Sunspot equilibria with persistent unemployment fluctuations*

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Abstract

We provide a business cycle model able to replicate the large amount of persistence in output and unemployment fluctuations found in the data. These variations in the unemployment rate are the result of self-fulfilling changes in expectations about future inflation in the wage bargaining process between workers of firms.

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1 Introduction

Persistent unemployment affecting Europe since the mid 1970s remains one of the most discussed subjects in economics. Several alternative explanatory theories have been proposed. According to the structuralist approach the observed persistent increase in unemployment is explained by permanent shocks that raised the natural rate of unemployment.[1] In contrast, Blanchard and Summers (1986, 1987) introduced the hysteresis hypothesis, where unemployment persistence is explained by the strong dependence of unemployment on the history of transitory shocks. This can be due either to the existence of multiple equilibria (unit roots)[2], or to the existence of an extremely slow speed of convergence to a unique equilibrium (quasi unit roots).[3] In this paper we simulate a model able to reproduce the observed persistence in unemployment that uses this last mechanism.

Recent studies, that have also used the second version of hysteresis to explain unemployment fluctuations, have in general introduced non competitive features in the labour market in an otherwise standard Real Business Cycles (RBC) model, and considered serially correlated exogenous shocks on fundamentals (productivity or government spending shocks). See for example Mertz (1995), Andolffato (1996), Maffezzoli (2001), Chéron and Langot (2004) and Chiarini and Piselli (2005). These works were able to match successfully some labour market stylized facts, and were as successful as previous standard RBC models in replicating the main variables comovements with output and the relatively variability of some series. However they were unable to replicate both the rich dynamic response of output and unemployment to transitory shocks and the autocorrelation function for output growth found in the data.

In this paper we offer an alternative explanation to unemployment persistence that does not rely on RBC exogenous shocks. We rely instead on sunspot shocks, i.e. shocks on expectations, an approach closely related to Keynes' idea that "animal spirits" may generate waves of optimism and pessimism leading to aggregate fluctuations and unemployment. Indeed, in the presence of indeterminacy of equilibria and/or bifurcations, sunspots shocks are able to generate endogenous cycles in economic variables. See Azariadis (1981), Cass and Shell (1983), Azariadis and Guesnerie (1986), Woodford

[1] See Phelps (1994), Phelps and Zoega (1998).

[2] See for example Rocheteau (1999), Raurich et al (2006) for the first line of research.

[3] Empirical studies

(1986), Grandmont *et al.* (1998). Recent research has also been showing that market imperfections may be responsible for the occurrence of indeterminacy and bifurcations leading to the existence of cycles driven by self fulfilling volatile expectations.

Accordingly in this paper we consider a parameterized version of the model we developed in Dufourt *et al.* (2005, forthcoming), where the Woodford (1986) framework of segmented asset markets with financially constrained workers is extended to account for several labor market imperfections. We consider efficient bargaining between unions and firms and an imperfect insurance scheme in which the government ensures a fixed minimum real income to those unemployed, financed by taxing employed workers. Since unions are able to set wages above the income received when unemployed, unemployment is costly. An interesting feature, proved to be satisfied in our model, is that, due to the financial constraint they face, both employed and unemployed workers end up consuming all there income and save nothing. This feature explains why, when we simulate the model with sunspots shocks, we are able to generate simultaneously procyclical movements of aggregate consumption, investment and output without introducing increasing returns to scale. It also explains why our model performs better than standard RBC models in replicating the relative volatility of consumption to output.

However, the most important results we obtain are the following. First, without relying on exogenous sources of dynamics, we are able to replicate the high persistence in unemployment fluctuations observed in many real economies, as the model replicates closely the autocorrelation function of the unemployment rate of the French economy. Secondly, again without exogenous sources of dynamics, we are able to replicate the two stylized facts about output dynamics emphasized in Cogley and Nason (1995). Indeed, our model is able to generate hump shaped, trend reverting, output and employment impulse response functions consistent to those observed in the data. Moreover, the model's autocorrelation function for output growth shows that output growth is positively and significantly autocorrelated over short horizons, as observed in the data.

