distribution that assigns values in an interval, the Kumaraswamy distribution, which is essentially a BETA distribution with a different parametrization. The pdf is the following:

$$g(x; a, b, \bar{x}) = \frac{ab}{\bar{x}}\left(\frac{x}{\bar{x}}\right)^{a-1}\left[1-\left(\frac{x}{\bar{x}}\right)^{a}\right]^{b-1} \qquad 0 < x < \bar{x} \tag{9}$$

depending on the value of the parameters, we can have an increasing pdf with most of the probability mass corresponding to high values of tax morale, a decreasing pdf, where the opposite is true, or a peak corresponding to intermediate values. We consider the same distribution of tax morale for both the buyer and the seller ($\bar{s} = \bar{v}$), since there is no theoretical or empirical reason to believe that the sellers (for instance) are, a priori, more honest than the buyers.[11] To choose the value of the parameters of the distribution we use data from the World Value Survey (WVS henceforth). This survey is part of an ongoing Worldwide research project whose goal is to

[11] Obviously we can argue that the profession choice is also dictated by the different opportunities to evade taxes, so that less honest individuals, more prone to tax evasion because of moral considerations, typically choose to work where it is easier to evade taxes (Pestieau and Possen 1991). Nevertheless, since there is no robust empirical evidence that confirms these speculations (Parker 2003), we decided to abstract from these issues.

compare several aspects of culture among different countries. Among the questions administered to a significant number of individuals in several different countries, there is one specifically related to tax morale, namely "Do you consider justifiable cheating on taxes?" Respondents are asked to pick a number between 0 and 10, where 0 means always justifiable while 10 never justifiable.

The core of the calibration procedure is choosing the parameters $a$, $b$, together with upper bound $\bar{s} = \bar{v}$ to match the empirical shape of the distribution of the answers and to match the observed level of tax evasion. We run a simple grid search procedure: for each upper bound of tax morale $\bar{s} = \bar{v}$ we divide the interval between 0 and $\bar{s} = \bar{v}$ into 9 equally spaced subintervals. We consider the threshold values of these intervals as corresponding to the 1-10 scale of the answers of the WWS. We then take couples of $a$ and $b$ and, for each couple, we compute the value of the model-based distribution at the threshold values. We then compute, for each couple, the sum of squares distances between the model based distribution and the empirical distribution, which is equal to the observed average relative frequencies from the empirical answers to the questionnaire (where the average is with respect to all countries included in the WWS sample, not weighted). We choose $a$, $b$ and $\bar{s} = \bar{v}$ to minimize this sum of square residuals for the target calibrated level of tax evasion, so to have the closest possible match between the model and the data. For a target evasion level of 30%, we end up with $b = 1$, $a = 5.93$ and $\bar{s} = \bar{v} = 1.748$ for $t = 0.3$ and $b = 1$, $a = 5.93$ and $\bar{s} = \bar{v} = 0.551$ for $t = 0.1$

The 30% baseline choice for the evasion level is in line with what Pissarides and Weber (1989) find in the UK for self employed individuals, but it is sensibly smaller than the 57% tax evasion on business income for self employed individuals in the US (Slemrod 2007, using data from the Tax Compliance Measuring Program implemented by the IRS). As robustness, checks we also consider two alternative scenarios of, respectively, high tax evasion (50%), and low tax evasion (15%).

## 4.2 Comparative Statics

To understand the functioning of the model, we start by describing the comparative statics of the three main policy instruments for the baseline model specification. In Section 4.2.1 we describe the baseline results, which are also plotted in figures (1), (2), (3) and (4). In Section 4.2.2 we explore the robustness of the results to different model parameterizations and calibrations. In all exercises we focus the attention to one instrument only and, therefore, we set the other two to their baseline value.

### 4.2.1 Comparative Statics: The Prototype Economy

Figure (1) reports the comparative static for different tax rates and for different calibration target for the tax evasion level, 15%, 30% and 50%. In the left panel we report the tax evasion rate and in the right panel the normalized net government revenue, which is equal to 100 in the benchmark model parametrization for each single evasion level. This normalization implies that, subtracting 100 from the net government revenue values, we have their variation from the non policy benchmark mode. It also implies that the three lines are not expressed in the same unit of measurement and, therefore, they are not directly comparable. Evasion is increasing in taxes and we have a standard Laffer curve for the government revenue. An increase in the tax rate determines an increase in tax evasion, with less individual that pay taxes, but those that continue paying tax pay more. The numerical result tells us that the first effect prevails for low tax rate up until a threshold, after which the second effect prevails. The picture shows that this threshold tax rate, which is also the one that maximizes the revenue, is smaller the higher the starting level of tax evasion.

Figure (2) analyzes the effect of the tax rebate $\tau$. As shown in the left panel, evasion decreases with $\tau$, since it reduces the incentive to cooperate for the buyer. There are two contrasting effects on the government revenue: on the one hand, the decreasing evasion increases revenue. On the other the higher $\tau$ and the lower the evasion, the higher the transfer from the government to the non cooperating buyers. The right panel shows that there is a threshold level for the tax rebate such that if the rebate is below the threshold the first effect prevails while, if the rebate is above, the second prevails. For a calibrated tax evasion level of 30%, the optimal tax rebate is $\tau = 3.5\%$. If the calibrated tax evasion rate was, for instance, 50%, the optimal $\tau$ would be 6.5%, while it would be only 2% in case of a 15% evasion level.

Figure (3) analyzes the effects of the cash tax $\vartheta$. As previously stressed, there are two effects on tax evasion of an increasing tax on cash: for the individuals with high cost $c$ (the cash users), increasing $\vartheta$ increases the incentive to cooperate, increasing tax evasion. For the individuals with low cost $c$ (the card users), increasing $\vartheta$ decreases the incentive to cooperate, decreasing tax evasion. Moreover, the number of card users increases and the number of cash users decreases. The picture shows that evasion and net government revenue are first increasing and then decreasing. The effect of the TCW on the net government revenue is twofold: on the one hand an increase in the TCW rate affects the cooperation rate and, therefore, the level of tax evasion. On the other, it affects the total cost of credit card use that must be subtracted from the gross revenue. For low levels of the TCW rate cooperation and, therefore, evasion, are increasing, which means a lower total cost of credit card use that can determine an increase in net government revenue even if tax evasion is increasing. Viceversa, for high values of the TCW we have a decreasing tax evasion, but the net government revenue can be decreasing in the TCW rate because the total credit card cost is very high. Tax evasion is lower than the benchmark for a very high tax on cash, but the net government revenue might be lower or higher than the benchmark for such values. In addition the gain in government revenue obtained with the optimal tax rate is higher the higher the starting level of tax evasion. There is no gain if the starting evasion level is low. Summing up, we have a numerical analog of Proposition 1 but also an additional result. We summarize all the previous results in the next proposition:

