Learning About Which Measure of Inflation to Respond to in an Interest Rate Rule*

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First Draft: April 2005
This Draft: November 2005

Abstract

Which measure of inflation should an active interest rate rule (i.e. a rule where the nominal interest rate responds more than proportionally to changes in inflation) respond to in order to guarantee a unique equilibrium whose Minimal State Variable (MSV) representation is learnable in the E-stability sense proposed by Evans and Honkapohja (2001)? Using a closed economy model with a flexible-price good and a sticky-price good we find that the measure corresponds to the sticky-price inflation To obtain this result we analyze separately forward-looking, contemporaneous and backward-looking rules that can respond to three possible measures of inflation: the flexible-price inflation, the sticky price inflation and the full-inflation, i.e. a convex combination of the previous two. Although whether the flexible-price good and the sticky-price good are Edgeworth complements, substitutes or utility separable plays an important role in the analysis, we find in particular the following. Active forward-looking and contemporaneous rules that respond to either the flexible-price inflation or the full-inflation are more prone to induce multiple equilibria and E-instability of the MSV solution. More importantly active backward-looking rules that react to either the flexible-price inflation or the full-inflation may guarantee a unique equilibrium but in these cases the MSV solution is not learnable in the E-stability sense. Only responding actively to the sticky-price inflation seems to be a robust policy recommendation across timings and types of goods, in order to guarantee a unique and learnable equilibrium.

Keywords: Learning; Expectational Stability; Interest Rate Rules; Multiple Equilibria; Determinacy
JEL Classifications: E4, E5

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*The authors benefited from comments and conversations with Mark Bils, David Bowman, Jim Bullard, Jane Ihrig, Chris Erceg, Jordi Galí, and the participants of the Federal Reserve Board IF workshop, the SCE 2005 conference, and the 2005 seminar about "Learning in Macroeconomic Models: Recent Advances and Policy Applications." All errors remain ours. The views expressed in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the view of the Board of Governors of the Federal Reserve System or of any other person associated with the Federal Reserve System.

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1 Introduction

Since the seminal work by Taylor (1993) it has become more common to think about monetary policy in terms of interest rate rules whereby the government maneuvers the nominal interest rate in response to inflation and output. Existing works in standard New Keynesian models have found that particular specifications of these rules may generate aggregate instability in the economy by inducing locally multiple equilibria, or in other words, local real indeterminacy.[1] To avoid this, some of the works have argued that a government should implement rules that guarantee a unique equilibrium.

Nevertheless this policy prescription is usually derived assuming that agents in the economy can coordinate their actions and learn the equilibria (unique or multiple) induced by the rule. But if agents cannot learn the unique equilibrium targeted by the rule then there are some rules that although guaranteeing a unique equilibrium, might not ensure that the economy will reach it.[2] Hence a "good" rule should in principle satisfy at least two criteria: uniqueness and learnability of the equilibrium.

This argument has motivated an important part of the interest rate rules literature that has tried to disentangle the specific features of a rule that are essential to guarantee a unique and learnable equilibrium. This part has mainly focused on the timing of the rule and on the degrees of responsiveness to inflation, to the output gap, and to past interest rates.[3] In contrast, the question of which measure of inflation should be included in the rule has received less attention. This is to some extent surprising if one takes into account the following two observations.

First, most of the works of the interest rate literature assume that the government targets and responds to the CPI-inflation. But in the practice of monetary policy, governments may respond to a different measure. For instance, they may consider measures that exclude significant impacts from terms of trade movements, changes in energy and food prices or some other prices that are determined in an *ad hoc* basis.[4]

Second, in defining this measure there are different characteristics of the prices of the goods included in this measure that are probably studied and taken into account. One of this characteristics is associated with the frequency of price changes. This seems to be a relevant characteristic since there is empirical evidence suggesting that the frequency of price changes differ dramatically across different goods: some prices seldom

[1] This means that there are rules that open the possibility of having fluctuations in the economy that are mainly driven by people's self-fulfilling expectations. To the extent that these fluctuations are usually characterized by a large degree of volatility in real and nominal macroeconomic aggregates, then a central bank should avoid, in principle, rules that induce multiple equilibria. See Benhabib et al. (2001), Bernanke and Woodford (1997), Clarida et al. (2000), Carlstrom and Fuerst (1999) and Woodford (2003), among others.

[2] See Bullard and Mitra (2002).

[3] For works that pursue a determinacy of equilibrium analysis see Batini and Pearlman (2002), Benhabib et al. (2001), Carlstrom and Fuerst (1999), Dupor (2001) and Levin et al. (2003), among others. For works that pursue a determinacy of equilibrium analysis in tandem with a learnability analysis see Bullard and Mitra (2002,2003), and Evans and Honkapohja (2003), among others.

[4] See for instance Bernanke et al. (1999) that documents the experiences and lessons of some countries that have adopted inflation targeting.

change for some goods, but some prices change very frequently for other goods.[5]

Based on these observations it seems relevant to answer the following question: if there are different goods in the economy, some with flexible prices and some with sticky prices, which measure of inflation should a government respond to in an active interest rate rule in order to guarantee a unique and learnable equilibrium? We find that the answer to this question is the inflation associated with the sticky-price goods.[6]

To obtain this answer we use a New Keynesian closed economy general equilibrium model with two types of goods: a flexible-price good and a sticky-price composite good. We analyze separately forward-looking, contemporaneous and backward-looking rules that respond more than proportionally to one of the following measures of inflation: the flexible-price inflation, the sticky price inflation and the full-inflation, i.e. a convex combination of the previous two.[7] We select the measure of inflation utilizing the following specific criteria: 1) the rule must guarantee a unique equilibrium and 2) the Minimal State Variable (MSV) representation of this unique equilibrium must be learnable in the E-stability sense proposed by Evans and Honkapohja (1999, 2001).[8,9]

Using these criteria we find that rules that respond to the sticky-price inflation are more prone to deliver a unique and learnable equilibrium than rules that respond to any other measure of inflation (i.e. the full-inflation and the flexible-price inflation.) In particular, although whether the flexible-price good and the sticky-price good are Edgeworth complements, substitutes or utility separable plays an important role in the analysis, we find the following. Forward-looking and contemporaneous rules that respond to either the flexible-price inflation or the full-inflation are more prone to induce multiple equilibria and E-instability of the MSV solution. More importantly backward-looking rules that react to either the flexible-price inflation or the full-inflation may guarantee a unique equilibrium but in these cases the fundamental solution (MSV representation) is not learnable in the E-stability sense.

Our main finding is to some extent reminiscent of previous results from the optimal monetary policy literature that recommend to target the inflation of the good (sector) with the higher degree of price stickiness.[10] But our selection criteria are very different.

Our main result is also related to previous contributions by Carlstrom, Fuerst and Ghironi (2004) and Zanna (2003). Both works analyze, in the contexts of a two-sector models for a closed economy and for a small

[5] See Bils and Klenow (2004).

[6] By an active rule we mean a rule that in response to a one percent increase in inflation, increases the nominal interest rate by more than one percent. In this sense it satisfies the Taylor Principle.

[7] The CPI-inflation can be understood as a full-inflation measure where the weights are associated with the shares of each good determined by the households preferences.

[8] Henceforth we will use the terms "learnability", "E-stability" and "expectational stability" interchangeably in this paper.

[9] Evans and Honkapoja (1999, 2001) have argued that a unique equilibrium is not "fragile" if it is learnable in the sense of E-stability. They assume that agents in the model do not have rational expectations but are endowed with a mechanism to form forecasts using recursive learning algorithms and previous data from the economy. Then they develop some E-stability conditions which govern whether or not a given rational expectations equilibrium is aymptotically stable under least squares learning.

[10] See Aoki (2001) and Mankiw and Reis (2002) among others.

open economy respectively, whether it is relevant for equilibrium determinacy which measure of inflation is included in an active rule. The first work assumes that labor is immobile and the degree of nominal price stickiness can vary across the sectors. It shows that a contemporaneous rule that responds actively and solely to the sticky-price inflation of one of the sectors is sufficient to guarantee a unique equilibrium in the whole economy. The second work shows that in order to avoid real indeterminacy problems the rule should respond to the sticky-price (non-traded) inflation instead of to the flexible-price (traded) inflation. Nevertheless both works do not consider the learnability of equilibrium criterion as we do in the present paper.

In addition and in comparison to the work by Carlstrom et al., our results suggest that assuming that all the prices in the economy are sticky, even in a different degree, is not an innocuous assumption. As mentioned before we find that the extreme asymmetry in terms of price adjustment, i.e. flexible prices vs sticky prices, has important consequences for the dynamics of the economy. In particular an active rule that responds, directly or indirectly, to the flexible-price inflation can make the rule more prone to induce multiple equilibria. In this sense our results have at least two important implications from the perspective of the determinacy and learnability of equilibrium analyses. First they imply a preference to a particular measure of inflation. Second they suggest that the Taylor Principle of increasing the nominal interest rate proportionally more than the increase in inflation does not necessarily apply at the sectoral level as well as at the aggregate level.

The remainder of this paper is organized as follows. Section 2 presents the closed economy set-up with its main assumptions. Section 3 pursues the determinacy of equilibrium and learning (E-stability) analyses for forward-looking, contemporaneous and backward-looking rules that respond exclusively to one of the previously mentioned measures of inflation. Finally Section 4 concludes.

2 The Model

In this section we develop a simple infinite-horizon closed economy model. The economy is populated by a continuum of identical household-firm units and a government. Before we describe in detail the behavior of these agents we state a few general assumptions and definitions.

There are two consumption goods: a flexible-price good and a composite sticky-price good whose prices are denoted by P_t^F and P_t^S respectively. The relative price of the flexible-price good to the sticky price (composite) good is defined as $q_t = P_t^F / P_t^S$ and its dynamics is determined by

$$q_t = q_{t-1} \left(\frac{\pi_t^F}{\pi_t^S} \right) \tag{1}$$

where $\pi_t^F = P_t^F / P_{t-1}^F$ is the gross flexible-price inflation and $\pi_t^S = P_t^S / P_{t-1}^S$ is the gross sticky-price inflation.

2.1 The Government

The government issues two nominal liabilities: money, M_t^g, and a domestic bond, B_t^g, that pays a gross nominal interest rate R_t. It also makes transfers to household-firm units, $P_t^F T_t^g$, pays interest on its debt, $(R_t - 1)B_t^g$, and receives revenues from seigniorage. By letting $n_t^g = m_t^g + b_t^g = \frac{M_t^g + B_t^g}{P_t^F}$ denote the real government liabilities at the beginning of period we can write the government budget constraint in real terms as $n_t^g = \frac{R_{t-1}}{\pi_t^F} n_{t-1}^g - \left[\frac{(R_{t-1}-1)}{\pi_t^F} m_{t-1}^g - T_t^g\right]$.

We assume that the government follows a generic Ricardian fiscal policy. It picks the path of transfers, T_t^g, in order to satisfy the intertemporal version of its budget constraint in conjunction with the transversality condition $\lim_{t\to\infty} \frac{n_t^g}{\prod_{j=0}^{t-1}\left(R_j/\pi_{j+1}^F\right)} = 0$.

We define monetary policy as an interest rate feedback rule whereby the government sets the gross nominal interest rate, R_t, as an increasing and continuous function, $\rho\,(.)$, of the deviation of inflation with respect to a target. That is

$$R_t = \bar{R}\rho\left(E_t\left(\frac{\pi_{t+j}}{\bar{\pi}}\right)\right) \qquad \text{with} \;\; j = -1, 0, 1; \quad \text{and} \quad \rho\,(1) = 1 \tag{2}$$

where $\bar{\pi}$ and $\bar{R}$ correspond to the inflation and the nominal interest rate target that the government wants to achieve.[11] We will consider three different timings. In other words the rule may be forward-looking responding to the deviation of the expected inflation with respect to the target, $E_t\left(\frac{\pi_{t+1}}{\bar{\pi}}\right)$; contemporaneous, when the rule reacts to the current inflation deviation, $\left(\frac{\pi_t}{\bar{\pi}}\right)$, or backward-looking, when it responds to the past inflation deviation $\left(\frac{\pi_{t-1}}{\bar{\pi}}\right)$. The general measure of inflation that we consider in (2) corresponds to

$$\pi_t = \omega\pi_t^F + (1-\omega)\pi_t^S \tag{3}$$

where $\omega \in [0, 1]$ is the weight that the government puts on the flexible-price inflation. We will consider three possible measures: the full-inflation, when $\omega \in (0, 1)$, the flexible-price inflation, when $\omega = 1$, and the sticky-price inflation, when $\omega = 0$.

To conclude the description of monetary policy we assume that the government responds aggressively to inflation. In terms of Leeper (1991) we assume that the rule is "active" satisfying the following Taylor Principle.

Assumption 0. *The rule is active:* $\rho_\pi = \rho'\,(1) > 1$

Loosely speaking this means that in response to a one percent increase in inflation the government will raise the nominal interest by more that one percent.

[11] For simplicity we also assume that these targets correspond to the steady-state levels of these variables.

2.2 The Household-Firm Unit

There exists a large number of identical and infinitely lived household-firm units, each of whom derives utility from consuming, not working and liquidity services of money. The preferences of the representative unit are described by

$$E_0 \sum_{t=0}^{\infty} \beta^t [U(c_t^F, c_t^S) + H(h_t^F) + L(h_t^S) + J(m_t)] \tag{4}$$

where E_0 is the expectations operator conditional on the set of information available at time 0, $\beta \in (0,1)$ represents the subjective discount factor, c_t^F and c_t^S denote consumptions of a flexible-price good and a sticky-price good respectively, h_t^F and h_t^S denote labor efforts required to produce these goods and $m_t = \frac{M_t}{P_t^F}$ corresponds to real money balances measured with respect to the flexible price.

By the specification in (4) it is clear that we assume separability in the single period utility function among consumption, real money balances and labor. By doing this we remove the distortionary effects of transactions money demand.[12] More formally we assume the following.

Assumption 1. *The functions in (4) satisfy: a) $U(.)$, $H(.)$, $L(.)$ and $J(.)$ are continuous and twice differentiable; b) $U(.)$, is strictly increasing ($U_F \equiv \frac{dU}{dc_t^F} > 0$, $U_S > 0$) and strictly concave ($U_{FF} < 0$, $U_{SS} < 0$) and $U_{FS} = U_{SF}$, $U_{SS} - U_{SF}\frac{U_S}{U_F} < 0$ and $U_{FF}U_{SS} - (U_{FS})^2 > 0$; c) $H(.)$ and $L(.)$ are strictly decreasing ($H_h \equiv \frac{dH}{dh_t^F} < 0$, $L_h \equiv \frac{dL}{dh_t^S} < 0$) and concave ($H_{hh} \leq 0$, $L_{hh} \leq 0$); and d) $J(.)$ is strictly increasing ($J_m > 0$) and strictly concave ($J_{mm} < 0$).*

The particular assumptions about the instantaneous utility function of consumption, $U(.,.)$, guarantee that both goods are normal. But notice that we are not imposing any sign restrictions in the cross derivatives U_{FS} and U_{SF}. We assume that they are equal but in this paper we will consider three cases: c_t^F and c_t^S can be Edgeworth complements ($U_{FS} > 0$), Edgeworth substitutes ($U_{FS} < 0$) or they may be utility separable ($U_{FS} = 0$).[13]

The representative household-firm unit is engaged in the production of the flexible-price good and the sticky-price good by employing labor from a perfectly competitive market. The technologies are described by

$$y_t^F = z_t^F f\left(\check{h}_t^F\right) \qquad \text{and} \qquad y_t^S = z_t^S g\left(\tilde{h}_t^S\right)$$

[12] This assumption also allows us to write the real money balances that enter the utility of the agent in terms of the price of the flexible-price good, $m_t = \frac{M_t}{P_t^F}$, without consequences for our results.