These results suggest that, unlike previous standard RBC models that were unable to replicate all these stylized facts, our model contains an internal propagation mechanisms that is able to generate, solely via its internal structure, output and unemployment fluctuations consistent with the empirical ones. In fact previous standard RBC models with a saddle-path equilibrium are notoriously known to have very weak endogenous persistent mechanisms.[4] In our model, on the contrary, we exploit the existence of indeterminacy to generate fluctuations endogenously. Remark that Benhabib and Wen (2004) also concluded that indeterminacy held the key for the propagation mechanism of their model. In fact our work is closely related to theirs. Although the models considered are quite different,[5] both papers are concerned with the issue of producing models with an internal source of propagation that generates dynamics consistent with empirical stylized facts. Like us, they are able to generate hump-shaped output dynamics with indeterminacy. However, they need serially correlated demand shocks to either consumption or government spending to obtain this type of dynamics, whereas our model is successful even when *i.i.d.* sunspot shocks are considered. As we discuss, the main explanation for this success is that the estimated values for the unobserved structural parameters imply that the economy is located close to the point where a Hopf bifurcation occurs in the parameters space. In this case, the two eigenvalues of the Jacobian matrix of the dynamic system come as complex conjugates with modulus close to one, implying a non-monotonous and very persistent process of convergence to the steady state.

The rest of the paper is organized as follows. In the next section we present the model used. In section 3 we obtain the equilibrium and discuss the local dynamic properties of the model. In section 4 we report our simulation results. We present and discuss the impulse response functions, the autocorrelated functions and standard business cycle statistics. Finally in section 5 we conclude.

2 The Model

We consider a one sector heterogenous agents economy with segmented asset markets and costly unemployment, as developed in Dufourt et al. (2007). The basis of this model is the finance constrained economy considered in Grandmont et al. (1998) and first proposed by Woodford (1986). In that economy all markets are perfectly competitive and there are two assets - money and productive capital, and two types of households, "workers" and "capitalists". Capitalists do not work and discount the future less than work-

[4]See Cogley and Nason (1995) and Rotemberg and Woodford (1996) on this issue.

[5]They consider an RBC model with variable capacity utilization and production externalities were all market are competitive.

ers. The latter face a borrowing constraint implying that they cannot borrow against current and future income to finance current consumption. Dufourt et al. (2007) obtain costly equilibrium unemployment in this framework introducing an (imperfect) insurance scheme provided by the government in a economy where, due to union power, wages are set above the reservation wage. Below we provide a brief description of this model, where for simplicity we consider a CES parameterized version of the production function.

Capitalists are identical and maximize $E\sum_{t=0}^{\infty}\beta^t Log c_t^c$, where $0<\beta<1$ is the discount factor, and c_t^c is consumption in period t. They face the following budget constraint $p_t c_t^c + p_t k_{t+1}^c + m_{t+1}^c = p_t R_t k_t^c + m_t^c$, where p_t is the price of output, k_{t+1}^c and m_{t+1}^c are respectively the capital stock and money holdings at the outset of period $t+1$, $R_t = (r_t + 1 - \delta)$ is the real gross rate of return on capital, r_t is the real rental rate of capital, and $0 \leq \delta \leq 1$ is the capital depreciation rate.

It can easily be verified that under the condition $R_{t+1} > E_t\{p_t/p_{t+1}\}$, capitalists only save in the form of capital and hold no money ($m_{t+1}^c = 0$). Hence, from the solution of the capitalists' problem, we have (see Woodford (1986))

$$c_t^c = (1-\beta)R_t k_t^c \tag{1}$$
$$k_{t+1}^c = \beta R_t k_t^c. \tag{2}$$

In addition to these optimality conditions, the following transersality condition must be verified:

$$\lim_{t\to\infty} \underset{0}{E}\left\{\beta^t \frac{1}{c_t^c}\left(k_{t+1}^c + \frac{m_{t+1}^c}{p_t}\right)\right\} = 0 \tag{3}$$

There is a continuum of workers. Workers preferences are represented by the following utility function $E\sum_{t=0}^{\infty}\gamma^t u(c_t^w)$, where c_t^w is consumption in period t and $0<\gamma<\beta$ is the discount factor of workers. Each period a worker supplies inelastically one unit of labor and may be either employed (state e) receiving in cash, at the beginning of next period, a nominal wage w_t, or unemployed (state u). If unemployed in t a worker receives from the government, at the beginning of period $t+1$, a constant real unemployment benefit b, financed by collecting, in period $t+1$, a given real amount from each worker employed at t. The budget constraint of a worker who was in state $i \in \{e, u\}$ in period $t-1$ can be written as $m_{t+1}^i + p_t k_{t+1}^i = m_t + p_t y_t^i + p_t R_t k_t - p_t c_t^i$, where m_t^w denotes money held at the beginning of period t, and where