**Proposition 2.** *i)Evasion is increasing in taxes and we have a standard Laffer curve for the government revenue. The optimal tax is smaller the higher the baseline tax evasion; ii) A moderate tax rebate can increase government revenue and reduce evasion. The optimal tax rebate is higher the higher the baseline tax evasion; iii) The optimal tax on cash withdrawals that maximizes net government revenues is higher the higher the baseline tax evasion.*

In Figure (4) we plot the joint effect of tax rebates and cash on tax for a fixed tax rate level equal to 30%. From the left panel, it is easy to see that, for a fixed tax rebate, there is a hump shaped response of the TCW that maximizes the net government revenue. Similarly, fixing the TCW there is hump shaped response of net revenue to tax rebates. The interaction of these two results implies that there is an interior solution when maximizing the net government revenue. Tax evasion is low for high tax rebates and it responds less to TCW the higher the tax rebate.

### 4.2.2 Comparative Statics: Robustness

We now consider various robustness exercises starting from a higher target value for the credit card users, $\lambda = 0.5$. In this scenario tax evasion is always decreasing in $\vartheta$ and, for a 30% benchmark calibrated evasion rate, $\vartheta = 0.15$ maximizes the government revenue net of the cost. Since there is a higher mass of individuals with a small cost of using alternative payment instruments, the government requires a smaller tax on cash to prevent cooperation from the buyers to reduce evasion. Reduced evasion, in turn, raises the tax collection and decreases the collection of the tax on cash withdrawals but the first effect prevails and revenues are increasing. Moreover, the use of cash is less common in sectors of the economy where the average transaction amount is higher (remember that the price $p$ and the value $u$ are just scalings in the model, so we would have exactly the same results if we redid the model parametrization and calibration for a different price level).

The comparative static results are similar in case of a lower baseline tax rate, $t = 0.1$ for the sale/vat tax interpretation of the model. The only difference is a slightly smaller value of the optimal $\tau$ and $\vartheta$. Changing the bargaining power of the seller $\beta$ results in a different distribution of the gains from evasion but a similar effect of policies on the equilibrium quantities. We also tried a rather extreme value for the enforcement probability, $\pi = 0.3$.[12] In this scenario, the comparative static results are similar, except that the optimal levels of $t$, $\tau$ and $\vartheta$ that maximize the net government revenue (everything else equal) are now smaller. Therefore enforcement is a substitute for all these policies. The problem is that, in our model, enforcement is costless, so we cannot really evaluate the impact of enforcement on the government revenue and, therefore, we cannot single out the optimal enforcement level.

We also tried risk aversion, assuming a CRRA utility function for both the seller and the buyer and risk aversion parameter equal to 3. The comparative static results are essentially similar to the benchmark. As already said, the reason is that the tax rebate and the TCW affect the buyer's incentive to cooperate rather than the terms of the gamble faced by the tax evader.[13] The previous results are summirized in the following proposition:

**Proposition 3.** *i) The larger the mass of individuals with high cost of using payment instruments alternative to cash (the smaller $\lambda$), the higher the optimal tax on cash withdrawals; ii) Tax rebate and tax on cash withdrawals are a substitute for enforcement; iii) Risk aversion is crucial in the*

[12] As already stressed in the previous section, for some businesses the marginal probability of detection can be very high, for instance in case of big firms with high level of evasion or in case there has already been evasion in the past.

[13] We also consider the comparative static results with respect to the fine $f$. Overall a higher fine results in a smaller level of tax evasion and in a higher government revenue, but the quantitative effect is very small. If $f = 3$, ten times bigger than the baseline value, evasion is only 1% lower than the benchmark and the revenue 2.5% higher.

*analysis of the equilibrium effects of fines and enforcement, but marginal in the analysis of the effects of the tax rate, tax rebates and TCW.*

### 4.3 Optimal Policy

We perform two different exercises. The first entails fixing the tax rate to find the optimal combination of $\tau$ and $\vartheta$ to reach a desired objective. The second is finding the optimal combination of $t$, $\tau$ and $\vartheta$ to reach the same objectives. The motivation behind the first exercise is that, since we are considering an extremely stylized microeconomic model, we are not able to capture all the possible macro effects of a change in the tax rate and, therefore, we are not in a good position to evaluate the consequences of different tax rates.[14]

We consider two different possible objectives of the policy. The first is the maximization of the net government revenue. The second entails the maximization of the same objective but conditional on the reduction of tax evasion below 1%. The reason for this second objective is that, as noted by Slemrod and Yitzhaki (1987), the social benefit of tax evasion reductions is not well measured by tax revenue increases only. Additional benefits include, among others, reduced risk bearing, increased efficiency and better competition among businesses. For both policy exercises, we stress the gain in gross government revenue with respect to the corresponding benchmark with fixed tax rate, no tax rebates and no tax on cash. This measure gives an idea of the possibility of the government to compensate honest taxpayers for the side effects introduced by the TCW. We propose some discussion about possible compensation schemes in Section 5.

A caveat before proceeding to the exercises: here we pick some numbers, but we do not intend to take the numbers very seriously. Instead, we think of them as suggestive of the direction that the policy should follow.

In Section 4.3.1 we summarize the optimal policy for the benchmark prototype economy, while in Section 4.3.2 we explore the robustness of the results.

[14]For instance, increasing taxes might induce a recession, which is also part of the reason why politicians are typically reluctant to implement them.

### 4.3.1 Optimal Policy: The Prototype Economy

Fixing the tax rate at the benchmark 30%, the policy that maximizes the net government revenue is $\tau = 3\%$ and $\vartheta = 15\%$. The level of tax evasion associated to this policy is rather high, 12.5%, even if significantly lower than the 30% benchmark. The gross government revenue is 50% larger than the benchmark. Conditional on keeping evasion below 1%, the policy that maximizes the net government revenue entails a higher tax rebate and a higher cash tax, specifically $\tau = 10\%$ and $\vartheta = 18\%$. The gross government revenue in this case is just 6% smaller than in the previous exercise and, more interestingly, 38% bigger than the benchmark value with no tax rebates and no tax on cash. This result stress that the cost of fighting evasion in term of forgone tax revenue is quite small.