[13] For instance the commonly used utility function

$$U(c_t^F, c_t^S) = \frac{\left[(\alpha_p)^{\frac{1}{a}}\left(c_t^F\right)^{\frac{a-1}{a}} + (1-\alpha_p)^{\frac{1}{a}}\left(c_t^S\right)^{\frac{a-1}{a}}\right]^{\left(\frac{a}{a-1}\right)(1-\sigma)} - 1}{1-\sigma}$$

with $\alpha_p \in (0,1)$ and $\sigma, a > 0$ satisfies Assumption 1a)-b) and the sign of U_{FS} is determined by the values of the intratemporal elasticity of substitution, a, and the intertemporal elasticity of substitution, $\frac{1}{\sigma}$. Specifically $U_{FS} \gtreqless 0$ if and only if $\frac{1}{\sigma} \gtreqless a$.

where $\breve{h}_t^F$ and $\tilde{h}_t^S$ denote the labor hired by the household-firm unit for the production of the flexible-price and the sticky-price goods respectively; z_t^F and z_t^S are productivity shocks whose logarithms, $\hat{z}_t^F = \ln(z_t^F)$ and $\hat{z}_t^S = \ln(z_t^S)$, follow stationary AR(1) stochastic processes:

$$\hat{z}_t^F = \phi^F \hat{z}_{t-1}^F + \xi_t^F \qquad\qquad \hat{z}_t^S = \phi^S \hat{z}_{t-1}^S + \xi_t^S \tag{5}$$

with $\phi^F, \phi^S \in (0,1)$ and $\xi_t^F \sim \mathcal{N}(0, (\sigma^F)^2)$ and $\xi_t^S \sim \mathcal{N}(0, (\sigma^S)^2)$. Technology shocks are the only source of fundamental intrinsic uncertainty and they are not correlated. The description of the technologies is completed by the following assumption.

Assumption 2. *$f(.)$ and $g(.)$ are continuous and twice differentiable, strictly increasing ($\frac{df}{d\breve{h}_t^F} \equiv f_h > 0$, $g_h > 0$), and strictly concave ($f_{hh} < 0$ and $g_{hh} < 0$.)*

The market for the flexible-price good is perfectly competitive.[14] In contrast there is imperfect competition in the market of the sticky-price good. The consumption of the stick-price good, c_t^S, is assumed to be a composite good made of a continuum of intermediate differentiated goods. The aggregator function is described of the Dixit-Stiglitz type. We assume that each household-firm unit is the monopolistic producer of one variety of sticky-price intermediate goods. The demand for the intermediate good is of the form $C_t^S d\left(\frac{\tilde{P}_t^S}{P_t^S}\right)$ satisfying $d(1) = 1$ and $d'(1) = -\mu$, with $\mu > 1$ where C_t^S denotes the level of aggregate demand for the sticky-price good, $\tilde{P}_t^S$ is the nominal price of the intermediate sticky-price good produced by the household-firm and P_t^S is the price of the composite sticky-price good. The household-firm unit that behaves as a monopolist in the production of the sticky-price good, sets the price of the good it supplies, $\tilde{P}_t^S$, taking the level of aggregate demand for the sticky-price good as given. Specifically the monopolist is constrained to satisfy demand at that price. That is

$$z_t^S g\left(\tilde{h}_t^S\right) \geq C_t^S d\left(\frac{\tilde{P}_t^S}{P_t^S}\right). \tag{6}$$

The way in which we introduce nominal price rigidities for the intermediate sticky-price good follows Rotemberg (1982). Then we assume the household-firm unit faces a resource cost of the type $\frac{\gamma}{2}\left(\frac{\tilde{P}_t^S}{\tilde{P}_{t-1}^S} - \bar{\pi}^S\right)^2$, reflecting that it is costly having the price of the good that it sets grow at a different rate from $\bar{\pi}^S$, the steady-state level of the gross sticky-price inflation rate.

The representative household-firm unit can invest in a bond issued by the government, B_t, that pays a gross nominal interest rate, R_t. This representative unit also receives a wage income from working, $W_t(h_t^F + h_t^S)$, transfers from the government T_t^g and dividends from selling the different consumption goods.

[14] Our results do not hinge on this assumption. The crucial assumption is that there are some goods that have flexible prices.

Then its flow budget constraint in units of the flexible-price good can be written as

$$n_t \leq \frac{R_{t-1}}{\pi_t^F} n_{t-1} + \frac{(1-R_{t-1})}{\pi_t^F} m_{t-1} + w_t (h_t^F + h_t^S) + T_t^g + \Delta_t - c_t^F - \frac{c_t^S}{q_t} \tag{7}$$

where $n_t = m_t + b_t$, $b_t = \frac{B_t}{P_t^F}$, $w_t = \frac{W_t}{P_t^F}$ and

$$\Delta_t = \left[z_t^F f\left(\check{h}_t^F\right) - \frac{W_t}{P_t^F} \check{h}_t^F \right] + \frac{1}{q_t} \left[\frac{\tilde{P}_t^S}{P_t^S} C_t^S d\left(\frac{\tilde{P}_t^S}{P_t^S}\right) - w_t q_t \tilde{h}_t^S - \frac{\gamma}{2}\left(\frac{\tilde{P}_t^S}{\tilde{P}_{t-1}^S} - \bar{\pi}\right)^2 \right]. \tag{8}$$

Equation (7) says that the end-of-period real financial domestic assets (money plus domestic bond) can be worth no more than the real value of financial domestic wealth brought into the period plus the sum of wage income, transfers and dividends (Δ_t) net of consumption. The dividends described in (8) correspond to the difference between sale revenues and costs from selling the flexible-price good and the sticky-price good.

Besides the budget constraint the household-firm unit is subject to an Non-Ponzi game condition

$$\lim_{t\to\infty} \frac{n_t}{\prod_{j=0}^{t-1} \left(R_j / \pi_{j+1}^F\right)} \geq 0. \tag{9}$$

The representative household-firm unit chooses the set of processes $\{c_t^F, c_t^S, h_t^F, h_t^S, \check{h}_t^F, \tilde{h}_t^S, \tilde{P}_t^S, m_t, n_t\}_{t=0}^{\infty}$ in order to maximize (4) subject to (6)-(9), given the initial condition n_{-1} and the set of processes $\{R_t, P_t^F, P_t^S, W_t, T_t^g, C_t^S, z_t^F, z_t^S\}$. Note that since the utility function specified in (4) implies that the preferences of the agent display non-sasiation then constraints (7) and (9) both hold with equality. The Appendix contains a detailed derivation of the necessary conditions for optimization. Imposing these conditions along with the market clearing conditions in the labor market ($h_t^F = \check{h}_t^F$ and $h_t^S = \tilde{h}_t^S$), the equilibrium symmetry ($\tilde{P}_t^S = P_t^S$ and $\tilde{h}_t^S = h_t^S$), the market clearing conditions for the sticky-price good

$$c_t^S = C_t^S = z_t^S g\left(h_t^S\right) - \frac{\gamma}{2}\left(\pi_t^S - \bar{\pi}\right)^2 \tag{10}$$

and the flexible-price good

$$c_t^F = z_t^F f\left(h_t^F\right) \tag{11}$$

and the definitions $\pi_t^S = P_t^S / P_{t-1}^S$, $d(1) = 1$ and $d'(1) = -\mu$ we obtain

$$-\frac{H_h(h_t^F)}{U_F(c_t^F, c_t^S)} = z_t^F f_h\left(h_t^F\right) \tag{12}$$

$$U_F(c_t^F, c_t^S) = \beta E_t \left[\frac{U_F(c_{t+1}^F, c_{t+1}^S) R_t}{\pi_{t+1}^F} \right] \tag{13}$$

$$U_S(c_t^F, c_t^S) = \beta E_t \left[\frac{U_S(c_{t+1}^F, c_{t+1}^S) R_t}{\pi_{t+1}^S} \right] \tag{14}$$

and

$$E_t \left[\frac{U_S(c_{t+1}^F, c_{t+1}^S)(\pi_{t+1}^S - \bar{\pi}^S)\pi_{t+1}^S}{U_S(c_t^F, c_t^S)} \right] = \frac{(\pi_t^S - \bar{\pi}^S)\pi_t^S}{\beta} - \frac{\mu c_t^S}{\beta\gamma} \left(\frac{\mu - 1}{\mu} - mc_t^S \right) \tag{15}$$

where $mc_t^S = -\frac{L_h(h_t^S)}{U_S(c_t^F, c_t^S) z_t^S g_h\left(h_t^S\right)}$ corresponds to the marginal cost of producing the sticky-price good.

The interpretation of these equations is straightforward. Condition (12) makes the marginal rate of substitution between labor (assigned to the production of the flexible-price good) and consumption of the flexible-price good equal to the marginal product of labor in the production of the flexible-price good. Equations (13) and (14) are the standard Euler equations for consumption of the flexible-price good and consumption of the sticky-price good. And equation (15) corresponds to the augmented Phillips curve for the sticky-price inflation.[15]

2.3 The Equilibrium

We are ready to provide a definition of the type of equilibrium in this economy that we are interested in studying.

Definition 1 *Given the inflation target, $\bar{\pi}$ and the exogenous stochastic processes $\left\{z_t^F, z_t^S\right\}_{t=0}^{\infty}$, a symmetric equilibrium is defined as a set of stochastic processes $\{c_t^F,\ c_t^S,\ h_t^F,\ h_t^S,\ \pi_t^F,\ \pi_t^S,\ \pi_t,\ R_t\}_{t=0}^{\infty}$ satisfying a) the market clearing conditions for the sticky-price good and the flexible-price good, (11) and (10); b) the intratemporal efficient condition (12); c) the Euler equations for consumption of the flexible-price good and consumption of the sticky-price good, (13) and (14); d) the augmented Phillips curve, (15), e) the interest rate rule (2); and f) definition (3).*

Note that this definition ignores the budget constraint of the government and its transversality condition. The reason is that by following a Ricardian fiscal policy the government guarantees that the intertemporal version of its budget constraint in conjunction with its transversality condition will be always satisfied. In addition real money balances do not appear in the definition. This is because monetary policy is described as an interest rate rule and real balances enter in the utility function in a separable way. In fact once we solve for $\{c_t^F,\ c_t^S,\ h_t^F,\ h_t^S,\ \pi_t^F,\ \pi_t^S,\ \pi_t,\ R_t\}_{t=0}^{\infty}$ it is possible to retrieve the set of stochastic processes $\{q_t,$ $m_t,\ b_t,\ w_t,\ mc_t^S,\ \lambda_t\}_{t=0}^{\infty}$ using (1), (7), and equations (52), (56), (57), and (59) that are presented in the Appendix.

[15] We would have derived a similar augmented Phillips curve if we had follow Calvo's (1983) approach.

2.4 The Log-linearized Economy

In order to pursue the determinacy of equilibrium analysis and the learnability analysis we log-linearize the system of equations that describe the dynamics of this economy around a steady state $\{\bar{c}^F,\ \bar{c}^S,\ \bar{h}^F,\ \bar{h}^S,\ \bar{\pi}^F,\ \bar{\pi}^S,\ \bar{\pi},\ \bar{R}\}$. In the Appendix we characterize this steady state.

Log-linearizing the equations of Definition 1 around the steady-state and manipulating them yields

$$\bar{c}^F \hat{c}_t^F = \alpha \bar{c}^S \hat{c}_t^S + \varkappa_1 \hat{z}_t^F \tag{16}$$

$$\hat{R}_t - E_t\left(\hat{\pi}_{t+1}^F\right) = \theta\left[E_t\left(\hat{c}_{t+1}^S\right) - \hat{c}_t^S\right] + \varkappa_2 \hat{z}_t^F \tag{17}$$

$$\hat{c}_t^S = E_t\left(\hat{c}_{t+1}^S\right) - \epsilon\left[\hat{R}_t - E_t\left(\hat{\pi}_{t+1}^S\right)\right] - \varkappa_3 \hat{z}_t^F \tag{18}$$

$$\hat{\pi}_t^S = \beta E_t(\hat{\pi}_{t+1}^S) + \beta\delta\hat{c}_t^S + \beta\varkappa_4\hat{z}_t^S + \beta\varkappa_5\hat{z}_t^F \tag{19}$$

$$\hat{R}_t = \rho_\pi E_t \hat{\pi}_{t+j} \qquad \textit{with } j = -1, 0, 1 \tag{20}$$

$$\hat{\pi}_t = \omega\hat{\pi}_t^F + (1-\omega)\hat{\pi}_t^S \tag{21}$$

where

$$\alpha = -\frac{(f_h)^2 U_{SF}}{f_{hh}U_F + (f_h)^2 U_{FF} + H_{hh}} \tag{22}$$

$$\epsilon = -\frac{U_S}{(\alpha U_{FS} + U_{SS})\,\bar{c}^S} \qquad\qquad \theta = -\frac{(\alpha U_{FF} + U_{SF})\,\bar{c}^S}{U_F} \tag{23}$$

$$\varkappa_1 = \frac{\bar{c}^F H_{hh} - (f_h)^2 U_F + f_{hh}U_F\bar{c}^F}{f_{hh}U_F + (f_h)^2 U_{FF} + H_{hh}} \qquad \varkappa_2 = \frac{\varkappa_1 U_{FF}(1-\varphi^F)}{U_F} \qquad \varkappa_3 = \frac{\varkappa_1 U_{FS}(1-\varphi^F)}{(\alpha U_{FS} + U_{SS})\,\bar{c}^S} \tag{24}$$

$$\delta = \frac{(\mu-1)(\bar{c}^S)^2}{\beta\gamma\bar{\pi}^2}\left[\frac{L_{hh}}{L_h g_h} - \frac{g_{hh}}{(g_h)^2} + \frac{1}{\epsilon\bar{c}^S}\right] \tag{25}$$

$$\varkappa_4 = \frac{(\mu-1)\bar{c}^S}{\beta\gamma\bar{\pi}^2}\left[\left(\frac{g_{hh}}{g_h} - \frac{L_{hh}}{L_h}\right)\frac{\bar{c}^S}{g_h} - 1\right] \qquad \varkappa_5 = -\frac{(\mu-1)\bar{c}^S}{\beta\gamma\bar{\pi}^2}\left(\frac{\varkappa_1 U_{FS}}{U_S}\right) \tag{26}$$

where for all the variables $\hat{x}_t = \log(\frac{x_t}{\bar{x}})$ and all the derivatives of the functions $U(.,.)$, $H(.)$, $L(.)$, $f(.)$ and $g(.)$ are evaluated at the steady state. To complete the system of log-linearized equations that describes the dynamics of the economy we have to consider the processes (5).

In addition for subsequent analysis it is useful to characterize the sign of some of the coefficients of the log-linearized system. The following Lemma accomplishes this goal.

Lemma 1 *Under Assumptions 1 and 2 it follows that a)* $\alpha \gtreqqless 0$ *if and only if* $U_{SF} \gtreqqless 0$; *b)* $\epsilon > 0$; *c)* $\theta \gtreqqless 0$ *if and only if* $U_{SF} \lesseqqgtr 0$; *and d)* $\delta > 0$

Proof. See Appendix. ■

We are now ready to pursue the determinacy and learnability of equilibrium analyses.

3 The Determinacy and Learning of Equilibrium Analyses

In this section we will pursue the determinacy and learnability of equilibrium analyses for three different rules (forward-looking, contemporaneous and backward-looking rules) that may react to different measures of inflation. The reason of pursuing determinacy of equilibrium and learnability analyses for these rules lies on two intertwined arguments.

The determinacy of equilibrium analysis will help us to discard rules that may generate aggregate instability in the economy by inducing multiple equilibria. Nevertheless even if we only consider rules that guarantee a unique equilibrium it does not follow necessarily that they will lead the economy to the targeted equilibrium. In particular it is possible that agents may not be able to coordinate their actions and learn the unique equilibrium (unique or multiple) induced by these rules. Hence a determinacy of equilibrium analysis for interest rate rules should in principle be accompanied by a learnability of equilibrium analysis in order to characterize rules that guarantee a unique and learnable equilibrium. To pursue both analyses we will focus on (5) and (16)-(21) and apply the following methodology.