$y_t^i \in \{w_{t-1} - p_t \tau_{t-1}, p_t b\}$. Additionally they face the borrowing constraint $m_{t+1}^i \geq 0$, and $k_{t+1}^i \geq 0$ for all t.[6] Denoting by λ_t^i, υ_t^i and η_t^i the Lagrange multipliers associated respectively with these three constraints, the first order conditions for the workers' problem with a positive level of consumption are given by:

$$u'\left(c_t^i\right) = p_t \lambda_t^i \tag{4}$$
$$\lambda_t^i - \upsilon_t^i = \gamma E_t \left\{ l_t \lambda_{t+1}^e + (1 - l_t) \lambda_{t+1}^u \right\} \tag{5}$$
$$p_t \lambda_t^i - \eta_t^i = \gamma E_t \left\{ p_{t+1} R_{t+1} \left[l_t \lambda_{t+1}^e + (1 - l_t) \lambda_{t+1}^u \right] \right\} \tag{6}$$

where l_t denotes the employment rate in period t.

As in Dufourt et al. (2007) we focus on equilibria along which both types of workers, employed and unemployed, choose $m_{t+1}^i = 0$ and $k_{t+1}^i = 0$, for $i = \{e, u\}$ and consume all their available income, i.e.

$$c_t^i = \frac{y_t^i}{p_t} \tag{7}$$

As proved below this is indeed the solution, provided the following condition is verified at the steady state

$$\gamma < \beta \left\{ u'\left(y^e\right) / \left[l u'\left(y^e\right) + (1 - l) u'\left(y^u\right) \right] \right\}. \tag{8}$$

Notice that $m_{t+1}^i = 0$ and $k_{t+1}^i = 0$ are chosen by workers if sequences of revenues and probability distributions over employment and unemployment satisfy the following conditions

$$u'\left(c_t^i\right) > \gamma E_t \left\{ \frac{p_t}{p_{t+1}} \left[l_t u'\left(c_{t+1}^e\right) + (1 - l_t) u'\left(c_{t+1}^u\right) \right] \right\} \tag{9}$$
$$u'\left(c_t^i\right) > \gamma E_t \left\{ R_{t+1} \left[l_t u'\left(c_{t+1}^e\right) + (1 - l_t) u'\left(c_{t+1}^u\right) \right] \right\} \tag{10}$$

In this case $\upsilon_t^i > 0$, so that $m_{t+1} = 0$, and $\eta_t^i > 0$, so that $k_{t+1} = 0$, for all $t = 0, ..., \infty$ and all $i \in \{e, u\}$. Since, from (2), $R = 1/\beta > 1$ at the steady-state, the inequality $R_{t+1} > E_t \{p_t / p_{t+1}\}$ holds in a neighborhood of this steady-state, and condition (10) is more restrictive than (9). Condition (10) is in particular verified at the steady state if (8) and $u'\left(y^u\right) > (\gamma/\beta) \left[l u'\left(y^e\right) + (1 - l) u'\left(y^u\right) \right]$, with $\{y^e, y^u\} = \{w/p - \tau, b\}$, are verified. Due to concavity of u, only (8) is actually required, as long as

[6] For simplicity of notation, we dropped the superscrit w.

$y^e \geq y^u$, a condition that as we shall see is implied by the wage bargaining process.[7]

Each period wages and employment are bargained between unions and firms. All workers are unionized and unions are firm-specific, i.e., we have one union per firm and each union represents the same mass of workers normalized to one.[8] Unions wish to maximize the sum of discounted consumptions of their representative member, and firms wish to maximize the expected value of discounted profits, Π_t. Firms operate under a CES production function with constant returns to scale. Accordingly, we have that

$$F(k_t, l_t) = A_t l_t f(x_t) = A l_t \left[\varphi x_t^{\frac{\sigma-1}{\sigma}} + (1-\varphi)\right]^{\frac{\sigma}{\sigma-1}} \quad ; \quad 0 < \varphi < 1 \tag{11}$$

where $x_t \equiv k_t/l_t$, k_t and l_t represent respectively capital and labour employed in each firm[9], $\sigma > 0$ is the constant factor elasticity of substitution and $A > 0$ is a scale parameter. Each period t events follow the following sequence. First, firms pay in cash last period wages, so that firms will have to hold, at the end of period t, $m^f_{t+1} \geq w_t l_t$. Then firms rent capital, k_t, at a given nominal rental rate $p_t r_t$. Next, wages, w_t, and employment, l_t, are negotiated between unions and firms. Finally, firms decide the level of money holdings and then production takes place. In order to ensure time consistency, the problem of the firm must be solved backwards, starting with the money holdings decision. In Dufourt et al (2005) we show that the firms cash constraint is always binding, i.e. $m^f_{t+1} = w_t l_t$. We proceed now with the wage-employment bargain and then with capital decisions.