Using also the tax rate as policy instrument, the policy that maximizes the net government revenue entails fixing $t$ and $\vartheta$ as high as possible and $\tau$ to zero. The cost of this policy is a high level of tax evasion, mainly induced by the increase in taxes. Conditional on keeping evasion below one percent, the optimal policy is $t = 24\%$, $\tau = 3\%$ and $\vartheta = 18\%$. Again the optimal policy entails a mix of TCW and of the tax rebate. For this policy the gross revenue is still almost 42% higher than the benchmark value where no cash tax and tax rebates are used, although significantly smaller than what could be achieved without fighting evasion. Nevertheless we still find that tax evasion can be eliminated while generating an hefty amount of extra tax revenue. The following proposition summarizes the main result of the optimal policy exercise:

**Proposition 4.** *An appropriate mix of tax rebates and cash tax can curb tax evasion while, at the same time, raising additional tax revenue.*

This additional revenue can be also rebated back to the taxpayers, to compensate the side effects induced by the TCW.

The TCW is arguably difficult to implement and more so the higher the rate (see Section 5 for a discussion). Therefore we perform an additional exercise: we find the optimal policy conditioning on keeping the tax on cash below 5%. The optimal policy to maximize tax revenue is $t = 30\%$, $\tau = 3\%$ and $\vartheta = 5\%$, with a 9% evasion and a 33% bigger gross government revenue. Conditional on keeping evasion below 1% the optimal policy entails the same tax on cash and tax rebate but a smaller tax rate, $t = 27\%$, for a 26% bigger revenue. Thus it is still possible to curb tax evasion and raise additional resources with the only difference of a more modest increase in government revenues. Proposition 4 still holds.

#### 4.3.2 Optimal Policy: Robustness

We now consider four robustness exercises: a higher level of tax evasion, 50%; economies with less widespread use of cash, $\lambda = 0.5$; a lower tax rate, $t = 0.1$; risk aversion for both the seller and the buyer, with a CRRA utility function and a risk aversion parameter of 3. The first exercise is meant to be representative of countries where tax evasion is higher. The second exercise is representative of countries with more developed and competitive financial systems or with a more financially literate population, where the use of electronic currency is more widespread; alternatively, it is representative of sectors of the economy where, given the higher value of the typical transactions, using cash is less practical. The third exercise is designed for the sale/vat tax interpretation of our tax evasion model.[15]

We start from the higher baseline tax evasion, 50%. Again the conclusion is that a mixture of cash taxes and tax rebates can virtually eliminate tax evasion and raise a substantial amount of additional resources. The additional piece of information that we have with this robustness check is that, if the tax evasion is already very high, the tax rebate must be higher.

The second robustness check entails a higher use of payment instruments alternative to cash, with $\lambda = 0.5$. The conclusion from this exercise is that the tax on cash must be smaller the smaller the cost of using payment instruments alternative to cash. Therefore in sectors of the economy where the typical transaction amount is bigger, so where it is easier to avoid paying cash, the optimal policy would entails a lower TCW.

The third robustness check entails the sale tax interpretation of the model, with $t = 0.1$. Again it is possible to curb evasion while raising a substantial amount of additional government resources. The difference with the benchmark model parametrization is a lower level of the tax rebate, which is a consequence of the lower tax rate from which we started.

The final robustness test entails risk aversion for both the seller and the buyer. Instead of the linear utility of the baseline model, we use a CRRA utility function with risk aversion parameter equal to 3. We found that the optimal policy is almost indistinguishable from the prototype

[15]The exact figures of the following exercises are available upon request.

economy, with only very small differences in the value of the optimal policy instruments. Once again, our main policy instruments, the tax on cash and the tax rebate affect significantly the buyer's incentive to cooperate rather than the terms of the gamble between that the tax evader faces. As in the prototype economy, it is still possible to raise additional government revenue while reducing evasion below 1% even with a 5% upper bound to the TCW. If it is not possible to use the TCW, however, the government revenue associated to the zero tolerance policy is almost equal to the non policy benchmark.

# 5 Cash Tax Implementation

We start the section with a brief account of how the tax on cash withdrawals could be managed. We then discuss some challenges to its implementation and propose solutions to partially overcome them.

Since it is a tax on cash withdrawals from a bank account, either a checking or a saving account, it can be naturally implemented by banks. The bank can collect the tax from the public and then transfer the money to the tax administration, with or without a documentation that might include the identity of the individual and the amount of the transaction. Importantly, the TCW is intrinsically different from a ban on non traceable payments instruments that, although ideal from the tax authority perspective, would most likely be upsetting to many: first because it would entail a partial loss of privacy; second because imposing the use of credit cards, cheques or bank transfers for transactions of small amount, can be too cumbersome. In this perspective, the TCW can be seen as putting a price on privacy and transaction ease: it is allowed to use non traceable payment instruments, to ensure privacy or to speed up the transaction, but there is a cost associated with these benefits.

The first, and foremost, challenge to the implementation of the cash tax is the possible emergence of a parallel cash economy: both sellers and buyers might start using whatever cash they already have for the transactions and accumulate it to avoid using the banking system and paying the TCW.

To credibly implement the TCW, the government must stifle the cash economy.

The first simpler possibility is to keep the cash tax rate small, in order to drastically reduce the risk of a bank run and to downsize the dimension of the parallel cash economy. We already stressed that, even with this constraint, it is still possible to reduce tax evasion and raise additional tax revenue, although less than in the unconstrained case.

A second possibility is the contemporaneous introduction of a tax on cash bank deposits, together with a ban on cash purchases of financial instruments (treasury bonds, corporate bonds or stocks) and of sanctions for the individuals caught exchanging cash amounts above a certain threshold. The idea is to lower the incentives of the sellers to accept cash payments and hoard cash, making difficult to use it. Consider, for instance, a small shopkeeper that evades taxes by selling for cash. He can use the accumulated cash to pay the suppliers and to pay the wages to the employees, fostering the cash economy. But he can also save them, perhaps for the down payment of a car or house or perhaps to improve the shop in the future. The sanctions for cash transactions of big amounts are meant exactly to limit the use of this accumulated savings. In addition, a ban on cash purchases of financial instruments greatly reduces the possibility to accumulate savings. A rational investor will decide to deposit if the tax is lower than the capitalized interests from the financial instruments. Overall, these sellers see a reduction in the value of their savings for all the cash transactions and this greatly reduces their incentive to accept cash payments (Morse et al. 2009).