3.1 The Methodology

For the determinacy of equilibrium analysis we will apply the results from Blanchard and Kahn (1980) that help to characterize whether a rational expectations linear system of equations such as

$$\hat{x}_t = \Lambda E_t(\hat{x}_{t+1}) + \Xi \hat{u}_t \tag{27}$$

has a unique equilibrium, multiple equilibria or no equilibrium, where $\hat{x}_t$ is a $s \times 1$ vector of endogenous variables, $\hat{u}_t$ is a $s \times 1$ vector that contains exogenous variables and forecast errors of endogenous variables, and Λ and Ξ are conformable matrices of constants. As is well known the analysis consists on comparing the number of roots of the matrix Λ that lie inside the unit circle with the number of non-predetermined variables. Then for the determinacy of equilibrium analysis of a specific rule that responds to a particular measure of inflation we only need to reduce the log-linearized system described by (16)-(21) and (5) to the representation in (27) in order to apply Blanchard and Kahn's results.[16]

To pursue the learnability analysis we follow Evans and Honkapohja (1999, 2001) and assume that agents in our model no longer are endowed with rational expectations. Instead they have adaptive rules whereby agents form expectations using recursive least squares updating and data from the system. In particular we will focus on the concept of E-stability as a criterion of learnability of an equilibrium. That is, an equilibrium is learnable if it is E-stable.

We will concentrate on this concept since, for models that display a unique equilibrium, Marcet and Sargent (1989) and Evans and Honkapohja (1999, 2001) have shown that under some general conditions, the notional time concept of expectational stability of a rational expectations equilibrium governs the local convergence of real time adaptive learning algorithms. This implies that under E-stability, recursive least-squares learning is locally convergent to the rational expectations equilibrium.

To define the concept of E-stability we proceed to explain the methodology proposed by Evans and Honkapohja (1999, 2001). Consider the model

$$\hat{y}_t = \eta + \Omega E_t \hat{y}_{t+1} + \Gamma \hat{y}_{t-1} + \Psi \hat{z}_t \qquad \text{and} \qquad \hat{z}_t = \Phi \hat{z}_{t-1} + \xi_t \tag{28}$$

where $\hat{y}_t$ is a $s \times 1$ vector of endogenous variables, $\hat{z}_t$ is a $s \times 1$ vector of exogenous variables which follows a stationary VAR whose $s \times 1$ vector of shocks consists of white noise terms, and η, Ω, Γ, Ψ, and Φ are conformable matrices of constants. In addition E_t denotes in general (non-rational) expectations.[17] Next, assume that agents follow a Perceived Law of Motion (PLM) that in the case of real determinacy corresponds to the fundamental solution or Minimal State Variable (MSV) representation[18]

$$\hat{y}_t = k + P\hat{y}_{t-1} + Q\hat{z}_t \tag{29}$$

where k, P, Q are conformable vectors and matrices and are to be derived by the method of undetermined coefficients. Iterating forward this law of motion and using it to eliminate all the forecasts ($E_t \hat{y}_{t+1} =$

[16] See also Farmer (1999).

[17] We use the notation E_t to denote both rational expectations in the determinacy of equilibrium analysis and possibly non-rational expectations in the learnability analysis.

[18] See McCallum (1983) and Uhlig (1997).

$k + P\hat{y}_t + Q\Phi\hat{z}_t$) in the model specified in (28) we can derive the implied Actual Law of Motion (ALM)

$$\hat{y}_t = k^A + P^A\hat{y}_{t-1} + Q^A\hat{z}_t,$$

which in tandem with (29) defines the T-mapping $T(k, P, Q) = (k^A, P^A, Q^A)$. The fixed points of this mapping correspond to the rational expectations equilibrium.

We say that an equilibrium described by the MSV representation is E-stable if the T-mapping is stable at the equilibrium in question. More formally a fixed point of the T-mapping is E-stable provided that the differential equation

$$\frac{d(k, P, Q)}{d\tau_n} = T(k, P, Q) - (k, P, Q) \tag{30}$$

is locally asymptotically stable at that particular fixed point, where τ_n is defined as the "notional" time.

We proceed to derive the E-stability conditions of the system (28). Consider the ALM of this system which corresponds to

$$\hat{y}_t = (I - \Omega P)^{-1}\left[\eta + \Omega k + \Gamma\hat{y}_{t-1} + (\Omega Q\Phi + \Psi)\hat{z}_t\right]$$

where I is the identity matrix. Using this ALM and the PLM in (29) we find the T-Mapping

$$T(k, P, Q) = (k^A, P^A, Q^A) = [(I - \Omega P)^{-1}(\eta + \Omega k), (I - \Omega P)^{-1}\Gamma, (I - \Omega P)^{-1}(\Omega Q\Phi + \Psi)] \tag{31}$$

whose fixed point correspond to the rational expectation equilibrium and can be used to determine the coefficients matrices $\bar{k}, \bar{P}$, and $\bar{Q}$ of the MSV solutions. That is $\bar{k}, \bar{P}$, and $\bar{Q}$ are the solutions to

$$(I - \Omega P - \Omega)k = \eta \qquad \Omega P^2 - P + \Gamma = 0 \qquad \text{and} \qquad (I - \Omega P)Q - \Omega Q\Phi = \Psi$$

Using (31) the "E-stability" conditions under which the differential equation (30) is locally asymptotically stable are derived and stated in Proposition 10.3 in Evans and Honkapohja (2001). They basically say that an MSV solution (29) to the system (28) is E-stable if all the eigenvalues of the matrices

$$DT_k = (I - \Omega\bar{P})^{-1}\Omega \qquad DT_P = \left[(I - \Omega\bar{P})^{-1}\Gamma\right]' \otimes [(I - \Omega\bar{P})^{-1}\Omega] \qquad \text{and} \qquad DT_Q = \Phi' \otimes [(I - \Omega\bar{P})^{-1}\Omega] \tag{32}$$

evaluated at $\bar{k}, \bar{P}$, and $\bar{Q}$ have real parts less than one. On the contrary the MSV solution is not E-stable if any of the eigenvalues has real part larger than one.

A fundamental part in the learnability analysis consists of making explicit what agents know when they form their forecasts. In the E-stability analysis literature it is common to assume that when agents form their expectations $E_t\hat{y}_t$, they do not know $\hat{y}_t$. In this paper this assumption may be inconsistent with the assumptions that we use to derive the equations of the model.[19] Henceforth for the learnability analysis we will assume that when forming expectations agents know $\hat{y}_t$.

To summarize, in order to determine whether a specific rule that responds to a particular measure of inflation induces a learnable MSV representation of the equilibrium we proceed as follows. First we reduce the log-linearized version of the economy described by (16)-(21) and (5) to a system similar to (28). Then we calculate the MSV solution of this system and check if all the eigenvalues of the matrices in (32) have real parts less than one.

3.2 Forward-Looking Rules

We start our analysis by studying rules that react to one-period ahead (expected) inflation, i.e. forward looking rules. Our analysis is motivated by the estimates of these rules provided by Clarida et al. (1998, 2000) and Corbo (2000) for industrialized and developing economies. Specifically we focus on rules whose log-linear representation corresponds to

$$\hat{R}_t = \rho_\pi E_t \hat{\pi}_{t+1} \qquad \text{with} \qquad \rho_\pi > 1. \tag{33}$$

Using equations (5), (16)-(19), (21) and (33) we obtain the system

$$\hat{y}_t = \eta + \Omega E_t \hat{y}_{t+1} + \Psi \hat{z}_t \qquad \text{and} \qquad \hat{z}_t = \Phi \hat{z}_{t-1} + \xi_t \tag{34}$$

where $\hat{y}_t = [\hat{\pi}_t^S, \hat{c}_t^S]'$, $\hat{z}_t = [\hat{z}_t^F, \hat{z}_t^S]'$, $\xi_t = [\xi_t^F, \xi_t^S]'$, $\eta = [0, 0]'$,

$$\Omega = \begin{bmatrix} \beta \left[1 + \frac{\delta\epsilon(1-\rho_\pi)}{(1-\tau\omega\rho_\pi)}\right] & \beta\delta \\ \frac{(1-\rho_\pi)\epsilon}{(1-\tau\omega\rho_\pi)} & 1 \end{bmatrix}, \tag{35}$$

$\Phi = \begin{bmatrix} \phi^F & 0 \\ 0 & \phi^S \end{bmatrix}$, and the form of Ψ is omitted since it is not required for the following analysis. Moreover

$$\tau = 1 - \theta\epsilon \tag{36}$$

and it is characterized by the following Lemma.

[19] In particular notice that for the derivation of the first order conditions of the representative agent we assume that $E_t \tilde{P}_t^S = \tilde{P}_t^S$ (or in a symmetric equilibrium $E_t P_t^S = P_t^S$.) Therefore assuming in the learnability analysis that the agents do not know P_t^S when forming expectations would have some implications for the specification of the model. Specifically it would require to replace $\hat{\pi}_t^S$ in (19) with the expectations of $\hat{\pi}_t^S$.

Lemma 2 *Under Assumptions 1 and 2 a) if* $U_{SF} \geqq 0$ *(*$\hat{c}_t^F$ *and* $\hat{c}_t^S$ *Edgeworth complements or utility separable) then* $\tau \geqq 1$ *and b) if* $U_{SF} < 0$ *(*$\hat{c}_t^F$ *and* $\hat{c}_t^S$ *are Edgeworth substitutes) then* $0 < \tau < 1$.

Proof. See Appendix. ■

The MSV solution of (34) corresponds to

$$\hat{y}_t = \bar{k} + \bar{Q}\hat{z}_t \quad \text{where} \quad \bar{k} = 0 \quad \text{and} \quad \bar{Q} \quad \text{solves} \quad Q - \Omega Q \Phi = \Psi. \tag{37}$$

Using this solution and (5), (16), (17), (21), and (33) we can find the set of processes $\{\hat{c}_t^F,\ \hat{\pi}_t^F,\ \hat{\pi}_t,\ R_t\}$.

We start by analyzing rules that respond to the full-inflation defined by (21) with $\omega \in (0,1)$. The following Proposition summarizes the main results . To save some space we state the results of the determinacy of equilibrium analysis together with the results from the learning analysis. Nevertheless from our previous description of the methodology, it should be clear that the determinacy analysis is pursued under the assumption of rational expectations whereas the learning analysis consider possibly non-rational expectations.

Proposition 1 *Consider the system defined in (34). Let* $\rho_\pi^\omega = \frac{\beta\delta\epsilon + 2(1+\beta)}{\beta\delta\epsilon + 2(1+\beta)\tau\omega}$ *where* $\rho_\pi^\omega > 0$, *and assume that the government follows an active forward-looking rule,* $\hat{R}_t = \rho_\pi E_t \hat{\pi}_{t+1}$ *with* $\rho_\pi > 1$, *that responds solely to full-inflation,* $\hat{\pi} = \omega\hat{\pi}_t^F + (1-\omega)\hat{\pi}_t^S$ *with* $\omega \in (0,1)$.

> **a)** *Assume* $U_{SF} \leq 0$ *(*$\hat{c}_t^F$ *and* $\hat{c}_t^S$ *are either Edgeworth substitutes or utility separable). For any* $\omega \in (0,1)$, *if* $1 < \rho_\pi < \rho_\pi^\omega$ *then there exists a unique rational expectations equilibrium and, under possibly non-rational expectations, the MSV solution* $\hat{y}_t = \bar{k} + \bar{Q}\hat{z}_t$ *is E-stable.*
>
> **b)** *Assume* $U_{SF} > 0$ *(*$\hat{c}_t^F$ *and* $\hat{c}_t^S$ *are Edgeworth complements). For any* $\omega \in (0, 1/\tau)$, *if* $1 < \rho_\pi < \rho_\pi^\omega$ *then there exists a unique rational expectations equilibrium and, under possibly non-rational expectations, the MSV solution* $\hat{y}_t = \bar{k} + \bar{Q}\hat{z}_t$ *is E-stable.*

Proof. See the Appendix. ■

Proposition 1 says that conditions under which active forward-looking rules deliver a unique and learnable equilibrium depend not only on the interest rate response coefficient to inflation, but also on the weight ω that the government puts on the flexible-price inflation to construct the full-inflation, and on whether $\hat{c}_t^F$ and $\hat{c}_t^S$ are Edgeworth substitutes, complements or utility separable.

The importance of ω can be grasped by realizing that the threshold ρ_π^ω is a decreasing function of ω. Which means that as the government puts more weight on the flexible-price inflation then the range $1 < \rho_\pi < \rho_\pi^\omega$ under which an active forward-looking rule will deliver a unique and learnable equilibrium will be reduced. In fact it is possible to prove that as long as $\rho_\pi > \rho_\pi^\omega$ then the forward-looking rule will always deliver multiple equilibria.[20] This observation has important consequences for the performance of forward-looking rules when $\hat{c}_t^F$ and $\hat{c}_t^S$ are either Edgeworth complements ($U_{SF} > 0$) or utility separable ($U_{SF} = 0$). To understand this we can study thoroughly the properties of the function ρ_π^ω and construct Figure 1.

The top, middle, and bottom panels of this Figure correspond to the cases when the goods are Edgeworth substitutes, utility separable and Edgeworth complements, respectively. These panels show the combinations of ρ_π and ω for which active forward-looking rules that respond to full inflation will deliver a unique and learnable equilibrium (real determinacy and E-stability) as well as the combinations for which there exist multiple equilibria (real indeterminacy). Within the regions for which there exist multiple equilibria we also characterize whether the MSV solution is E-stable or E-unstable. In the figures "D" and "I" stand for real determinacy (unique equilibrium) and real indeterminacy (multiple equilibria) respectively. Whereas "ES" stands for E-stable and "EU" for E-unstable.

Starting from the bottom panel, that corresponds to the case in which $\hat{c}_t^F$ and $\hat{c}_t^S$ are Edgeworth complements ($U_{SF} > 0$), we can observe the following. Active forward-looking rules, satisfying $\rho_\pi > 1$, will always lead to real indeterminacy either for $\omega \in (0, \frac{1}{\tau})$ and $\rho_\pi > \rho_\pi^\omega$ or for $\omega > \frac{1}{\tau}$ and any $\rho_\pi > 1$. Only rules satisfying $1 < \rho_\pi < \rho_\pi^\omega$ and $\omega \in (0, \frac{1}{\tau})$, as stated in part b) of Proposition 1), will deliver a unique and learnable equilibrium. Now consider the middle panel associated with the case in which $\hat{c}_t^F$ and $\hat{c}_t^S$ are utility separable ($U_{SF} = 0$). In this case the region of real determinacy and E-stability increases with respect to the previous case of Edgeworth complements. Nevertheless the range $1 < \rho_\pi < \rho_\pi^\omega$ under which there exist real determinacy and E-stability still decreases as the weight ω increases. In fact for ω very close to one we can conclude that any active rule with $\rho_\pi > 1$ will induce multiple equilibria.

Finally in the top panel, associated with the case when $\hat{c}_t^F$ and $\hat{c}_t^S$ are Edgeworth substitutes ($U_{SF} < 0$), we can see that the problems of real indeterminacy seem to subside. Although it is still true that the region of real determinacy and E-stability characterized by part a) of Proposition 1 decreases as ω increases.

The dependence of these results on ω suggests that it is important to study active forward-looking rules in the extreme cases when $\omega = 1$ and $\omega = 0$. According to (21) these cases correspond to react exclusively to either the flexible-price inflation, $\hat{\pi}^F$, or the sticky-price inflation, $\hat{\pi}^S$. By studying these two cases we will be also able to provide an economic intuition of the results from Proposition 1 and Figure 1.

The following Proposition summarizes the determinacy of equilibrium and learnability results for an active rule that responds solely to the flexible-price inflation.

Proposition 2 *Consider the system defined in (34) evaluated at $\omega = 1$. Let $\rho_\pi^1 = \frac{\beta\delta\epsilon+2(1+\beta)}{\beta\delta\epsilon+2(1+\beta)\tau\omega}$ where $\rho_\pi^1 > 0$, and assume that the government follows an active forward-looking rule that responds to the flexible-price inflation ($\hat{\pi}^F$) and is described by $\hat{R}_t = \rho_\pi E_t \hat{\pi}_{t+1}^F$ with $\rho_\pi > 1$.*

[20] Results are available from the authors upon request.