A second, related, challenge concerns the dynamic of the introduction of the tax. If the tax is announced and then implemented, it is likely that a bank run will take place, with individuals withdrawing cash to avoid paying the tax and/or to facilitate tax evasion in the future.

The probability associated with these two scenarios is higher the higher the TCW. It is difficult to think of a massive bank run as a result of a small TCW, since the cost of managing all the payments in cash will most likely be higher than the cost imposed by the tax. In addition, operating exclusively in the cash economy hampers or precludes the possibility of obtaining mortgages or even short term financing for the sellers, imposing a very high cost to them (Straub 2005, Antunes and Cavalcanti 2007, Gordon and Li 2009, Capasso and Jappelli 2013). Also, some credit card issuers already charge a small fee for ATM withdrawals, but many individuals use them anyway.

A third problem is related to the costs of the electronic payment system. One the one hand, the TCW forces many individuals to pay the cost of using debit and credit cards. On the other, the sellers might accept electronic payments to avoid loosing customers (in favor of establishments where they can pay with credit cards) and, since this services are costly, they will face lower profits. In the end, the TCW will create rents for the banks. To neutralize this effect part of the extra government revenue can be used to provide tax rebates to consumers and sellers that help pay for the credit and debit card fees.[16]

More generally, compensating honest taxpayers (both sellers and buyers) that do not use cash is a crucial element of our policy proposal, since it will boost its acceptability by the general public, maximizing the probabilities that it will be successful. If it is perceived as unfair, on the other hand, it is more likely doomed to be ineffective, as it will strengthen a social norm against it (Bordignon 1993, Falkinger 1995, Torgler 2003 [1], Slemrod 2003).[17] Particularly challenging is to design the compensation for those who have a cost of using the electronic payment instruments other than the banking fees. We think in particular about specific categories of individuals, like the elderly, who, being less familiar with new technologies, might find it hard to cope with an electronic payment system. However, it is difficult, both from a technical and from a legal perspective, to link a monetary compensation to the cost, since beside old age and disability, there is not really much more information that can help discriminate between individuals.

A fourth implementation problem is specific to currency areas: the TCW should be implemented in all the countries at once and, to avoid arbitrage, the tax rate should be the same in all countries. This will hamper the possibility of tailoring the policy to the specific needs of a country. One plausible argument against this objection is that the possibilities of arbitrage are not widespread. For individuals that typically live and work in one country only, that is the majority of them, there is not really the possibility of frequent cross border travels just to withdraw cash or, alternatively, the possibility of opening a bank account in another country. Probably the cost of these operations, which includes travel expenses and banking fees, will reduce the gain from arbitrage at least for a small tax.

[16] To avoid overpricing of the bank's financial services, the Government can also regulate the market imposing maximum prices or other regulatory schemes.

[17] The compensation is also crucial since many of the optimal policies that we isolated feature a TCW higher than the tax rebate and this might decrease the purchasing power of the individuals if they perform many expenses in cash.

# 6 Incomplete Information

One of the simplifying assumption of our model is the complete information on both sides of the market: the buyer perfectly knows the tax morale of the seller and the seller knows perfectly both the tax morale of the buyer and his cost of using payment instruments alternative to cash. This assumption is reasonable for repeated interactions. Think about the relationship with a family doctor or with the neighborhood grocer, for instance. However it is hardly representative of all market transactions, that often are occasional. To analyze how the results change in a more reasonable incomplete information scenario, we consider a different model set-up following the double auction model of Chatterjee and Samuelson.

We consider a seller and a buyer that submit a bid for a price discount, respectively $\psi_s(\upsilon)$ and $\psi_b(s, c)$. If the bid submitted by the seller, which is the maximum he is willing to accept to induce cooperation from the buyer, is bigger than the bid submitted by the buyer, which is the minimum he is willing to accept to allow tax evasion, then we assume that there is cooperation and tax evasion, with a discount that is a weighted average of these two bids. Thus if $\psi_s(\upsilon) \geq \psi_b(s, c)$ then $d = \beta\psi_s(\upsilon) + (1 - \beta)\psi_b(s, c)$. In this case the seller fully evades taxes and the buyer cooperates, accepting the discount. If $\psi_s(\upsilon) < \psi_b(s, c)$ there is no deal, no tax evasion and the utilities of both seller and buyer are unaffected. Instead, in case of evasion, the seller's expected profit becomes

$$V_s^e = p(1-t) + \{pt(1-\pi) - [1-\pi(1-ft)]E[d|\psi_s(\upsilon) \geq \psi_b(s,c)] - \upsilon\}Prob[\psi_s(\upsilon) \geq \psi_b(s,c)] \quad (10)$$

while the buyer's is

$$V_b^e = u - p(1-t) - \min[p\vartheta, c] + \{(1+\vartheta)[d|\psi_s(\upsilon) \geq \psi_b(s,c)] - p(\tau+\vartheta) + \min[p\vartheta, c] - s\}Prob[\psi_s(\upsilon) \geq \psi_b(s,c)]. \quad (11)$$

Both the seller and the buyer choose the bid that maximizes the expected revenue. A pair of best response offer strategies constitute a Nash (or Bayesian) equilibrium. By definition, neither player can increase his expected profit by unilaterally altering his chosen strategy.

In case of uniform distribution of $\upsilon$, $s$ and $c$ there is a simple analytical expression for the equilibrium strategies and discount. In appendix we discuss such a result. However, for the actual distributions that we use, we don't know the analytical expression for the optimal strategies.

Therefore we consider polynomial approximations. The solution algorithm entails guessing an optimal strategy for the sellers, which is a polynomial in his type $v$, to then compute the best response for the buyers, which is a function of both $c$ and $u$. Given this best response, we compute the buyer's best response and then the difference between this last function and the guessed strategy (we consider the sum of square distances as a metric). We repeat this procedure for different guesses, in the form of different parameters of the polynomial, and we stop when we find the Bayes-Nash equilibrium.