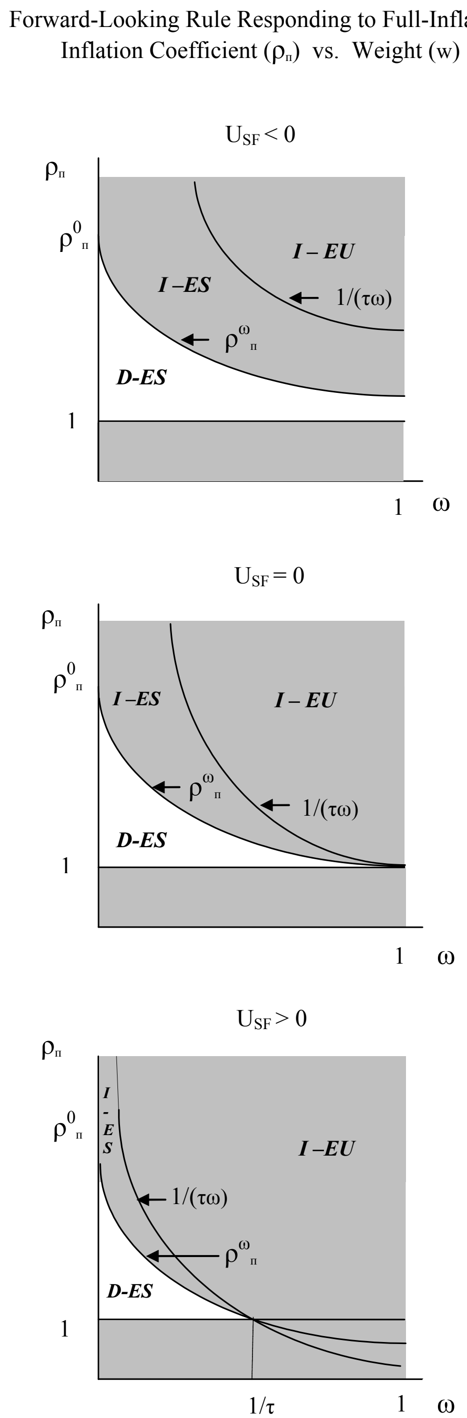

Figure 1: Forward-looking rules responding to full-inflation. This Figure shows the possible combinations of the interest rate response coefficient (ρ_π) and weight on the flexible-price inflation (ω) under which there is real determinacy (D) or real indeterminacy (I). It also shows the combinations of these parameters under which the MSV solution is E-stable (ES) or E-unstable (EU). The top, middle and bottom panels corresponds to the cases of Edgeworth substitutes ($U_{SF} < 0$), ($U_{SF} = 0$) and complements ($U_{SF} > 0$).

a) *Assume $U_{SF} < 0$ ($\hat{c}_t^F$ and $\hat{c}_t^S$ are Edgeworth substitutes). If $1 < \rho_\pi < \rho_\pi^1$ then there exists a unique rational expectations equilibrium and, under possibly non-rational expectations, the MSV solution $\hat{y}_t = \bar{k} + \bar{Q}\hat{z}_t$ is E-stable.*

b) *Assume $U_{SF} \geq 0$ ($\hat{c}_t^F$ and $\hat{c}_t^S$ are either Edgeworth complements or utility separable). If $\rho_\pi > 1$ then there exist multiple rational expectations equilibria and, under possibly non-rational expectations, the MSV solution $\hat{y}_t = \bar{k} + \bar{Q}\hat{z}_t$ is E-unstable.*

Proof. See the Appendix. ■

The results of Proposition 2 show that conditions under which active forward-looking rules that respond exclusively to the flexible-price inflation deliver a unique and learnable equilibrium depend strongly on whether are $\hat{c}_t^F$ and $\hat{c}_t^S$ are Edgeworth substitutes, Edgeworth complements or utility separable. In particular when $\hat{c}_t^F$ and $\hat{c}_t^S$ are either Edgeworth complements or utility separable, then these rules will always induce aggregate instability by generating multiple equilibria. More interestingly even if we focus on the MSV solution, we find that this representation is never E-stable.

To understand this real indeterminacy result it is useful to derive the following equation from (16)-(18), (20) and (21) evaluated at $\omega = 1$,

$$E_t\hat{\pi}_{t+1}^F = \left(\frac{1-\tau}{1-\tau\rho_\pi}\right) E_t\hat{\pi}_{t+1}^S \tag{38}$$

where we have ignored the terms associated with $\hat{z}_t^F$. With this equation and equations (18) and (19), it is possible to construct a self-fulfilling equilibrium when $\hat{c}_t^F$ and $\hat{c}_t^S$ are either Edgeworth complements or utility separable. But this is not feasible when they are substitutes.

Consider the cases of Edgeworth complementarity or separability first and assume that people expect a higher sticky-price inflation. Since $\hat{c}_t^F$ and $\hat{c}_t^S$ are Edgeworth complements or utility separable then by Lemma 2 we know that $\tau \geqq 1$. If in addition the rule is active $\rho_\pi > 1$, then by (38) we see that higher expectations of the sticky price-inflation will be associated with higher expectations of the flexible price inflation satisfying $E_t\hat{\pi}_{t+1}^F < E_t\hat{\pi}_{t+1}^S$. But this means that if the rule responds actively and exclusively to the flexible-price inflation $\hat{R}_t = \rho_\pi E_t\hat{\pi}_{t+1}^F$ then the government in response to people's expectations of a higher sticky-price inflation can decrease the real interest rate measured with respect to the expected sticky-price inflation, $\hat{R}_t - E_t\hat{\pi}_{t+1}^S$. This stimulates consumption of the sticky-price good (see 18) and as a response to this, firms raise the price of the sticky-price good inducing a higher sticky-price inflation (see 19.) Then the original expectations of a higher sticky-price inflation are validated.

Next consider the case when $\hat{c}_t^F$ and $\hat{c}_t^S$ are Edgeworth substitutes. From Lemma 2 we know that $0 < \tau < 1$. If in addition the rule is active $\rho_\pi > 1$ and satisfies $\rho_\pi < \frac{1}{\tau}$, then by (38) we see that higher expectations of the sticky price-inflation will be associated with higher expectations of the flexible price inflation but in this case we have that $E_t\hat{\pi}_{t+1}^F > E_t\hat{\pi}_{t+1}^S$. Since the rule responds actively ($\rho_\pi > 1$) and exclusively to the flexible-price inflation $\hat{R}_t = \rho_\pi E_t \hat{\pi}^F_{t+1}$, then the government will increase the real interest rate measured with respect to the expected sticky-price inflation, $\hat{R}_t - E_t \hat{\pi}^S_{t+1}$. Consumption of the sticky-price good declines (see 18) and as a response to this firms decrease the price of the sticky-price good inducing a lower sticky-price inflation (see 19.) Then the original expectations of a higher sticky-price inflation are not validated.

In a similar way it is possible to grasp the intuition of why the MSV solution is not learnable (E-stable) when $\hat{c}^F_t$ and $\hat{c}^S_t$ are Edgeworth complements or utility separable whereas it is learnable if $\hat{c}^F_t$ and $\hat{c}^S_t$ are Edgeworth substitutes. In order to do so we use (38) and the rule $\hat{R}_t = \rho_\pi E_t \hat{\pi}^F_{t+1}$ to derive the real interest rate measured with respect to the expected sticky-price inflation

$$\hat{R}_t - E_t \hat{\pi}^S_{t+1} = -\left(\frac{1-\rho_\pi}{1-\tau\rho_\pi}\right) E_t \hat{\pi}^S_{t+1} \tag{39}$$

Consider the case in which they are Edgeworth complements or utility separable ($\tau \geq 1$) and recall that $\rho_\pi > 1$. According to (39) a deviation of people's expected sticky-price inflation from the rational expectations value will always lead to a decrease in the real interest rate measured with respect to the expected sticky-price inflation. But this will stimulate consumption of the sticky-price good by (18) which in turn will increase the sticky-price inflation by (19.) Over time this mechanism leads to upward revisions of both the expected sticky-price inflation and the expected consumption of the sticky-price goods. Therefore the policy of targeting actively the flexible-price inflation will not off-set the initial deviation from the rational expectations equilibrium. It will move the economy further away from it.

On the contrary assume that the goods are Edgeworth substitutes. In this case $0 < \tau < 1$. Using (39) we can see that a deviation of people's expected sticky-price inflation from the rational expectations value may induce an increase in the real interest rate measured with respect to the expected sticky-price inflation, as long as $1 < \rho_\pi < \frac{1}{\tau}$. But this will decrease consumption of the sticky-price good by (18) which in turn will decrease the sticky-price inflation by (19). Over time this mechanism leads to downward revisions of both the expected sticky-price inflation and the expected consumption of the sticky-price goods. Hence in this case the policy of targeting actively the flexible-price inflation is able to lead the original people's expectations towards the the rational expectations value.

We continue studying rules that respond to the sticky-price inflation. In contrast to the previous results, we show that the results for these rules do not depend on the joint characteristics of $\hat{c}^F_t$ and $\hat{c}^S_t$.

Proposition 3 *Consider the system defined in (34) with* $\omega = 0$. *Let* $\rho^0_\pi = 1 + \frac{2(1+\beta)}{\beta\delta\epsilon}$ *and assume that the government follows an active forward-looking rule that responds to the sticky-price inflation* ($\hat{\pi}^S$) *and is described by* $\hat{R}_t = \rho_\pi E_t \hat{\pi}^S_{t+1}$ *with* $\rho_\pi > 1$. *Regardless of whether* $U_{SF} \gtreqqless 0$ *(whether* $\hat{c}^F_t$ *and* $\hat{c}^S_t$ *are Edgeworth substitutes, complements or utility separable), if* $1 < \rho_\pi < \rho^0_\pi$ *then there exists a unique rational expectations* *equilibrium and, under possibly non-rational expectations, the MSV solution* $\hat{y}_t = \bar{k} + \bar{Q}\hat{z}_t$ *is E-stable.*

Proof. See the Appendix. ■

It can be also proved that when $\rho_\pi > \rho_\pi^0$ then the rule induces multiple equilibria. In fact the results of Proposition 3 are equivalent to the ones in Bullard and Mitra (2002) and therefore we omit an explanation.

Figure 1 can be also used to understand graphically the results for the analysis of forward-looking rules that respond to either the flexible-price inflation or the sticky-price inflation. These cases correspond to $\omega = 1$ and $\omega = 0$. From this Figure and regardless of whether $\hat{c}_t^F$ and $\hat{c}_t^S$ are Edgeworth substitutes, complements or utility separable, we can conclude the following. In general forward-looking rules that respond to either the full-inflation, with $\omega \in (0, 1)$ or to the flexible-price inflation are more prone to induce multiple equilibria and E-instability than rules that respond to the sticky-price inflation. We proceed to verify whether this statement is still valid for contemporaneous rules.

3.3 Contemporaneous Rules

Now we study contemporaneous rules. The motivation for studying these rules stems from empirical evidence provided by Lubik and Schorfheide (2003) among others. Specifically we focus on rules of the following type

$$\hat{R}_t = \rho_\pi \hat{\pi}_t \qquad \text{with} \qquad \rho_\pi > 1 \tag{40}$$

and, as before, we start by analyzing rules that respond to the full-inflation defined by (21) with $\omega \in (0, 1)$.

Using equations (5), (16)-(19), (21) and (40) we obtain the system

$$\hat{y}_t = \eta + \Omega E_t \hat{y}_{t+1} + \Psi \hat{z}_t \qquad \text{and} \qquad \hat{z}_t = \Phi \hat{z}_{t-1} + \xi_t \tag{41}$$

where $\hat{y}_t = [\hat{\pi}_t^F, \hat{\pi}_t^S, \hat{c}_t^S]'$, $\hat{z}_t = [\hat{z}_t^F, \hat{z}_t^S]'$, $\xi_t = [\xi_t^F, \xi_t^S]'$, $\eta = [0, 0, 0]'$,

$$\Omega = \begin{bmatrix} \frac{1}{\tau\omega\rho_\pi} + \frac{\beta\delta\epsilon(1-\omega)}{\tau\omega} & -\frac{(1-\tau)}{\tau\omega\rho_\pi} - \beta\left(\frac{1-\omega}{\omega}\right)\left(1 + \frac{\delta\epsilon}{\tau}\right) & -\frac{\beta\delta(1-\omega)}{\omega} \\ -\frac{\beta\delta\epsilon}{\tau} & \beta\left(1 + \frac{\delta\epsilon}{\tau}\right) & \beta\delta \\ \frac{\epsilon}{\tau} & \frac{\epsilon}{\tau} & 1 \end{bmatrix}, \tag{42}$$

$\Phi = \begin{bmatrix} \phi^F & 0 \\ 0 & \phi^S \end{bmatrix}$ and the form of Ψ is omitted since it is not required for the following analysis.

We find that active contemporaneous rules that react to the current full-inflation always deliver real indeterminacy regardless of whether $\hat{c}_t^F$ and $\hat{c}_t^S$ are Edgeworth substitutes, complements or utility separable. Moreover although the MSV solution maybe learnable when $\hat{c}_t^F$ and $\hat{c}_t^S$ are Edgeworth substitutes, it is not

E-stable when these goods are complements or utility separable, for any $\rho_\pi > 1$. The following Proposition formalizes these statements.

Proposition 4 *Consider the system defined in (41). Assume that the government follows an active contemporaneous rule,* $\hat{R}_t = \rho_\pi \hat{\pi}_t$ *with* $\rho_\pi > 1$, *that responds solely to full-inflation,* $\hat{\pi} = \omega \hat{\pi}_t^F + (1-\omega)\hat{\pi}_t^S$ *with* $\omega \in (0,1)$.

a) *Regardless of whether* $U_{SF} \gtreqless 0$ *(whether* $\hat{c}_t^F$ *and* $\hat{c}_t^S$ *are Edgeworth substitutes, complements or utility separable), there exist multiple rational expectations equilibria.*

b) *Under possibly non-rational expectations, the MSV solution* $\hat{y}_t = \bar{k} + \bar{Q}\hat{z}_t$ *is E-unstable if* $U_{SF} \geq 0$ *(Edgeworth complements or utility separable), and it may be E-stable if* $U_{SF} < 0$ *(Edgeworth substitutes.)*

Proof. See Appendix. ■

It is surprising that active contemporaneous rules with respect to the full-inflation will always deliver multiple equilibria. To understand this point it is useful to study rules that respond exclusively to either the flexible-price inflation or the sticky-price inflation.

The next two Propositions show that the previous results for active contemporaneous rules that respond to the full-inflation are mainly explained by the fact that by reacting to the full-inflation the government is indirectly responding to the flexible-price inflation. In particular, we find that regardless of whether $\hat{c}_t^F$ and $\hat{c}_t^S$ are either Edgeworth substitutes, complements or utility separable, an active contemporaneous rule that responds to the flexible-price inflation will induce multiple equilibria. In addition when $\hat{c}_t^F$ and $\hat{c}_t^S$ are either complements or utility separable then the MSV solution is not learnable. These results are the same as the ones presented for rules that react to full-inflation.

Proposition 5 *Consider the system defined in (41) with* $\omega = 1$. *Assume that the government follows an active contemporaneous rule that responds to the flexible-price inflation* ($\hat{\pi}^F$) *described by* $\hat{R}_t = \rho_\pi \hat{\pi}_t^F$ *with* $\rho_\pi > 1$.

a) *Regardless of whether* $U_{SF} \gtreqless 0$ *(whether* $\hat{c}_t^F$ *and* $\hat{c}_t^S$ *are Edgeworth substitutes, complements or utility separable), there exist multiple rational expectations equilibria.*

b) *Under possibly non-rational expectations, the MSV solution* $\hat{y}_t = \bar{k} + \bar{Q}\hat{z}_t$ *is E-unstable if* $U_{SF} \geq 0$ *(Edgeworth complements or utility separable), and it may be E-stable if* $U_{SF} < 0$ *(Edgeworth substitutes.)*

Proof. See Appendix. ■

On the contrary rules that respond to the sticky-price inflation guarantee a unique and learnable equilibrium. To derive this we cannot use the system (41) with Ω defined in (42) since some entries of Ω are not well defined when $\omega = 0$. But we can use equations (5), (16)-(19), (21) and the rule $\hat{R}_t = \rho_\pi \hat{\pi}_t^S$ to obtain the new system

$$\hat{y}_t = \eta + \Omega E_t \hat{y}_{t+1} + \Psi \hat{z}_t \qquad \text{and} \qquad \hat{z}_t = \Phi \hat{z}_{t-1} + \xi_t \tag{43}$$

where $\hat{y}_t = [\hat{\pi}_t^S, \hat{c}_t^S]'$, $\hat{z}_t = [\hat{z}_t^F, \hat{z}_t^S]', \xi_t = [\xi_t^F, \xi_t^S]'$, $\eta = [0, 0]'$

$$\Omega = \begin{bmatrix} \frac{\beta(1+\delta\epsilon)}{1+\beta\delta\epsilon\rho_\pi} & \frac{\beta\delta}{1+\beta\delta\epsilon\rho_\pi} \\ \frac{(1-\beta\rho_\pi)\epsilon}{1+\beta\delta\epsilon\rho_\pi} & \frac{1}{1+\beta\delta\epsilon\rho_\pi} \end{bmatrix}, \tag{44}$$

$\Phi = \begin{bmatrix} \phi^F & 0 \\ 0 & \phi^S \end{bmatrix}$ and the form of Ψ is omitted since it is not required for the following analysis. The following Proposition is based on this system.