Overall the numerical results are substantially similar to the complete information case, with only two major differences. The first is that the gains from trade are distributed differently between the buyer and the seller. Second, the uncertainty about the identity of the buyer in terms of the cost $c$ leads the seller to submit a bid based on the expected, not actual, cost $c$, so the individuals with high cost in a sense matter less for the model equilibrium. Therefore under asymmetric information we have results that resemble the one that we would have obtained under complete information but with a higher value of $\lambda$ or, in general, with a distribution with lower probability mass on high cost $c$ individuals. For instance, we mentioned in the previous section that if the measure of individuals with high cost $c$ is small, evasion is always decreasing in the cash tax. Under incomplete information, evasion is decreasing in the tax cash also with a mixture exponential distribution of $c$ and a small $\lambda$, a scenario that delivered an inverse u-shape relationship under complete information.

# 7 Conclusion

We found that an appropriate mix of tax rebates and taxes on cash withdrawals, together with an appropriate choice of a tax rate, can curb tax evasion while, at the same time, raising additional revenue. We showed that the tax rebates must be higher the higher the tax rate and the higher the tax evasion rate. The cash tax, on the other hand, must be smaller the smaller the cost of using payment instruments alternative to cash. The additional revenue generated by this policy can be partly used to subsidize the use of credit cards and POS machines, in order to reduce the cost of managing electronic payment systems both for the buyer and the seller. The foremost challenge to the implementation of this policy scheme is the possibility of emergence of a parallel cash economy.

We propose to keep the cash tax rate low to decrease the probability associated to this scenario. This will reduce the amount of additional tax revenue collected, but it will still result in a sharp tax evasion reduction.

# 8 Appendix A: The Case of Italy

In this section we consider actual empirical evidence for Italy as an example of the optimal policy discussed in the main body of the paper. We choose Italy because, beside being the country in which we live and work, it is also one of the countries where tax evasion is particulary pernicious and where it attracts a lot of attention from citizens and media.

For the tax rate we choose $t = 0.35$ and the income/profit interpretation of the model[18]. This number is a (rounded) weighted average of the different rates with weights equal to the percentage of income in the bracket. Obviously we are simplifying, perhaps too much, the complexity of the Italian tax system by picking this number. Firms are in fact required to pay a conspicuous number of taxes: some are fixed and required to stay open, some of which depend on the revenues, some depend on the profits, some on the number of workers. Different assumptions will lead us to include or exclude some of them in the computation of the marginal tax and, inevitably, to different possible numbers and long discussions to defend our assumptions. Instead of doing some complicated computation of the average tax rate, we decided to just stick to this simple benchmark, taking into account that the results that we would obtain in case of a smaller or bigger tax rate can be inferred from the discussion in Section 4.3.

For the enforcement probability $\pi$, we divide the number of tax audits made in 2011 by the "Guardia di Finanza", the main tax enforcement authority in Italy, by the number of economic units (firms, entrepreneurs, individual professionals) operating in Italy in 2011. There are two kinds of audits implemented by this tax enforcement authority: a more throughout one, which is less frequent but that detects evasion with certainty (a careful screening of all the fiscal documents, together with a detailed analysis of the economic activity) and a more superficial one, much more frequent but less effective (a simple spot control where the agents monitor the day to day activity

[18] For the sale/value added tax interpretation $t = 0.21$, which is the VAT tax rate on most manufacturing goods and personal and professional services. We omit the results for this alternative case as they are a straightforward consequence of what we already discussed in the previous section.

and step in if there is a violation). In the first case we have $\pi = 0.0067$ while in the second $\pi = 0.172$. We present the results for the first value and we assume that all audits are random and independent on sellers subjective characteristics. While this is certainly true for the spot controls, it is not for the more throughout controls, which are typically the final step of some monitoring activity that takes into account the business characteristics, also based on the past tax reports. For the fine we use the value $f = 0.3$ according to the Italian tax law that prescribes a fine from 6% to 30% to be paid on the evaded amount. In Italy, tax evasion is also subject to jail sentences, in addition to the fines, but only in extreme cases (very high amount), which makes them extremely unlikely. Thus we focus on pecuniary fines only.

Since there is no tax on cash withdrawals in Italy, we set $\vartheta = 0$. For the distribution of the cost of payment instruments other than cash $c$, we consider data on payment instruments from the ECB (gathered through the national central banks). We divide the sum of all transactions made with credit cards and debit cards by the consumption component of GDP (goods, including durable, and services). We obtain $\lambda = 0.127$. We stick to the assumption of $\beta = 0.5$ and to $p = 10$ and $u = 1.5p$.

To calibrate the model, we need a target value for tax evasion and we need a shape of the distribution of the tax morale to match. For the tax morale we take the values from the WWS website. For tax evasion, we consider two different sources. The first is the Study by EURES (2012), an Italian independent research institute. Total tax evasion in Italy is estimated to be between 16.3% and 17.5% of the GDP. Both numbers are obtained averaging over different sectors (32.8% agriculture, 12.4% Industry and between 20% and 27% for services ) and across different geographic areas. The other source is the ISTAT, the Italian statistical institute, that reports an average of 12.7%, also obtained averaging over different sectors (22% agriculture, 6% industry, 11% construction, 14% services) and geographic areas (9% North, 11% center, 20% South). We set our benchmark close to the average of the two numbers at 15%. We understand that this average can generate a lot of disagreement and, quite frankly, we don't really know if this is the best possible number that we can use. The main issue here is the interpretation of the model and the sector of the economy for which it is more representative. Perhaps the cooperative tax evasion phenomenon is more widespread in the service sector or among professionals, so we should probably use a higher target level. In addition, there is some direct survey evidence reported by the EURES (2012) that highlights much higher evasion rates among specific professions like, for instance, a 63% evasion rate for baby sitters and housekeepers, 33% for hairdressers, 67% for gardeners etc. Here we present the results only for the baseline average evasion level, but we stress that, in case of a higher evasion level, the same consideration that we made in the robustness test of Section 4.3 apply.

For the 15% benchmark tax evasion level, we end up with $b = 1$ and $a = 5.87$ and $\bar{s} = \bar{\upsilon} = 2.025$.

Fixing the tax rate to the benchmark 35%, the policy that maximizes the net government revenue is $\tau = 4\%$ and $\vartheta = 30\%$, with a tax evasion level of 10.7%. and a 45% bigger gross government revenue. Conditional on keeping evasion below 1%, the policy that maximizes the net government revenue entails a higher tax rebate and a lower cash tax, specifically $\tau = 15\%$ and $\vartheta = 15\%$. The gross government revenue in this case is approximately equal to the benchmark value. Again the result is that it is possible to achieve a virtual elimination of tax evasion by using an appropriate mix of cash taxes and tax rebates. Using also the tax rate as policy instrument, the policy that maximizes the net government revenue entails again a high tax rate and a high cash tax and zero rebates, but it induces a high level of tax evasion. Conditional on keeping evasion below 1%, the optimal policy is $t = 25\%$, $\tau = 1\%$ and $\vartheta = 35\%$, with a 37% bigger gross government revenue that the benchmark. If the maximum feasible cash tax rate is 5%, the optimal policy is $t = 35\%$, $\tau = 2\%$ and $\vartheta = 5\%$, with a 13% bigger gross government revenue and a 9% evasion rate. Conditional on keeping evasion below 1%, the optimal policy is $t = 30\%$, $\tau = 3\%$ and $\vartheta = 5\%$, with both a lower tax rate and a higher rebate. The gross government revenue is equal to the benchmark value. Without using the tax on cash the policy that maximizes the net government revenue entails again higher taxes and higher rebates, $t = 45\%$ ad $\tau = 12\%$. For this policy, the gross government revenue is just 1.5% higher than its benchmark value and evasion is slightly more than half the benchmark value, 8%. Conditional on keeping evasion below 1%, the optimal policy is $t = 35\%$ and $\tau = 13\%$, thus a lower tax and a higher deduction. The effect is a 26% lower gross government revenue. In this example, fighting evasion without using the tax on cash is costly.

# 9 Appendix B: Analytical solution with Uniform Distributions

In this Appendix we propose an analytical solution of the model for the case of incomplete information when the uncertainty is uniformly distributed. Namely $\upsilon \sim U[0, \bar{\upsilon}]$ $s \sim U[0, \bar{s}]$ and $c \sim U[0, u]$. We show that the solution has a structure similar to the one in the complete information benchmark.

In this section the double auction model of Chatterjee and Samuelson (1983) is used to model the bargaining between the seller and the buyer. Since all the heterogeneity is uniformly distributed, we guess a linear solution for the optimal strategies for both the seller and the buyer and we solve the model by guess and verify. We consider the following strategies

$$\psi_s(\upsilon) = a_1 + a_2\phi_s(\upsilon) \qquad \phi_s(\upsilon) = \frac{pt(1-\pi) - \upsilon}{1 - \pi(1 - ft)} \tag{12}$$

$$\psi_b(s, c) = a_3 + a_4\phi_b(s, c) \qquad \phi_b(s, c) = \frac{p(\tau + \vartheta) - \min[p\vartheta, c] + s}{1 + \vartheta} \tag{13}$$

The seller's optimal strategy solves the following maximization problem

$$\max_{\psi_s} p(1-t) + [1 - \pi(1 - ft)]\{\phi_s(\upsilon) - \beta\psi_s - (1-\beta)E[\psi_b(s, c)|\psi_s \geq \psi_b]\}Prob[\psi_s \geq \psi_b] \tag{14}$$

We have

$$Prob[\psi_s \geq \psi_b] = \frac{p\vartheta}{\bar{c}}Prob[\psi_s \geq \psi_b|p\vartheta > c] + \left(1 - \frac{p\vartheta}{\bar{c}}\right)Prob[\psi_s \geq \psi_b|p\vartheta < c] \tag{15}$$

if $p\vartheta > c$ the distribution of the buyer's optimal response is uniform:

$$\psi_b(s, c) \sim U\left[a_3 + a_4\frac{p(\tau + \vartheta) - \tilde{c}}{1 + \vartheta}\, ;\, a_3 + a_4\frac{p(\tau + \vartheta) - \tilde{c} + \bar{s}}{1 + \vartheta}\right] \tag{16}$$

where $\tilde{c} = E[c|p\vartheta > c] = p\vartheta/2$.

if $p\vartheta < c$ the distribution of the buyer's optimal response is again uniform:

$$\psi_b(s,c) \sim U\left[a_3 + a_4\frac{p\tau}{1+\vartheta}\,;\, a_3 + a_4\frac{p\tau+\bar{s}}{1+\vartheta}\right] \tag{17}$$

Thus

$$Prob[\psi_s \geq \psi_b] = \frac{1}{a_4\bar{s}}\left[(\psi_s - a_3)(1+\vartheta) - a_4 p\tau - a_4\frac{(p\vartheta)^2}{2\bar{c}}\right] \tag{18}$$

and

$$E[\psi_b(s,c)|\psi_s \geq \psi_b] = \frac{1}{2}\left[\psi_s + a_3 + a_4\frac{p\tau}{1+\vartheta} + a_4\frac{(p\vartheta)^2}{2\bar{c}(1+\vartheta)}\right]. \tag{19}$$

Plugging these last two expressions in the seller's optimization problem and differentiating with respect to $\psi_s$ we obtain the following first order condition

$$\psi_s(\upsilon) = \frac{1}{1+\beta}\phi_s(\upsilon) + \frac{\beta}{1+\beta}\left[a_3 + a_4\frac{p\tau}{1+\vartheta} + a_4\frac{(p\vartheta)^2}{2\bar{c}(1+\vartheta)}\right]. \tag{20}$$

The buyer optimal strategy is the solution to the following optimization problem

$$\max_{\psi_b} u - p(1-\tau) - min[p\vartheta, c] + (1+\vartheta)\{\beta E[\psi_s|\psi_s \geq \psi_b] + (1-\beta)\psi_b - \phi_b(s,c)\}Prob[\psi_s \geq \psi_b]. \tag{21}$$

The optimal strategy of the seller is uniformly distributed

$$\psi_s(\upsilon) \sim U\left[a_1 + a_2\,\frac{pt(1-\pi) - \bar{\upsilon}}{1-\pi(1-ft)}\,;\, a_1 + a_2\,\frac{pt(1-\pi)}{1-\pi(1-ft)}\right] \tag{22}$$

thus:

$$Prob[\psi_s \geq \psi_b] = \frac{1}{a_2\bar{\upsilon}}\left\{(a_1 - \psi_b)[1-\pi(1-ft)] + a_2 pt(1-\pi)\right\} \tag{23}$$

and

$$E[\psi_s|\psi_s \geq \psi_b] = \frac{1}{2}\left[\psi_b + a_1 + a_2\frac{pt(1-\pi)}{1-\pi(1-ft)}\right]. \tag{24}$$

Plugging these expression in the buyer'e expected profit and differentiating with respect to $\psi_b$

we obtain the following first order condition

$$\psi_b = \frac{1}{2-\beta}\phi_b(s,c) + \frac{1-\beta}{2-\beta}\left[a_1 + a_2\frac{pt(1-\pi)}{1-\pi(1-ft)}\right]. \tag{25}$$