Proposition 6 *Consider the system defined in (43). Assume that the government follows an active contemporaneous rule that responds to the sticky-price inflation ($\hat{\pi}^S$) and is described by $\hat{R}_t = \rho_\pi \hat{\pi}_t^S$ with $\rho_\pi > 1$. Regardless of whether $U_{SF} \gtreqless 0$ (whether $\hat{c}_t^F$ and $\hat{c}_t^S$ are Edgeworth substitutes, complements or utility separable) there exists a unique rational expectations equilibrium and, under possibly non-rational expectations, the MSV solution $\hat{y}_t = \bar{k} + \bar{Q}\hat{z}_t$ is E-stable.*

Proof. See the Appendix. ■

Our analyses for contemporaneous and forward-looking rules with different measures of inflation have some important policy implications. First forward-looking and contemporaneous rules that respond to either the full-inflation or the flexible-price inflation are more prone to deliver real indeterminacy than rules that respond exclusively to the sticky price inflation. This result is very clear when $\hat{c}_t^F$ and $\hat{c}_t^S$ are either Edgeworth complements or utility separable. More importantly under these assumptions about $\hat{c}_t^F$ and $\hat{c}_t^S$, the MSV solution is never learnable for active forward-looking and contemporaneous rules that respond to the flexible-price inflation. As a consequence of this, the MSV solution is less prone to be learnable for active forward-looking and contemporaneous rules that respond to the full-inflation.

These results in tandem with the results of Proposition 6 suggest that the measure of inflation that should be included in a rule is the sticky-price inflation. The natural question that arises is whether this policy recommendation is still valid for backward-looking rules. We proceed pursuing the analysis of these rules.

3.4 Backward-Looking Rules

In this subsection we pursue the determinacy of equilibrium and learning analyses for active backward-looking rules of the type

$$\hat{R}_t = \rho_\pi \hat{\pi}_{t-1} \qquad \text{with} \qquad \rho_\pi > 1. \tag{45}$$

The motivation for studying these rules comes not only from an empirical motivation such as Taylor (1993) but also from a theoretical motivation. In fact works by Benhabib et al. (2001), Bernanke and Woodford (1997) and Carlstrom and Fuerst (1999) among others, suggest that backward-looking rules are less prone to induce local multiple equilibria.

We start our analysis by studying rules that respond exclusively and actively to the full-inflation defined by (21) with $\omega \in (0,1)$. For the determinacy of equilibrium analysis we use equations (5), (16)-(19), (21) and (45) to obtain the system

$$\hat{x}_t = \Lambda E_t \hat{x}_{t+1} + \Sigma \hat{z}_t \qquad \textit{and} \qquad \hat{z}_t = \Phi \hat{z}_{t-1} + \xi_t \tag{46}$$

where $\hat{x}_t = [\hat{R}_t, \hat{\pi}_t^F, \hat{\pi}_t^S, \hat{c}_t^S]'$, $\hat{x}_t = [\hat{z}_t^F, \hat{z}_t^S]', \xi_t = [\xi_t^F, \xi_t^S]'$, $E_t \hat{R}_{t+1} = \hat{R}_{t+1}$ since $\hat{R}_t$ is a predetermined variable,

$$\Lambda = \begin{bmatrix} 0 & \frac{1}{\tau} & \frac{\tau-1}{\tau} & 0 \\ \frac{1}{\omega\rho_\pi} & \frac{(1-\omega)\beta\delta\epsilon}{\omega\tau} & -\beta\left(\frac{1-\omega}{\omega}\right)\left(1+\frac{\delta\epsilon}{\tau}\right) & -\frac{\beta\delta(1-\omega)}{\omega} \\ 0 & -\frac{\beta\delta\epsilon}{\tau} & \beta\left(1+\frac{\delta\epsilon}{\tau}\right) & \beta\delta \\ 0 & -\frac{\epsilon}{\tau} & \frac{\epsilon}{\tau} & 1 \end{bmatrix}, \tag{47}$$

$\Phi = \begin{bmatrix} \phi^F & 0 \\ 0 & \phi^S \end{bmatrix}$, and the form of Ψ is omitted since it is not required for the following analysis.

On the other hand since there is a predetermined variable (an endogenous state variable) then for the E-stability analysis we utilize (5), (16)-(19), (20) and (45) to derive the system

$$\hat{y}_t = \eta + \Omega E_t \hat{y}_{t+1} + \Gamma \hat{y}_{t-1} + \Psi \hat{z}_t \qquad \textit{and} \qquad \hat{z}_t = \Phi \hat{z}_{t-1} + \xi_t \tag{48}$$

where $\hat{x}_t = [\hat{R}_t, \hat{\pi}_t^F, \hat{\pi}_t^S, \hat{c}_t^S]'$, $\hat{x}_t = [\hat{z}_t^F, \hat{z}_t^S]', \xi_t = [\xi_t^F, \xi_t^S]'$, $\eta = [0,0,0,0]'$, $E_t \hat{R}_{t+1} = \hat{R}_{t+1}$ (since $\hat{R}_t$ is a predetermined variable),

$$\Omega = \begin{bmatrix} 0 & 1 & \tau - 1 & 0 \\ \frac{1}{\omega\rho_\pi} & 0 & -\beta\left(\frac{1-\omega}{\omega}\right)(1+\delta\epsilon) & -\frac{\beta\delta(1-\omega)}{\omega} \\ 0 & 0 & \beta(1+\delta\epsilon) & \beta\delta \\ 0 & 0 & \epsilon & 1 \end{bmatrix} \tag{49}$$

$$\Gamma = \begin{bmatrix} 0 & (1-\tau)\omega\rho_\pi & (1-\tau)(1-\omega)\rho_\pi & 0 \\ 0 & (1-\omega)\beta\delta\epsilon\rho_\pi & \frac{(1-\omega)^2\beta\delta\epsilon\rho_\pi}{\omega} & 0 \\ 0 & -\beta\delta\epsilon\omega\rho_\pi & -(1-\omega)\beta\delta\epsilon\rho_\pi & 0 \\ 0 & -\omega\epsilon\rho_\pi & -(1-\omega)\epsilon\rho_\pi & 0 \end{bmatrix} \tag{50}$$

$$\Psi = \begin{bmatrix} \frac{\varkappa_1 U_{FF}(1-\phi^F)}{U_F} & \frac{\theta\epsilon\varkappa_1 U_{FS}(1-\phi^F)}{U_S} \\ -\frac{(1-\omega)\beta\varkappa_5}{\omega} & -\beta\left(\frac{1-\omega}{\omega}\right)\left[\varkappa_4 + \frac{\delta\epsilon\varkappa_1 U_{FS}(1-\phi^F)}{U_S}\right] \\ \beta\varkappa_5 & \beta\left[\varkappa_4 + \frac{\delta\epsilon\varkappa_1 U_{FS}(1-\phi^F)}{U_S}\right] \\ 0 & \frac{\epsilon\varkappa_1 U_{FS}(1-\phi^F)}{U_S} \end{bmatrix}, \tag{51}$$

$\Phi = \begin{bmatrix} \phi^F & 0 \\ 0 & \phi^S \end{bmatrix}$, and $\varkappa_1$, $\varkappa_4$, and $\varkappa_5$ are constants defined in (24) and (26).

For the determinacy of equilibrium and learning analyses of backward-looking rules that respond to the full-inflation, it is not possible to derive analytical results. Therefore we have to simulate the model. We will assume some functional forms and assign some reasonable values for the parameters associated with these forms. Then we will apply the methodology described in 3.1 taking into account that for the E-stability analysis the MSV solution corresponds to $\hat{y}_t = k + P\hat{y}_{t-1} + Q\hat{z}_t$.

The functional forms are the following. For consumption and labor preferences

$$U(c_t^F, c_t^S) = \frac{\left[(\alpha^p)^{\frac{1}{a}}\left(c_t^F\right)^{\frac{a-1}{a}} + (1-\alpha^p)^{\frac{1}{a}}\left(c_t^S\right)^{\frac{a-1}{a}}\right]^{\left(\frac{a}{a-1}\right)(1-\sigma)} - 1}{1-\sigma} \qquad V(h_t^F, h_t^S) = -\frac{\left(h_t^F\right)^{1+\xi^p}}{1+\xi^p} - \frac{\left(h_t^S\right)^{1+\xi^p}}{1+\xi^p}$$

where $\alpha^p \in (0,1)$, $\sigma, a > 0$ and $\xi^p \geq 0$. Note that they satisfy Assumption 1. In particular the sign of U_{FS} is determined by the values of the intratemporal elasticity of substitution, a, and the intertemporal elasticity of substitution, $\frac{1}{\sigma}$. That is, $U_{FS} \gtreqless 0$ if and only if $\frac{1}{\sigma} \gtreqless a$.

The technologies are described by

$$y_t^F = (h_t^F)^{\theta^F} \qquad \text{and} \qquad y_t^S = (h_t^S)^{\theta^S}$$

where $\theta^F, \theta^S \in (0,1)$. They satisfy Assumption 2.

Our parametrization of the model does not pretend to be a calibration of a particular economy. It is only illustrative. The time unit is a quarter. Then we set $\beta = 0.98$. We will assume that the share of sticky-price goods is bigger than the share of flexible-price goods. Hence we set $\alpha^p = 0.2$. We set $\sigma = 1$. Since the relative magnitudes of σ and a determine the sign of U_{FS}, that is whether c_t^F and c_t^S are Edgeworth substitutes, complements or utility separable, we will vary a in some of the simulations. We pick $a = 0.8$ to represent goods that are complements and $a = 2$ to represent substitutes. In some other cases we will consider that $a \in (0, 4.5)$. We also set $\theta^F = \theta^S = 0.5$. In addition we use the following values $\bar{\pi} = 1.01$, $\xi^p = 0.5$, $\mu = 6$, $\gamma = 17.5$, $\phi^F = \phi^S = 0.82$ and $\xi^p = 0.5$ that agree with some of the parameter values used in the New Keynesian monetary rules literature.[21]

The results of the simulations are presented in Figure 2. It shows the combinations of the interest rate response coefficient to inflation, ρ_π, and weight on the flexible-price inflation, $\omega \in (0, 1)$, that lead to either real determinacy (D) or real indeterminacy (I)., That is, a unique equilibrium or multiple equilibria. This Figure also shows the combinations for these parameters under which the MSV solution is either E-stable (ES) or E-unstable (EU). In the figure the marker "$\otimes$" represents both indeterminacy and E-instability of the MSV solution. Whereas the marker "$\bigcirc$" represents determinacy and E-instability of the MSV solution. The right panel corresponds to the case of Edgeworth complements whereas the left panel corresponds to the case of substitutes.

The Figure shows that regardless of the type of goods, active backward-looking rules that respond to the full-inflation are more prone to induce indeterminacy as the weight ω is reduced. In addition whether the goods are complements or substitutes matter for the determinacy of equilibrium. In general if they are substitutes active backward-looking rules with respect to the full inflation are more prone to deliver multiple equilibria than if they are complements. More interestingly the figure also shows that regardless of the type of goods, the interest rate response coefficient to inflation ρ_π and the weight ω, the MSV solution is not learnable (E-stable).

As before, and in order to understand these results, we analyze active backward-looking rules that respond solely to the flexible-price inflation. To accomplish this goal we can use the systems in (46) and (48) evaluating the matrices of the system at $\omega = 1$. It is possible to derive analytical results for the determinacy of equilibrium analysis when the goods are utility separable; but when they are either Edgeworth complements or substitutes we have to rely on simulations. On the contrary for the learning analysis, it is feasible to derive analytical results for all the cases. The following proposition summarizes the analytical results.

Proposition 7 *Assume that the government follows an active backward-looking rules that responds to the flexible-price inflation ($\hat{\pi}^F$) described by $\hat{R}_t = \rho_\pi \hat{\pi}^F_{t-1}$ with $\rho_\pi > 1$.*

a) *Consider the system (46) with $\omega = 1$. If $U_{SF} = 0$ ($\hat{c}^F_t$ and $\hat{c}^S_t$ are utility separable) then the rule induces a unique rational expectations equilibrium.*

b) *Consider the system (48) with $\omega = 1$. Under possibly non-rational expectations and regardless of whether $U_{SF} \gtreqless 0$ (whether $\hat{c}^F_t$ and $\hat{c}^S_t$ are Edgeworth substitutes, complements or utility separable), the MSV solution $\hat{y}_t = k + P\hat{y}_{t-1} + Q\hat{z}_t$ is E-unstable.*

[21] See Schmitt-Grohé and Uribe (2004) and Woodford (2003) among others.

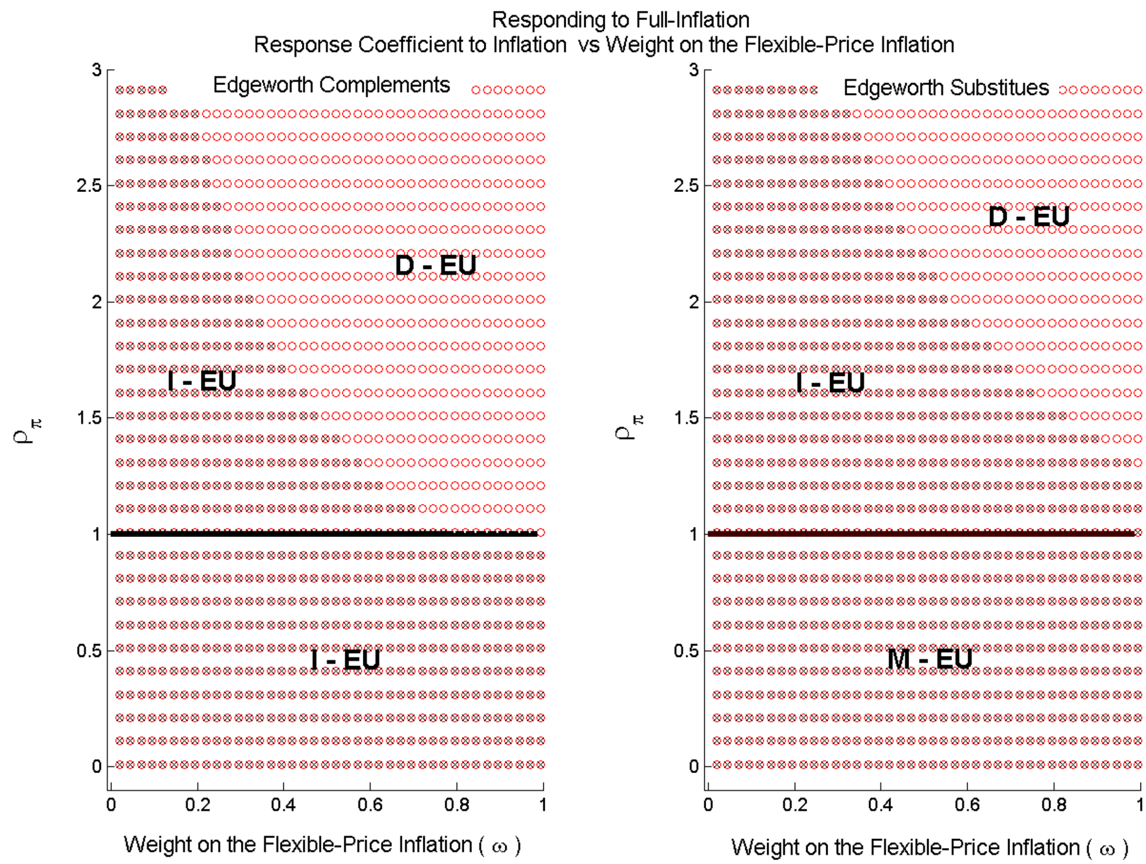

Figure 2: Backward-looking rules responding to full-inflation. This Figure shows the possible combinations of the interest rate response coefficient (ρ_π) and the weight on the flexible-price inflation (ω) under which there is real determinacy (D) or real indeterminacy (I). It also shows the combinations of these parameters under which the MSV solution is E-stable (ES) or E-unstable (EU). The marker "$\otimes$" represents both indeterminacy and E-instability of the MSV solution. Whereas the marker "$\bigcirc$" represents determinacy and E-instability of the MSV solution.