Comparing the first order conditions (20) and (25) with the guesses (12) and (13) we obtain a linear system of four equations in four unknowns $a_1$, $a_2$, $a_3$ and $a_4$. Solving the system and plugging the solution in (12) and (13) we obtain the following optimal strategies:

$$\psi_s^* = \frac{\beta}{2}\frac{1}{1+\vartheta}\left(p\tau + \frac{(p\vartheta)^2}{2\bar{c}}\right) + \frac{1}{1+\beta}\frac{1}{1-\pi(1-ft)}\left\{\left[1+\frac{\beta(1-\beta)}{2}\right]pt(1-\pi) - \upsilon\right\} \tag{26}$$

and

$$\psi_b^* = \frac{1}{2-\beta}\frac{1}{1+\vartheta}\left\{\left[1+\frac{\beta(1-\beta)}{2}\right]\left[p\tau + \frac{(p\vartheta)^2}{2\bar{c}}\right] + s - \min[p\vartheta, c]\right\} + \frac{1-\beta}{2}\frac{pt(1-\pi)}{1-\pi(1-ft)}. \tag{27}$$

The equilibrium discount is $d^* = \beta\psi_s^* + (1-\beta)\psi_b^*$ if $\psi_s^* \geq \psi_b^*$. We then compute the threshold values of the tax morale that define participation in this bargaining game $\tilde{s}_1(\upsilon, c)$ and $\tilde{s}_2(\upsilon)$ and the corresponding values $\tilde{\upsilon}_1$ and $\tilde{\upsilon}_2$ similarly to what we did for the model in complete information, with the only difference that now we have to consider the expected profit. Following the same steps of the integration to find total tax evasion we obtain the following expression:

$$\begin{aligned} E = & \frac{(2-\beta)(1-\beta)}{8}\,\frac{(1+\vartheta)[1-\pi(1-ft)]}{2\bar{c}\bar{s}\bar{\upsilon}}\Bigg\{\bar{c}B^2 \\ & - \frac{(p\vartheta)^2}{(2-\beta)(1+\vartheta)}\left[B - \frac{1}{3}\frac{p\vartheta}{(2-\beta)(1+\vartheta)}\right]\Bigg\} \end{aligned} \tag{28}$$

where

$$B = \frac{pt(1-\pi)}{1-\pi(1-ft)} - \frac{p\tau}{(1+\vartheta)} - \frac{p\vartheta}{1+\vartheta}\left[\frac{1}{2-\beta} - \frac{p\vartheta}{2\bar{c}}\right]. \tag{29}$$

To compare the expression for tax evasion in the incomplete information model with the one in the complete information benchmark we compute the integral (8) assuming uniform distributions and obtain the following expression:

$$E = \frac{(1+\vartheta)}{2\bar{c}\bar{s}\bar{v}} \left\{ \bar{c}A^2 - \frac{(p\vartheta)^2}{(1+\vartheta)} \left[ A - \frac{1}{3}\frac{p\vartheta}{(1+\vartheta)} \right] \right\} \tag{30}$$

where

$$A = pt(1 - \pi(1+f)) - \frac{p\tau}{(1+\vartheta)}. \tag{31}$$

Therefore the solution of the model under incomplete information has a very similar structure to the solution of the model under complete information except for the bargaining power that plays no role in the complete information benchmark.

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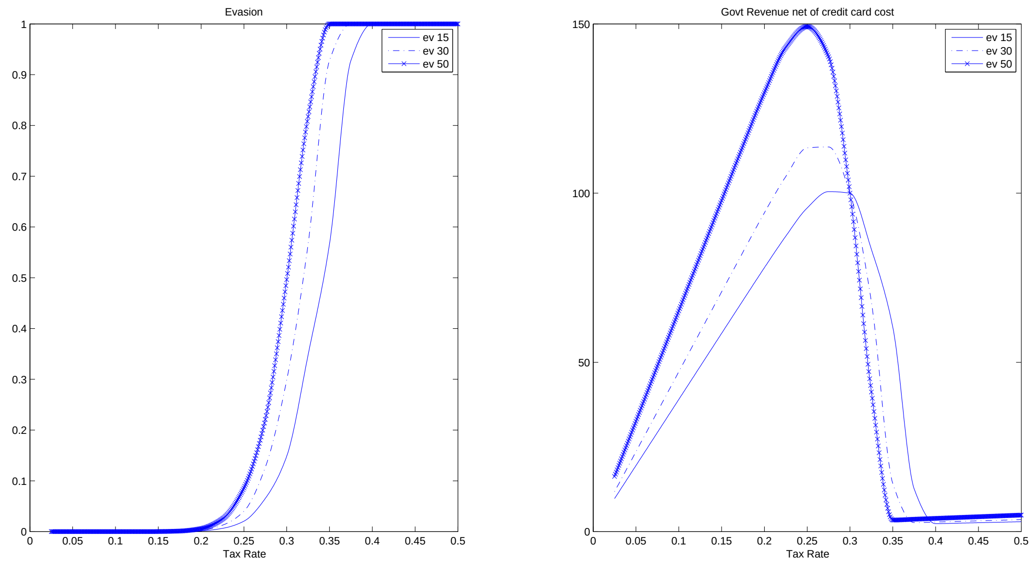
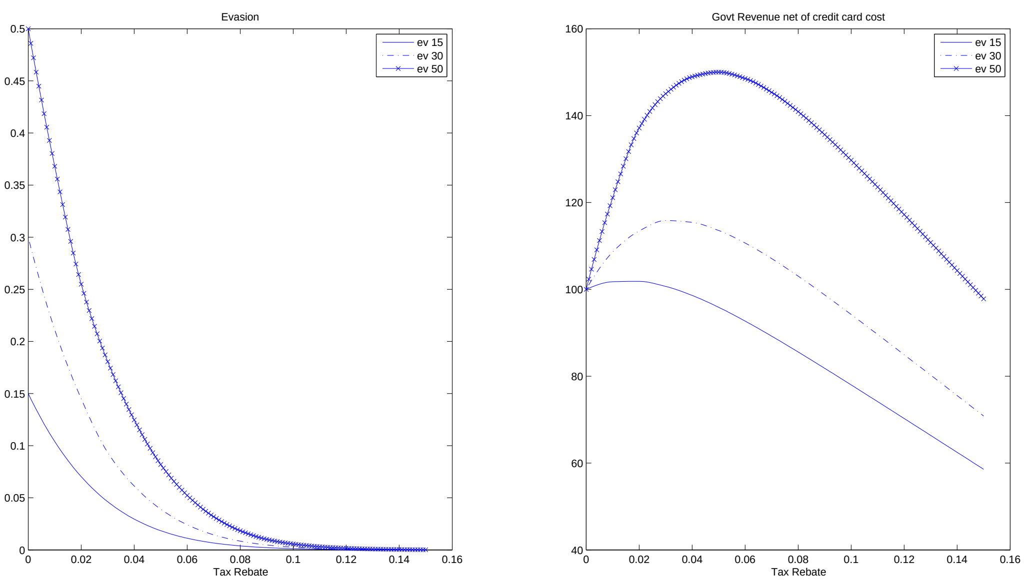
Figure 1: **Effect of Taxes**