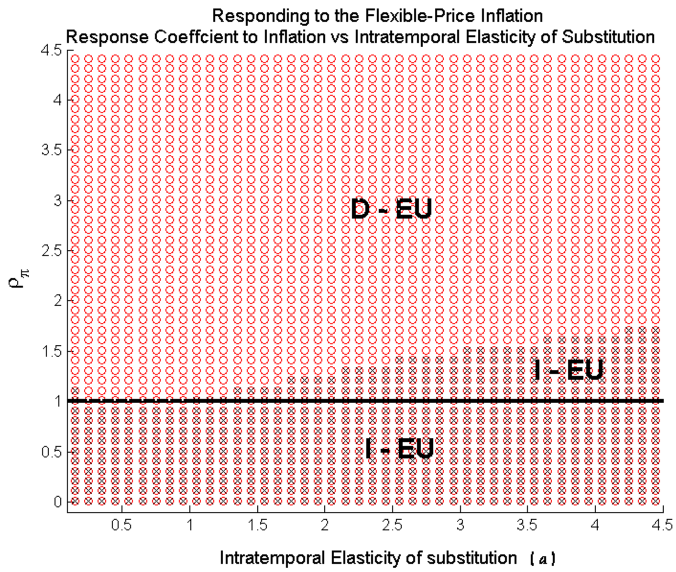

Figure 3: Backward-looking rules responding to the flexible-price inflation. This Figure shows the possible combinations of the interest rate response coefficient (ρ_π) and the intratemporal elasticity of substitution (a) under which there is real determinacy (D) or real indeterminacy (I). It also shows the combinations of these parameters under which the MSV solution is E-stable (ES) or E-unstable (EU). The marker "⊗" represents both indeterminacy and E-instability of the MSV solution. Whereas the marker "◯" represents determinacy and E-instability of the MSV solution.

Proof. See Appendix. ∎

Proposition 7 states one of the most important results of the paper: even if the rule is backward-looking, reacting to the flexible-price inflation will make the MSV solution E-unstable. Therefore it is not learnable. The proposition also points out that as long as the two goods are utility separable then the rule will guarantee a unique equilibrium. In order to complete the determinacy analysis for these rules, we use the aforementioned parametrization to simulate the model when the goods are substitutes and complements. The results are presented in Figure 3.

This Figure shows the combinations of the interest rate response coefficient to inflation (ρ_π) and the intratemporal elasticity of substitution (a) that lead to either real determinacy (D) or real indeterminacy (I). That is, a unique equilibrium or multiple equilibria. In addition it shows the combinations for these parameters under which the MSV solution is either E-stable (ES) or E-unstable (EU). The marker "⊗" represents both indeterminacy and E-instability of the MSV solution. Whereas the marker "◯" represents determinacy and E-instability of the MSV solution. As explained before the relation between a and σ determines the type of goods under consideration. For our parametrization we have that $\sigma = 1$. Hence the case of Edgeworth complements, substitutes and utility separable correspond to $a < 1$, $a > 1$ and $a = 1$, respectively.

Figure 3 confirms our results from Proposition 7: the type of goods under consideration (complements, substitutes and utility separable) does not affect the E-stability characterization of the MSV solution. This solution is never learnable. Nevertheless the type of goods may affect the determinacy results for active backward-looking rules. Interestingly when the goods are Edgeworth substitutes, active backward-looking rules may still induce multiple equilibria.

To conclude our analysis we focus on active backward-looking rules that respond to the sticky-price inflation. We cannot set $\omega = 0$ and use the systems (46) and (48) to study these rules. The reason is that by doing this some of the coefficients of Λ and Ω in (47) and (49) are not well defined. Hence we have to derive new systems to pursue the determinacy and learning analyses. In addition we cannot derive analytical results for the learning analysis. Hence we prefer to present some simulations that combine both analyses.

Figure 4 presents and summarizes our results for these rules. As in Figure 3, this figure shows the combinations of the interest rate response coefficient to inflation (ρ_π) and the intratemporal elasticity of substitution (a) that lead to either real determinacy (D) or real indeterminacy (I). In addition it shows the combinations for these parameters under which the MSV solution is either E-stable (ES) or E-unstable (EU). The marker "$\otimes$" represents both indeterminacy and E-instability of the MSV solution. Whereas no marker represents determinacy and E-stability of the MSV solution. As argued before $a < 1$, $a > 1$ and $a = 1$, correspond to assuming that the goods are Edgeworth complements, substitutes and utility separable respectively.

The main conclusion from Figure 4 is that active backward-looking rules with respect to the sticky-price inflation will deliver a unique and learnable equilibrium regardless of the type of goods under consideration. It is important to emphasize that this result cannot be derived setting $\omega = 0$ and using the systems (46) and (48) that were used for the analysis of rules that respond to the full inflation. In fact it is clear from Figure 2 that when $\omega = 0$, the determinacy and learning results do not coincide with those in Figure 4. The reason is that there is a discontinuity for the aforementioned systems at $\omega = 0$.

4 Conclusions

In this paper we develop a closed economy model with a flexible-price good and a sticky-price good to answer the following question: which measure of inflation should the government target in an active interest rate rule in order to guarantee a unique equilibrium whose MSV solution is learnable in the E-stability sense proposed by Evans and Honkapojha (2001)?. We find that the answer to this question is the sticky-price inflation.

In order to obtain this answer we study how the conditions under which interest rate rules lead to real determinacy and to E-stability of the MSV solution may depend not only on the interest rate response coef-

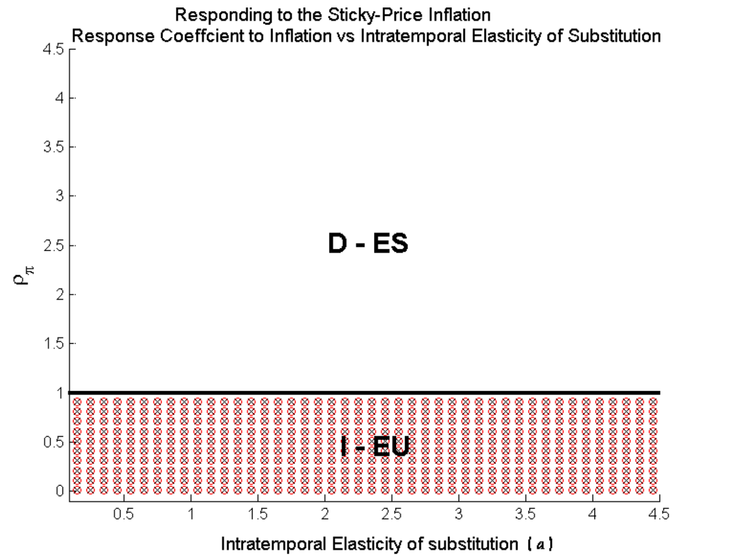

Figure 4: Backward-looking rules responding to the sticky-price inflation. This Figure shows the possible combinations of the interest rate response coefficient (ρ_π) and the intratemporal elasticity of substitution (a) under which there is real determinancy (D) or real indeterminacy (I). It also shows the combinations of these parameters under which the MSV solution is E-stable (ES) or E-unstable (EU). The marker "$\otimes$" represents both indeterminacy and E-instability of the MSV solution. Whereas no marker represents determinacy and E-stability of the MSV solution.

ficient to inflation but also on the timing of the rule; on whether the two consumption goods are Edgeworth substitutes, complements or utility separable; and more importantly on the measure of inflation included in the rule.

We find specifically that responding to either the flexible-price inflation or the full inflation (a convex combination of the sticky-price and the fexlible-price inflations) is more prone to induce multiple equilibria (real indeterminacy) and to make the MSV solution not learnable. Although backward-looking rules with respect to the flexible-price inflation or the full inflation may guarantee a unique equilibrium, they also make the MSV solution not learnable in the E-stability sense.

Only responding actively to the sticky-price inflation seems to be a robust policy recommendation across timings and types of goods, in order to guarantee a unique and learnable equilibrium. This result has at least two important implications from the perspective of the determinacy and learnability of equilibrium analyses. First they imply a preference to a particular measure of inflation. Second they suggest that the Taylor Principle of increasing the nominal interest rate proportionally more than the increase in inflation does not necessarily apply at the sectoral level as well as at the aggregate level.

A Appendix

A.1 The First Order Conditions of the Household-Firm Unit Problem in the Simple Model

The representative household-firm unit chooses the set of processes $\{c_t^F,\ c_t^S,\ h_t^F,\ h_t^S,\ \check{h}_t^F,\ \tilde{h}_t^S,\ \tilde{P}_t^S,\ m_t,\ n_t\}_{t=0}^{\infty}$ in order to maximize (4) subject to (6), (7) and (9), given the initial condition n_{-1} and the set of processes $\{R_t,\ P_t^F,\ P_t^S,\ W_t, T_t^g\ ,\ C_t^S,\ z_t^F,\ z_t^S\}$. The first order conditions correspond to (7) and (9) with equality and

$$U_F(c_t^F, c_t^S) = \lambda_t \tag{52}$$

$$\frac{U_F(c_t^F, c_t^S)}{U_S(c_t^F, c_t^S)} = q_t \tag{53}$$

$$-\frac{H_h(h_t^F)}{U_F(c_t^F, c_t^S)} = w_t \tag{54}$$

$$-\frac{L_h(h_t^S)}{U_S(c_t^F, c_t^S)} = w_t q_t \tag{55}$$

$$1 = \frac{w_t}{z_t^F f_h\left(\check{h}_t^F\right)} \tag{56}$$

$$mc_t^S = \frac{w_t}{q_t z_t^S g_h\left(\tilde{h}_t^S\right)} \tag{57}$$

$$\begin{aligned} 0 \quad = \quad & \frac{\lambda_t C_t^S}{q_t P_t^S} d\left(\frac{\tilde{P}_t^S}{P_t^S}\right) + \frac{\lambda_t \tilde{P}_t^S C_t^S}{q_t \left(P_t^S\right)^2} d'\left(\frac{\tilde{P}_t^S}{P_t^S}\right) - \frac{\gamma \lambda_t}{q_t \tilde{P}_{t-1}^S}\left(\frac{\tilde{P}_t^S}{\tilde{P}_{t-1}^S} - \bar{\pi}^S\right) - \frac{mc_t^S \lambda_t C_t^S}{q_t \tilde{P}_t^S} d'\left(\frac{\tilde{P}_t^S}{P_t^S}\right) \\ & + \beta\gamma E_t\left[\frac{\lambda_{t+1}}{q_{t+1}}\left(\frac{\tilde{P}_{t+1}^S}{\tilde{P}_t^S} - \bar{\pi}^S\right)\frac{\tilde{P}_{t+1}^S}{\left(\tilde{P}_t^S\right)^2}\right] \end{aligned} \tag{58}$$

$$\frac{1}{J_m(m_t)} = \frac{1}{U_F(c_t^F, c_t^S)}\left(\frac{R_t}{R_t - 1}\right) \tag{59}$$

$$\lambda_t = \beta E_t\left(\frac{\lambda_{t+1} R_t}{\pi_{t+1}^F}\right) \tag{60}$$

where $\frac{mc_t^S \lambda_t}{q_t}$ and λ_t correspond to the Lagrange multipliers of (6) and (7) respectively.

We will focus on a symmetric equilibrium in which all the monopolistic producers of sticky-price goods pick the same price. Hence $\tilde{P}_t^S = P_t^S$. Since all the monopolists face the same wage rate, W_t, and the same production function, $z_t^S g\left(h_t^S\right)$, then they will demand the same amount of labor $\tilde{h}_t^S = h_t^S$. In equilibrium the money market, the domestic bond market, the labor markets, the sticky-price goods market and the flexible-price good market clear. Therefore

$$m_t = m_t^g \tag{61}$$

$$b_t = b_t^g \tag{62}$$

$$h_t^F = \check{h}_t^F \tag{63}$$

$$h_t^S = \tilde{h}_t^S \tag{64}$$

$$z_t^S g\left(h_t^S\right) = c_t^S + \frac{\gamma}{2}\left(\pi_t^S - \bar{\pi}^S\right)^2 \tag{65}$$

and

$$c_t^F = z_t^F f\left(h_t^F\right) \tag{66}$$

Combining (54) and (56) yields equation (12). Using conditions (52) and (60) we obtain the Euler equation (13). Utilizing (1), (52), (53), and (60)we derive the Euler equation (14). And finally using the notion of a symmetric equilibrium, conditions (6), (53), (55), (57), (64), (58) and the definitions $\pi_t^S = P_t^S/P_{t-1}^S$, $d(1) = 1$ and $d'(1) = -\mu$ we can derive the augmented Phillips curve described by equation (15).

A.2 Characterization of The Steady State

We use (1), (10)-(15), and the condition that at the steady state $x_t = \bar{x}$ for all the variables to derive

$$\bar{\pi}^S = \bar{\pi}^F = \bar{\pi} \qquad \beta\bar{R} = \bar{\pi}^S$$

$$\left(\frac{\mu - 1}{\mu}\right) U_S(f(\bar{h}^F), g(\bar{h}^S)) = -\frac{L_h(\bar{h}^S)}{g_h\left(\bar{h}^S\right)} \qquad \bar{c}^S = g\left(\bar{h}^S\right)$$

$$U_F(f(\bar{h}^F), g(\bar{h}^S)) = -\frac{H_h(\bar{h}^F)}{f_h\left(\bar{h}^F\right)} \qquad \bar{c}^F = f\left(\bar{h}^F\right)$$

Then it is simple to prove that under some assumptions that include Assumptions 1 and 2, and given $\mu > 1$, $\beta \in (0, 1)$ and $\bar{\pi} > 1$, there exists a steady state $\{\bar{c}^F, \bar{c}^S, \bar{h}^F, \bar{h}^S, \bar{\pi}^F, \bar{\pi}^S, \bar{R}\}$ for the economy that satisfies these equations with $\bar{c}^F, \bar{c}^S, \bar{h}^F, \bar{h}^S > 0$ and $\bar{\pi}^F, \bar{\pi}^S, \bar{R} > 1$. In particular to guarantee that there exist $\bar{c}^F, \bar{c}^S, \bar{h}^F, \bar{h}^S > 0$ we need to impose some Inada-type assumptions such as $U_F(0, \bar{c}^S) = U_S(\bar{c}^F, 0) = \infty$, $U_F(\infty, \bar{c}^S) = U_S(\bar{c}^F, \infty) = 0$, $f_h(0) = g_h(0) = \infty$, and $f_h(\infty) = g_h(\infty) = 0$.[22]

A.3 Proof of Lemma 1

Proof. The proof is straightforward. a) follows from Assumptions 1 and 2 and the definition of α in (22). b) follows from using the definition of ϵ in (23), Assumptions 1a), 1b), 1c), and 2, $\bar{c}^S > 0$ and $\alpha U_{FS} + U_{SS} = \frac{f_{hh}U_F U_{SS} + (f_h)^2(U_{FF}U_{SS} - U_{FS}^2) + H_{hh}U_{SS}}{f_{hh}U_F + (f_h)^2 U_{FF} + H_{hh}} < 0$. Part c) can be proved by using the definition of θ in (23), Assumptions 1a), 1b), 1c), and 2, $\bar{c}^S > 0$ and $\alpha U_{FF} + U_{SF} = -\frac{f_{hh}U_F U_{SF}}{f_{hh}U_F + (f_h)^2 U_{FF} + H_{hh}}$. Finally d) follows from using Assumptions 1a), 1b), 1c), and 2, with $\epsilon > 0$, $\mu > 1$ and $\bar{c}^S, \beta, \gamma, \bar{\pi} > 0$. ■

[22] Details are available from the author upon request.