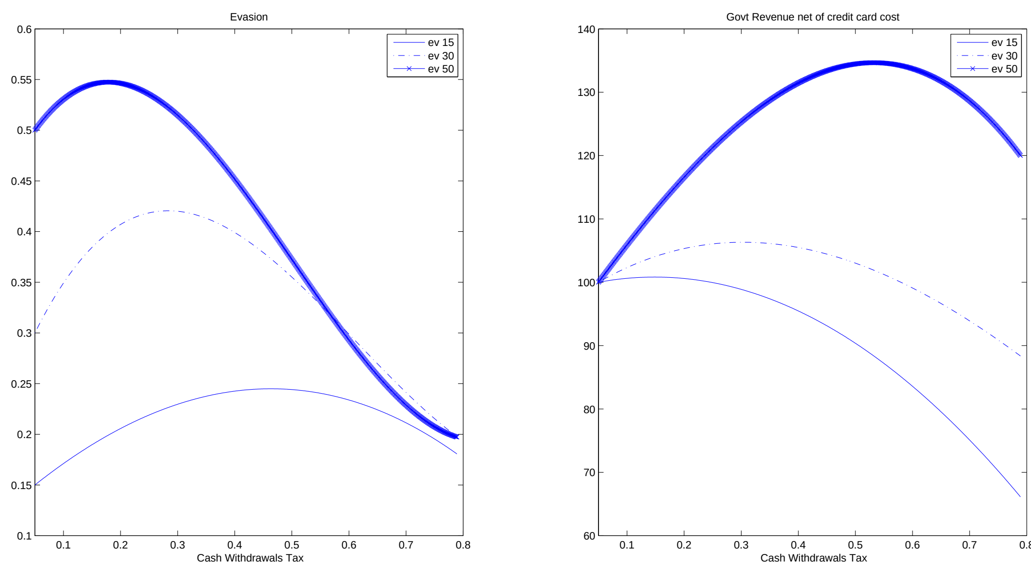

**Notes:** Left panel: tax evasion in percentage terms. Right panel: total government revenue minus the cost of payment instruments $c$ for all the buyers that do not cooperate with tax evaders, scaled to be equal to 100 if equal to the total government revenue in the baseline model specification ($t = 0.3$, $\tau = 0$ and $\vartheta = 0$) for each tax evasion level.

Figure 2: **Effect of Tax Rebate**



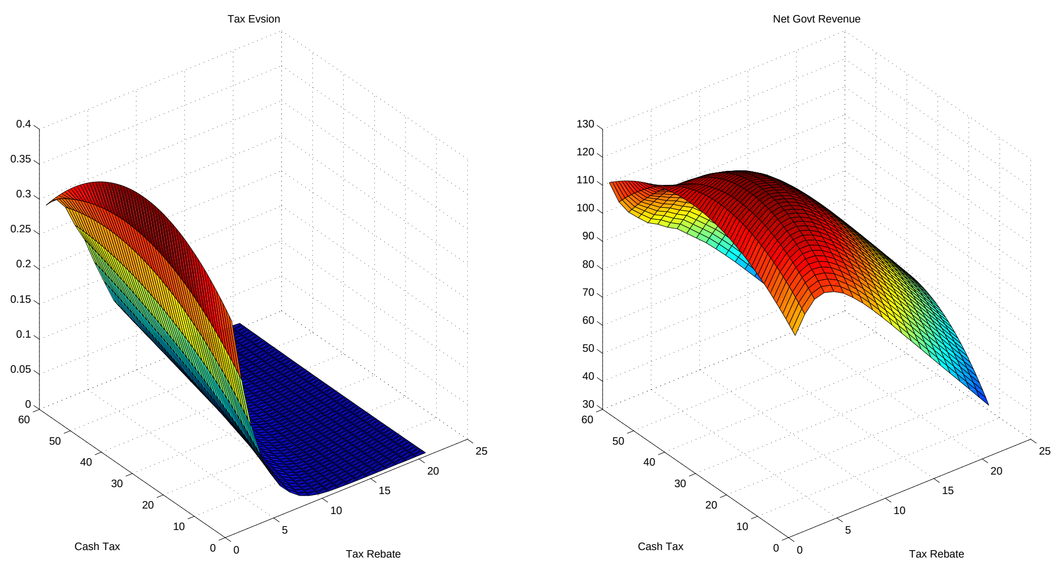

**Notes:** Left panel: tax evasion in percentage terms. Right panel: total government revenue minus the cost of payment instruments $c$ for all the buyers that do not cooperate with tax evaders, scaled to be equal to 100 if equal to the total government revenue in the baseline model specification ($t = 0.3$, $\tau = 0$ and $\vartheta = 0$) for each tax evasion level.

Figure 3: **Effect of Cash Tax**




**Notes:** Left panel: tax evasion in percentage terms. Right panel: total government revenue minus the cost of payment instruments $c$ for all the buyers that do not cooperate with tax evaders, scaled to be equal to 100 if equal to the total government revenue in the baseline model specification ($t = 0.3$, $\tau = 0$ and $\vartheta = 0$) for each tax evasion level.

Figure 4: **Effect of Tax Rebates and Cash Tax**




**Notes:** Left panel: tax evasion in percentage terms. Right panel: total government revenue minus the cost of payment instruments $c$ for all the buyers that do not cooperate with tax evaders, scaled to be equal to 100 if equal to the total government revenue in the baseline model specification ($t = 0.3$, $\tau = 0$ and $\vartheta = 0$). Tax rate is fixed to 30%