A.4 Proof of Lemma 2

Proof. To prove part a) we use parts part b) and c) of Lemma 1. If $U_{SF} \geqq 0$ then $\theta \leqq 0$ which together with $\epsilon > 0$ and the definition of τ imply $\tau \geqq 1$. On the other hand if $U_{SF} < 0$, we use the definitions of ϵ and θ in (23) to write $\tau = 1 - \frac{\theta}{\epsilon} = 1 - 1/\left[\frac{U_{SS}U_F}{U_{SF}U_S} + \frac{(U_{SS}U_{FF}-U_{FS}^2)f_h^2+H_{hh}U_{SS}}{U_{SF}U_S f_{hh}}\right]$. From Assumptions 1a), 1b), 1c), 2, and $U_{SF} < 0$ we have that $\frac{U_{SS}U_F}{U_{SF}U_S} > 1$ and $\frac{(U_{SS}U_{FF}-U_{FS}^2)f_h^2+H_{hh}U_{SS}}{U_{SF}U_S f_{hh}} > 0$. Then $0 < 1/\left[\frac{(U_{SS}U_{FF}-U_{FS}^2)f_h^2+H_{hh}U_{SS}}{U_{SF}U_S f_{hh}} + \frac{U_{SS}U_F}{U_{SF}U_S}\right] < 1$ and therefore $0 < \tau < 1$ if $U_{SF} < 0$. ■

A.5 Proof of Proposition 1

Proof. The characteristic polynomial for Ω defined in (35) is given by $\mathcal{P}(v) = v^2 - Trace(\Omega)v + Det(\Omega)$ where $Det(\Omega)$ refers to the determinant of Ω and

$$Trace(\Omega) = 1 + \beta\left[1 + \frac{\delta\epsilon(1-\rho_\pi)}{(1-\tau\omega\rho_\pi)}\right] \qquad \text{and} \qquad Det(\Omega) = \beta$$

Using these and the characteristic polynomial $\mathcal{P}(v)$ we can derive that

$$\mathcal{P}(1) = -\frac{\beta\delta\epsilon(1-\rho_\pi)}{(1-\tau\omega\rho_\pi)} \qquad \text{and} \qquad \mathcal{P}(-1) = \left[\frac{\beta\delta\epsilon + 2(1+\beta)\tau\omega}{(1-\tau\omega\rho_\pi)}\right](\rho_\pi^\omega - \rho_\pi) \tag{67}$$

We first prove a). Recall from Lemma 1 that $\epsilon > 0$ and $\delta > 0$. By Lemma 2 we know that if $U_{SF} = 0$ then $\tau = 1$ and if $U_{SF} < 0$ then $0 < \tau < 1$. Using these with $\delta > 0$, $\epsilon > 0$, $\beta \in (0,1)$ and $\omega \in (0,1)$ we can infer that $\rho_\pi^\omega = \frac{\beta\delta\epsilon+2(1+\beta)}{\beta\delta\epsilon+2(1+\beta)\tau\omega} > 1$ and that $\frac{\beta\delta\epsilon+2(1+\beta)}{\beta\delta\epsilon+2(1+\beta)\tau\omega} < \frac{\beta\delta\epsilon+2(1+\beta)}{[\beta\delta\epsilon+2(1+\beta)]\tau\omega}$. Hence $1 < \rho_\pi^\omega < \frac{1}{\tau\omega}$. This in tandem with $1 < \rho_\pi < \rho_\pi^\omega$, (67), $\delta > 0$, $\epsilon > 0$, $\beta \in (0,1)$ and $\omega \in (0,1)$ imply that $\mathcal{P}(1) > 0$ and $\mathcal{P}(-1) > 0$. Moreover $0 < Det(\Omega) < 1$ since $Det(\Omega) = \beta$ and $\beta \in (0,1)$. This together with $\mathcal{P}(1) > 0$ and $\mathcal{P}(-1) > 0$ imply that the two eigenvalues of Ω are inside the unit circle.[23] Since there are two non-predetermined variables, $\hat{\pi}_t^S$ and $\hat{c}_t^S$, then by Blanchard and Kahn (1980) we conclude that there exists a unique rational expectations equilibrium.

To prove that $1 < \rho_\pi < \rho_\pi^\omega$ for any $\omega \in (0,1)$ implies that the MSV solution $\hat{y}_t = \bar{k} + \bar{Q}\hat{z}_t$ is E-stable we verify that the E-stability conditions are satisfied. For $\hat{y}_t = \bar{k} + \bar{Q}\hat{z}_t$ the E-stability conditions in (32) are reduced to verify that all the eigenvalues of $DT_k = \Omega$ and $DT_Q = \Phi' \otimes \Omega$ have real parts less than one. Nevertheless by assumption the eigenvalues of Φ are less than one. This implies that the E-stability conditions will be satisfied whenever that all the eigenvalues of Ω have real parts less than one. To check this it is sufficient to verify that all the eigenvalues of $\Omega - I$ have negative real parts. We do so in the following way. The eigenvalues v_1 and v_2 of $\Omega - I$ satisfy $Trace(\Omega - I) = v_1 + v_2$ and $Det(\Omega - I) = v_1 v_2$.[24] Then sufficient and necessary conditions for all the eigenvalues of $\Omega - I$ to have negative real parts are $Trace(\Omega - I) < 0$ and $Det(\Omega - I) > 0$. We calculate

$$Trace(\Omega - I) = \beta\left[1 + \frac{\delta\epsilon(1-\rho_\pi)}{(1-\tau\omega\rho_\pi)}\right] - 1 \qquad \text{and} \qquad Det(\Omega - I) = -\frac{\beta\delta\epsilon(1-\rho_\pi)}{(1-\tau\omega\rho_\pi)} \tag{68}$$

As was proved before $\rho_\pi^\omega < \frac{1}{\tau\omega}$ which together with the assumption $1 < \rho_\pi < \rho_\pi^\omega$, the expression for

[23] See Azariadis (1993).
[24] See Horn and Johnson (1985).

$Det(\Omega - I)$ in (68), $\delta > 0$, $\epsilon > 0$, $\beta \in (0,1)$ and $\omega \in (0,1)$ imply that $Det(\Omega - I) > 0$. On the other hand, using (68) we can deduce that $Trace(\Omega - I) < 0$ is equivalent to $(1-\beta) > -Det(\Omega - I)$. This last inequality is satisfied since $\beta \in (0,1)$ and we proved that $Det(\Omega - I) > 0$. Then E-stability of the MSV solution follows.

Second we prove b). By Lemma 2 we know that if $U_{SF} > 0$ then $\tau > 1$. Define the function $\hat{\rho}_\pi = \frac{1}{\omega\tau}$. Using this definition and the definition of ρ_π^ω it is simple to show that $\rho_\pi^\omega < \hat{\rho}_\pi = \frac{1}{\omega\tau}$ for any $\omega \in (0, 1/\tau)$. Then using this together with the assumption $1 < \rho_\pi < \rho_\pi^\omega$ for any $\omega \in (0, 1/\tau)$ we can proceed as we did in part a) to prove the existence of a unique equilibrium and the E-stability of the MSV. ■

A.6 Proof of Proposition 2

Proof. We just need to consider (34) with $\omega = 1$. The characteristic polynomial for Ω defined in (35) is given by $\mathcal{P}(v) = v^2 - Trace(\Omega)v + Det(\Omega)$ where $Det(\Omega)$ refers to the determinant of Ω. By replacing $\omega = 1$ into Ω we can obtain

$$Trace(\Omega) = 1 + \beta\left[1 + \frac{\delta\epsilon(1-\rho_\pi)}{(1-\tau\rho_\pi)}\right] \qquad \text{and} \qquad Det(\Omega) = \beta.$$

Using these and the characteristic polynomial $\mathcal{P}(v)$ we can derive that

$$\mathcal{P}(1) = -\frac{\beta\delta\epsilon(1-\rho_\pi)}{(1-\tau\rho_\pi)} \qquad \text{and} \qquad \mathcal{P}(-1) = \left[\frac{\beta\delta\epsilon + 2(1+\beta)\tau}{(1-\tau\rho_\pi)}\right](\rho_\pi^1 - \rho_\pi). \tag{69}$$

We first prove a). Recall from Lemma 1 that $\delta > 0$ and $\epsilon > 0$. By Lemma 2 we know that $\tau > 0$ and if $U_{SF} < 0$ then $\tau < 1$. Using these and $\delta > 0$, $\epsilon > 0$, and $\beta \in (0,1)$ we can infer that $\rho_\pi^1 = \frac{\beta\delta\epsilon + 2(1+\beta)}{\beta\delta\epsilon + 2(1+\beta)\tau} > 1$ and $\frac{\beta\delta\epsilon+2(1+\beta)}{\beta\delta\epsilon+2(1+\beta)\tau} < \frac{\beta\delta\epsilon+2(1+\beta)}{[\beta\delta\epsilon+2(1+\beta)]\tau}$. Hence $\rho_\pi^1 < \frac{1}{\tau}$. This in tandem with $1 < \rho_\pi < \rho_\pi^1$,(69), $\delta > 0$, $\epsilon > 0$, and $\beta \in (0,1)$ imply that $\mathcal{P}(1) > 0$ and $\mathcal{P}(-1) > 0$. Moreover $0 < Det(\Omega) < 1$ since $Det(\Omega) = \beta$ and $\beta \in (0,1)$. This together with $\mathcal{P}(1) > 0$ and $\mathcal{P}(-1) > 0$ imply that the two eigenvalues of Ω are inside the unit circle.[25] Since $\hat{\pi}_t^S$ and $\hat{c}_t^S$ are non-predetermined variables then by Blanchard and Kahn (1980) we conclude that there exists a unique equilibrium.

To prove that $1 < \rho_\pi < \rho_\pi^1$ implies that the MSV solution $\hat{y}_t = \bar{k} + \bar{Q}\hat{z}_t$ is E-stable we verify that the E-stability conditions are satisfied. For $\hat{y}_t = \bar{k} + \bar{Q}\hat{z}_t$ the E-stability conditions in (32) are reduced to verify that all the eigenvalues of $DT_k = \Omega$ and $DT_Q = \Phi' \otimes \Omega$ evaluated at $\omega = 1$ have real parts less than one. Nevertheless by assumption the eigenvalues of Φ are less than one. This implies that the E-stability conditions will be satisfied whenever that all the eigenvalues of Ω have real parts less than one. Moreover the MSV is not E-stable if any of the eigenvalues of DT_k and DT_Q evaluated at $\omega = 1$ are bigger than one. Hence we proceed to characterize the eigenvalues of these matrices.

By assumption the eigenvalues of Φ are less than one. This implies that the E-stability conditions will be satisfied whenever that all the eigenvalues of Ω evaluated at $\omega = 1$ have real parts less than one. To check this it is sufficient to verify that all the eigenvalues of $\Omega - I$ with $\omega = 1$ have negative real parts. We do so in the following way. The eigenvalues v_1 and v_2 of $\Omega - I$ satisfy $Trace(\Omega - I) = v_1 + v_2$ and $Det(\Omega - I) = v_1 v_2$.[26] Then a sufficient and necessary conditions for the eigenvalues of $\Omega - I$ to have negative real parts are $Trace(\Omega - I) < 0$ and $Det(\Omega - I) > 0$. We calculate them taking into account that $\omega = 1$. Hence

[25] See Azariadis (1993).
[26] See Horn and Johnson (1985).

$$Trace(\Omega - I) = \beta\left[1 + \frac{\delta\epsilon(1-\rho_\pi)}{(1-\tau\rho_\pi)}\right] - 1 \qquad \text{and} \qquad Det(\Omega - I) = -\frac{\beta\delta\epsilon(1-\rho_\pi)}{(1-\tau\rho_\pi)} \tag{70}$$

As was proved before $\rho_\pi^1 < \frac{1}{\tau}$ which together with the assumption $1 < \rho_\pi < \rho_\pi^1$, the expression for $Det(\Omega - I)$ in (70) and $\delta > 0$, $\epsilon > 0$, $\beta \in (0,1)$ imply that $Det(\Omega - I) > 0$. On the other hand, using (70) we can observe that $Trace(\Omega - I) < 0$ is equivalent to $(1-\beta) < -Det(\Omega - I)$. This last inequality is satisfied since $\beta \in (0,1)$ and we proved that $Det(\Omega - I) > 0$. Then E-stability of the MSV solution follows.

Second we prove b). By Lemma 2 we know that $\tau > 0$ and if $U_{SF} \geq 0$ then $\tau \geq 1$. Therefore $\frac{1}{\tau} \leq 1$. Suppose that $\tau = 1$ then from (69), $\delta > 0$, $\epsilon > 0$, and $\beta \in (0,1)$ we can infer that $\mathcal{P}(1) < 0$. Now consider the case in which $\tau > 1$. Since the rule is active ($\rho_\pi > 1$) and $1 > \frac{1}{\tau}$ then we can see that $\rho_\pi > \frac{1}{\tau}$. Using this, $\rho_\pi > 1$, (69), $\delta > 0$, $\epsilon > 0$, and $\beta \in (0,1)$ we derive that $\mathcal{P}(1) > 0$. Hence regardless of whether $\tau = 1$ or $\tau > 1$ (equivalently $U_{SF} = 0$ or $U_{SF} > 0$) we have that $\mathcal{P}(1) < 0$. It is easy to prove that $\mathcal{P}(-1) = 2(1 + Det(\Omega)) - \mathcal{P}(1)$. Then using this, $\mathcal{P}(1) < 0$ and $Det(\Omega) = \beta > 0$ we can deduce that $\mathcal{P}(-1) > 0$. This and $\mathcal{P}(1) < 0$ are sufficient to conclude that Ω evaluated at $\omega = 1$ has one eigenvalue inside the unit circle and one eigenvalue outside the unit circle. Since $\hat{\pi}_t^S$ and $\hat{c}_t^S$ are non-predetermined variables then by Blanchard and Kahn (1980) we conclude that there exists multiple equilibria.

Furthermore to prove that the MSV solution is not E-stable we start by recalling there exists E-instability if any of the eigenvalues of $DT_k = \Omega$ and $DT_Q = \Phi' \otimes \Omega$ evaluated at $\omega = 1$ have real parts bigger than one. Hence we proceed to characterize the eigenvalues of these matrices. By assumption the eigenvalues of Φ are less than one. Then we can just focus on the eigenvalues of Ω. We want to prove that Ω evaluated at $\omega = 1$ has some eigenvalues with real parts bigger than one, or equivalently that $\Omega - I$ has some eigenvalues with positive real parts. When $\tau = 1$ then from (70) we can deduce that $Det(\Omega - I) = -\beta\delta\epsilon$. And using this and the facts that $\delta > 0$, $\epsilon > 0$, and $\beta \in (0,1)$, we deduce that $Det(\Omega - I) < 0$. On the other hand if $\tau > 1$ we already derived that in this case $\rho_\pi > \frac{1}{\tau}$. Using this, with $\rho_\pi > 1$, $\delta > 0$, $\epsilon > 0$, $\beta \in (0,1)$, and the expression of $Det(\Omega - I)$ in (70) allows to conclude that $Det(\Omega - I) > 0$. Hence regardless of whether $\tau = 1$ or $\tau > 1$ (equivalently $U_{SF} = 0$ or $U_{SF} > 0$) we have that $Det(\Omega - I) < 0$. The eigenvalues υ_1 and υ_2 of $\Omega - I$ satisfy $Det(\Omega - I) = \upsilon_1\upsilon_2$.[27] Therefore $Det(\Omega - I) < 0$ implies that there exists one eigenvalue with a positive real part and the E-instability of the MSV solution follows. ■

A.7 Proof of Proposition 3

Proof. We just need to consider (34) with $\omega = 0$. The characteristic polynomial for Ω defined in (35) is given by $\mathcal{P}(v) = v^2 - Trace(\Omega)v + Det(\Omega)$ where $Det(\Omega)$ refers to the determinant of Ω. By replacing $\omega = 0$ into Ω we can obtain

$$Trace(\Omega) = 1 + \beta\left[1 + \delta\epsilon(1-\rho_\pi)\right] \qquad \text{and} \qquad Det(\Omega) = \beta. \tag{71}$$

Using these and the characteristic polynomial $\mathcal{P}(v)$ we can derive that

$$\mathcal{P}(1) = -\beta\delta\epsilon(1-\rho_\pi) \qquad \text{and} \qquad \mathcal{P}(-1) = \beta\delta\epsilon(\rho_\pi^0 - \rho_\pi) \tag{72}$$

[27] See Horn and Johnson (1985).

Recall from Lemma 1 that $\delta > 0$ and $\epsilon > 0$. Using this and $\beta \in (0,1)$ it is clear that $\rho_\pi^0 > 1$. Using $1 < \rho_\pi < \rho_\pi^0$ in tandem with (72), $\delta > 0$, $\epsilon > 0$, and $\beta \in (0,1)$ we can infer that $\mathcal{P}(1) > 0$ and $\mathcal{P}(-1) > 0$. Moreover since $Det(\Omega) = \beta$ and $\beta \in (0,1)$ then $0 < Det(\Omega) < 1$. This together with $\mathcal{P}(1) > 0$ and $\mathcal{P}(-1) > 0$ imply that the two eigenvalues of Ω are inside the unit circle. Since $\hat{\pi}_t^S$ and $\hat{c}_t^S$ are non-predetermined variables then by Blanchard and Kahn (1980) we conclude that there exists a unique equilibrium.

Next we prove that $1 < \rho_\pi < \rho_\pi^0$ implies that the MSV solution $\hat{y}_t = \bar{k} + \bar{Q}\hat{z}_t$ is E-stable. The E-stability conditions correspond to verify that all the eigenvalues of $DT_k = \Omega$ and $DT_Q = \Phi' \otimes \Omega$ evaluated at $\omega = 0$ have real parts less than one. Nevertheless by assumption the eigenvalues of Φ are less than one. This implies that the E-stability conditions will be satisfied whenever that all the eigenvalues of Ω have real parts less than one or equivalently when all the eigenvalues of $\Omega - I$ have negative real parts. This will be satisfied if and only if $Trace(\Omega - I) < 0$ and $Det(\Omega - I) > 0$ hold since the eigenvalues υ_1 and υ_2 of $\Omega - I$ satisfy $Trace(\Omega - I) = \upsilon_1 + \upsilon_2$ and $Det(\Omega - I) = \upsilon_1 \upsilon_2$.[28] Using $\omega = 0$ we obtain

$$Trace(\Omega - I) = \beta\left[1 + \delta\epsilon(1 - \rho_\pi)\right] - 1 \qquad \text{and} \qquad Det(\Omega - I) = -\beta\delta\epsilon(1 - \rho_\pi) \tag{73}$$

The assumption $1 < \rho_\pi < \rho_\pi^0$, the expression for $Det(\Omega - I)$ in (73) and $\delta > 0$, $\epsilon > 0$, $\beta \in (0,1)$ imply that $Det(\Omega - I) > 0$. On the other hand, using (73) we can observe that $Trace(\Omega - I) < 0$ is equivalent to $(1 - \beta) > -Det(\Omega - I)$. This last inequality is satisfied since $\beta \in (0,1)$ and $Det(\Omega - I) > 0$. Then E-stability of the MSV solution follows. ■

A.8 Proof of Proposition 4

Proof. We will first prove that regardless of $U_{SF} \gtreqless 0$ the rule always deliver real indeterminacy. To do so we proceed as follows. Recall the Schur Theorem that states that the eigenvalues of a 3×3 matrix Ω are inside the unit circle if and only if having the characteristic polynomial $\mathcal{P}(\upsilon) = d_0\upsilon^3 + d_1\upsilon^2 + d_2\upsilon + d_3 = 0$ the following conditions are satisfied i) $d_0 + d_1 + d_2 + d_3 > 0$; ii) $d_0 - d_1 + d_2 - d_3 > 0$; iii) $d_0(d_0 + d_2) - d_3(d_1 + d_3) > 0$; iv) $d_0(d_0 - d_2) + d_3(d_1 - d_3) > 0$; v) $d_0 + d_3 > 0$, and vi) $d_0 - d_3 > 0$.[29] Using the definition of Ω in (42) we can derive its characteristic polynomial obtaining that $d_0 = -1$ and $d_3 = \frac{\beta}{\tau\omega\rho_\pi}$. Using these and $\beta \in (0,1)$, $\omega \in (0,1)$, $\tau > 0$ (see Lemma 2) and $\rho_\pi > 1$ then condition vi) of the Schur Theorem is violated given that $d_0 - d_3 = -\left(1 + \frac{\beta}{\tau\omega\rho_\pi}\right) < 0$. Hence the number of eigenvalues of Ω inside the unit circle is less than the number of non-predetermined variables ($\hat{\pi}_t^F$, $\hat{\pi}_t^S$, and $\hat{c}_t^S$). Applying the results of Blanchard and Kahn (1980) we conclude that there exist multiple equilibria.

Second we focus on the E-stability analysis. We need to find the E-stability conditions. For the MSV solution $\hat{y}_t = \bar{k} + \bar{Q}\hat{z}_t$ the E-stability conditions correspond to verify that all the eigenvalues of $DT_k = \Omega$ and $DT_Q = \Phi' \otimes \Omega$ have real parts less than one. Moreover the MSV solution is not E-stable if any of the eigenvalues of DT_k and DT_Q have real parts bigger than one. Hence we proceed to characterize the eigenvalues of these matrices.

By assumption the eigenvalues of Φ are less than one. Then we can just focus on the eigenvalues of Ω. We want to prove that Ω has some eigenvalues with real parts bigger than one, or equivalently that $\Omega - I$

[28] See Horn and Johnson (1985).
[29] See Lorenz (1993).

has some eigenvalues with positive real parts. We do so in the following way. Using Ω in (42) we obtain

$$Det(\Omega - I) = -\frac{\beta\delta\epsilon(1-\rho_\pi)}{\tau\omega\rho_\pi} + \beta\delta\epsilon\left(1 - \frac{1}{\tau^2}\right). \tag{74}$$

Consider the case $U_{SF} \geq 0$. By Lemma 2 we know that $\tau \geq 1$. This together with $\delta > 0$, $\epsilon > 0$, $\beta \in (0,1), \omega \in (0,1)$, $\rho_\pi > 1$, and (74) imply that $Det(\Omega - I) > 0$. But the eigenvalues υ_1, υ_2 and υ_3 of $\Omega - I$ satisfy $Det(\Omega - I) = \upsilon_1\upsilon_2\upsilon_3$.[30] Then $\Omega - I$ has at least one eigenvalue with a positive real part. Then E-instability of the MSV solution follows.

Now consider $U_{SF} < 0$. By Lemma 2 we know that $\tau < 1$. This together with $\delta > 0$, $\epsilon > 0$, $\beta \in (0,1), \omega \in (0,1)$, $\rho_\pi > 1$ and (74) imply that there might be some values of $\omega \in (0,1)$ and $\rho_\pi > 1$ for which $Det(\Omega - I) < 0$. Since the eigenvalues υ_1, υ_2 and υ_3 of $\Omega - I$ satisfy $Det(\Omega - I) = \upsilon_1\upsilon_2\upsilon_3$ then $\Omega - I$ may have either one eigenvalue or three eigenvalues with negative real parts. This implies that the MSV solution may be E-stable. ■

A.9 Proof of Proposition 5

Proof. The proof is very simple. It is the same as the Proof for Proposition 4 taking into account that $\omega = 1$. ■

A.10 Proof of Proposition 6

Proof. The characteristic polynomial for Ω defined in (44) is given by $\mathcal{P}(\upsilon) = \upsilon^2 - Trace(\Omega)\upsilon + Det(\Omega)$ where $Det(\Omega)$ refers to the determinant of Ω and

$$Trace(\Omega) = \frac{1+\beta(1+\delta\epsilon)}{1+\beta\delta\epsilon\rho_\pi} \qquad \text{and} \qquad Det(\Omega) = \frac{\beta}{1+\beta\delta\epsilon\rho_\pi} \tag{75}$$

Using these and the characteristic polynomial $\mathcal{P}(\upsilon)$ we can derive that

$$\mathcal{P}(1) = \frac{\beta\delta\epsilon(\rho_\pi - 1)}{1+\beta\delta\epsilon\rho_\pi} \qquad \text{and} \qquad \mathcal{P}(-1) = \frac{\beta\delta\epsilon(\rho_\pi + 1) + 2(1+\epsilon)}{1+\beta\delta\epsilon\rho_\pi}. \tag{76}$$

Recall from Lemma 1 that $\delta > 0$ and $\epsilon > 0$. Using this, assumption $\rho_\pi > 1$, definitions (76), and $\beta \in (0,1)$ we can infer that $\mathcal{P}(1) > 0$ and $\mathcal{P}(-1) > 0$. Moreover $0 < Det(\Omega) < 1$. This together with $\mathcal{P}(1) > 0$ and $\mathcal{P}(-1) > 0$ imply that the two eigenvalues of Ω are inside the unit circle.[31] Since $\hat{\pi}_t^S$ and $\hat{c}_t^S$ are non-predetermined variables then by Blanchard and Kahn (1980) we conclude that there exists a unique equilibrium.

Next we prove that $\rho_\pi > 1$ implies that the MSV solution $\hat{y}_t = \bar{k} + \bar{Q}\hat{z}_t$ is E-stable. For the system in (43), the E-stability conditions correspond to verify that all the eigenvalues of $DT_k = \Omega$ and $DT_Q = \Phi' \otimes \Omega$ have real parts less than one. However by assumption the eigenvalues of Φ are less than one. This implies that the E-stability conditions will be satisfied whenever that all the eigenvalues of Ω have real parts less than one. To check this it is sufficient to verify that all the eigenvalues of $\Omega - I$ have negative real parts. We do so

[30] See Horn and Johnson (1985).
[31] See Azariadis (1993).

in the following way. The eigenvalues υ_1 and υ_2 of $\Omega - I$ satisfy $Trace(\Omega - I) = \upsilon_1 + \upsilon_2$ and $Det(\Omega - I) = \upsilon_1\upsilon_2$.[32] Then sufficient and necessary conditions for all the eigenvalues of $\Omega - I$ to have negative real parts are $Trace(\Omega - I) < 0$ and $Det(\Omega - I) > 0$. We calculate

$$Trace(\Omega - I) = -\frac{(1-\beta) + \beta\delta\epsilon(2\rho_\pi - 1)}{1 + \beta\delta\epsilon\rho_\pi} \qquad \text{and} \qquad Det(\Omega - I) = \frac{\beta\delta\epsilon(\rho_\pi - 1)}{1 + \beta\delta\epsilon\rho_\pi}.$$

These, the assumption $\rho_\pi > 1$, and the facts that $\delta > 0$, $\epsilon > 0$, and $\beta \in (0,1)$ imply that $Trace(\Omega - I) < 0$ and $Det(\Omega - I) > 0$. Hence E-stability of the MSV solution follows. ■

A.11 Proof of Proposition 7

Proof. First we prove a) by considering the representation (46). To do so we derive the characteristic polynomial of the matrix Λ in (47) taking into account that $\omega = 1$ and that the two goods are utility separable. This means that $\tau = 1$ by Lemma 2. Then we obtain

$$\mathcal{P}(v) = \underbrace{\left(v^2 - \frac{1}{\rho_\pi}\right)}_{\mathcal{P}_1(v)} \underbrace{\left[v^2 - (1 + \beta + \beta\delta\epsilon)\, v + \beta\right]}_{\mathcal{P}_2(v)}. \tag{77}$$

This shows that the characteristic polynomial $\mathcal{P}(v)$ of Λ corresponds to the product of the polynomial $\mathcal{P}_1(v)$ and $\mathcal{P}_2(v)$. Hence the roots of these two polynomials will determine the eigenvalues of Λ (with $\omega = 1$ and $\tau = 1$). Consider the roots of $\mathcal{P}_1(v)$. It is simple to see that if $\rho_\pi > 1$ then the two roots of $\mathcal{P}_1(v)$ are inside the unit circle.

On the other hand the polynomial $\mathcal{P}_2(v)$ satisfies

$$\mathcal{P}_2(1) = -\beta\delta\epsilon \qquad \text{and} \qquad \mathcal{P}_2(-1) = 2(1+\beta) + \beta\delta\epsilon.$$

Using these expressions, $\beta \in (0,1)$ and the facts that $\epsilon > 0$ and $\delta > 0$ (see Lemma 1) we can infer that $\mathcal{P}_2(1) < 0$ and $\mathcal{P}_2(-1) > 0$. Which in turn implies that one of the roots of $\mathcal{P}_2(v)$ is inside the unit circle whereas the other one is outside of it.[33] Putting these results together we conclude that three of the roots of $\mathcal{P}(v)$ (or equivalently three of the eigenvalues of Λ with $\omega = 1$ and $\tau = 1$) are inside the unit circle while the fourth one is outside of it. Then since $\hat{\pi}_t^F$, $\hat{\pi}_t^S$ and $\hat{c}_t^S$ are the only non-predetermined variables from the results of Blanchard and Kahn (1980), it follows that there exists a unique equilibrium.

Next we prove part b) by considering the system (48). We need to prove that the MSV solution $\hat{y}_t = k + P\hat{y}_{t-1} + Q\hat{z}_t$ is E-unstable. To do so we recall from Evans and Honkapohja (2001) that the MSV solution is E-unstable if any of the eigenvalues of $DT_k = (I - \Omega\bar{P})^{-1}\Omega$, $DT_P = \left[(I - \Omega\bar{P})^{-1}\Gamma\right]' \otimes [(I - \Omega\bar{P})^{-1}\Omega]$, and $DT_Q = \Phi' \otimes [(I - \Omega\bar{P})^{-1}\Omega]$ have real parts bigger than one. Then we start by studying the eigenvalues of $DT_k = (I - \Omega\bar{P})^{-1}\Omega$. More specifically we will prove that $DT_k = (I - \Omega\bar{P})^{-1}\Omega$ has some eigenvalues with real parts bigger than one, or equivalently that $\left[(I - \Omega\bar{P})^{-1}\Omega - I\right]$ has some eigenvalues with real positive parts. To do so it is necessary to find the MSV solution. In particular we need to solve for $\bar{P}$ using the method of undetermined coefficients. From (**??**) we know that this matrix should satisfy $\Omega P^2 - P + \Gamma = 0$

[32] See Horn and Johnson (1985).
[33] See Azariadis (1993).

or equivalently $(I-\Omega P)^{-1}\Gamma = P$. However since $\omega = 1$ then the matrix Γ in the system (48) becomes

$$\Gamma = \begin{bmatrix} 0 & (1-\tau)\rho_\pi & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -\beta\delta\epsilon\rho_\pi & 0 & 0 \\ 0 & -\epsilon\rho_\pi & 0 & 0 \end{bmatrix}. \tag{78}$$

This help us to have an "educated" guess for P. We choose

$$P = \begin{bmatrix} 0 & p_1 & 0 & 0 \\ 0 & p_2 & 0 & 0 \\ 0 & p_3 & 0 & 0 \\ 0 & p_4 & 0 & 0 \end{bmatrix}.$$

Using this expression, the expression for Ω (with $\omega = 1$) in (49), (78) and $(I-\Omega P)^{-1}\Gamma = P$ we can prove that in this case $\bar{P} = \Gamma$ Utilizing this and the expression for Ω (with $\omega = 1$) in (49) we can find $Det\left[(I-\Omega\bar{P})^{-1}\Omega - I\right] = -\frac{\beta\delta\epsilon(\rho_\pi - 1)}{\rho_\pi\tau}$. Note that $Det\left[(I-\Omega\bar{P})^{-1}\Omega - I\right] < 0$ since by assumption $\rho_\pi > 1$, $\beta \in (0,1)$, $\epsilon > 0$, $\delta > 0$ and $\tau > 0$ (see Lemma 2).

Finally the eigenvalues υ_1, υ_2, υ_3, and υ_4 of $\left[(I-\Omega\bar{P})^{-1}\Omega - I\right]$ satisfy $Det\left[(I-\Omega\bar{P})^{-1}\Omega - I\right] = \upsilon_1\upsilon_2\upsilon_3\upsilon_4$.[34] Given that $Det\left[(I-\Omega\bar{P})^{-1}\Omega - I\right] < 0$, we know that $\left[(I-\Omega\bar{P})^{-1}\Omega - I\right]$ has at least one eigenvalue with a positive real part. Hence the E-instability of the MSV solution follows. ■

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