We model the bargaining process between each union and the respective firm using the standard generalized Nash bargaining solution. Let $0 < \alpha \leq 1$ be the firm's power in the bargain, then the solution of the Nash maximiza-

[7] Of course, since the expression between curled brackets in (8) is lower than 1, this last condition can only be verified if $\gamma < \beta$, as asumed above and in Woodford (1986).

[8] Workers are matched exogenously and uniformly with unions and cannot move between firms or unions

[9] As we have normalized the mass of workers per firm to one, l represents both the employment level in each firm and the employment rate in the economy at a symmetric equilibrium.

tion problem, assuming that $l_t < 1$ is given by:[10]

$$(b+\tau_t)E_t\pi_{t+1} = A\left[\varphi x_t^{\frac{\sigma-1}{\sigma}} + (1-\varphi)\right]^{\frac{1}{\sigma-1}}(1-\varphi) \tag{12}$$

$$\frac{w_t}{p_t} = A\left[\varphi x_t^{\frac{\sigma-1}{\sigma}} + (1-\varphi)\right]^{\frac{1}{\sigma-1}}\left[(1-\alpha)\varphi x_t^{\frac{\sigma-1}{\sigma}} + (1-\varphi)\right] \tag{13}$$

where $\pi_{t+1} \equiv p_{t+1}/p_t$ is the inflation factor. From (12) we see that employment is determined by the equality between the firm's marginal productivity of labor, $MPL = A\left[\varphi x_t^{\frac{\sigma-1}{\sigma}} + (1-\varphi)\right]^{\frac{1}{\sigma-1}}(1-\varphi)$, and the expected real reservation wage, $(b+\tau_t)E_t\pi_{t+1,}$, for all α. From (12) and (13) we see that, if $\alpha < 1$, unions are able to set wages above the reservation wage, i.e., $w_t/p_t > (b+\tau_t)E_t\pi_{t+1}$, with a markup factor $\left[1+(1-\alpha)(\varphi/1-\varphi)x_t^{\frac{\sigma-1}{\sigma}}\right]$ which is increasing with union bargaining power. Hence, given the absence of perfect redistributive schemes, unemployed workers are clearly worse off. Another issue worth noting is that the level of employment is nonpredetermined, being influenced by expectations of future prices (or inflation). A change in expected future prices changes the reservation wage, and thereby the equilibrium level of employment.

The firm, anticipating the result of the bargaining process, chooses $k_t > 0$ to maximize profits, which yields the following first order condition:

$$r_t = \varphi\alpha A\left[(1-\varphi)x_t^{-\frac{(\sigma-1)}{\sigma}} + \varphi\right]^{\frac{1}{\sigma-1}}. \tag{14}$$

i.e., r_t/α equals the marginal productivity of capital.[11]

3 Equilibrium and dynamics

Since the government balances its budget, this real tax τ_t, paid by each worker employed in period t, is determined endogenously by the balanced-budget condition

$$\tau_t = b(1-l_t)/l_t \tag{15}$$

[10]See Dufourt et al (2007) for the derivation.

[11]Note that we recover the competitive outcome when unions have no power in the bargaining process ($\alpha = 1$): i.e., the real rental cost of capital is identical to the marginal productivity of capital, and the real wage equals both the real reservation wage and the marginal productivity of labor.

Assuming, as in Woodford (1986), a constant (per firm) amount of outside money in the economy, m, and given that only firms hold money, money market equilibrium in every period implies that $m = w_t l_t = w_{t+1} l_{t+1}$, so that $p_{t+1}/p_t = (w_t l_t / p_t) / (w_{t+1} l_{t+1} / p_{t+1})$. Using this last relation, equations (15), (12) and (13) we obtain equation (17) below. Considering an identical number of firms and capitalists, and using the definition of R and equations (2) and (14), equilibrium in the capital services market implies equation (16) below. Accordingly we have:

Definition 1 *A rational expectations intertemporal equilibrium is a sequence* $(k_t,\, l_t) \in \Re^2_{++}$, $t = 1, 2, ...\infty$ *that solves the two-dimensional dynamic system, with* $x_t = k_t / l_t$

$$k_{t+1} = \beta \left[\varphi \alpha A_t \left[(1-\varphi) x_t^{-\frac{(\sigma-1)}{\sigma}} + \varphi \right]^{\frac{1}{\sigma-1}} + (1-\delta) \right] k_t \tag{16}$$

$$b \left[1 + \frac{(1-\alpha)\varphi x_t^{\frac{\sigma-1}{\sigma}}}{(1-\varphi)} \right] = E_t \left\{ l_{t+1} A_{t+1} \left[\varphi x_{t+1}^{\frac{\sigma-1}{\sigma}} + (1-\varphi) \right]^{\frac{1}{\sigma-1}} \left[(1-\alpha)\varphi x_{t+1}^{\frac{\sigma-1}{\sigma}} + (1-\varphi) \right] \right\} \tag{17}$$

Equation (16) represents the standard capital accumulation process, while equation (17) represents the equilibrium intertemporal arbitrage condition for workers.

To facilitate the discussion, it is useful to write this dynamic system under the following implicit form

$$E_t g(z_{t+1}, z_t) = 0 \tag{18}$$

where $z_t \equiv (k_t, l_t)$. It can be verified that, under non-restrictive conditions on parameters, this dynamic system has a unique steady state equilibrium $\overline{z}$ defined by $g(\overline{z}, \overline{z}) = 0$, and that the Jacobian matrix of the map $z \to g(z, \overline{z})$ evaluated at $\overline{z}$ is invertible. By the Implicit Function Theorem, this system can therefore be solved in z_{t+1} in the neighborhood of $\overline{z}$, leading to a solution of the form

$$z_{t+1} = h(z_t, \epsilon_{t+1}) \tag{19}$$

where ϵ_{t+1} is a vector of forecast errors.

3.1 Analysis of equilibria

From Definition 1, it is clear that k_t is a predetermined variable whose behavior is determined by past savings decisions of capitalists. However, l_t is a non-predetermined variable whose level is influenced by expectations. Hence, depending on the local stability properties of the steady-state, there is potentially the room for stationary stochastic equilibria driven by self-fulfilling changes in expectations (sunspot shocks). We now briefly analyze when such situations can occur.

Consider first the case where the steady state is a saddle (locally determinate).[12] In this case, it is easily verified that expectations-driven equilibria can never arise. This is because, given the initial capital stock $k_{0,}$ there is a unique locus defined by (19) which further satisfies the transversality condition (3) and is compatible with a long run convergence to the steady state. This implies that the forecast error ϵ_{t+1} is necessarily zero in the absence of exogenous shocks on fundamentals.

The situation is completely different, however, when the steady state is a sink (or locally indeterminate). In this configuration, given the initial value of the capital stock k_0, there are now infinitely many equilibrium trajectories $\{l_t, k_{t+1}\}_{t=0,...\infty}$ compatible with (3) and converging to the steady state. Also, as proved by Azariadis and Guesnerie (1986), there are also infinitely many nondegenerate stochastic equilibria driven by self-fulfilling changes on expectations (sunspots equilibria), that stay arbitrarily close to the steady state. In terms of equation (19), this implies that the forecast error ϵ_{t+1} may now act as an independent source of the business cycle, even in the absence of extrinsic uncertainty affecting fundamentals (see Benhabib and Farmer (1999) for further discussion on this point).

Finally, a last, but nonetheless interesting, type of equilibria is worth discussing. It can occur when the steady state is either a source or a sink, but that the economy is located near the point where a Hopf bifurcation occurs in the parameters space.[13] In this case, as discussed in Grandmont et al. (1998), deterministic and (possibly) stationary stochastic equilibria gener-

[12] Explain

[13] A bifurcation occurs when the local stability properties of the system are drastically affected by a small change in parameters. Technically, it occurs when one (or several) eigenvalues of the characteristic polynomial of the system crosses the unit circle through this change of parameters. For example, a *flip* bifurcation occurs when one eigenvalue crosses -1. A *Hopf* bifurcation occurs when two complex conjugate eigenvalues have their modulus crossing 1.

ated by periodic or quasi-periodic orbits appear, lying around an invariant closed curve that surrounds the steady state in the state space. Thus, in this configuration, the economy may very well exhibit infinitely recurrent fluctuations in the unemployment rate *even in the absence of any kind of stochastic shocks* (on fundamentals or on expectations) - a form of "hysteresis" which is relatively new compared to the traditional literature.[14]

3.2 Dynamic configurations

In Dufourt *et al.* (forthcoming), a complete analytical characterization of the local stability properties of a (more general) version of this model has been undertaken in terms of relevant parameters. Figure 1, which is easily computed as a direct application of this theoretical analysis, reports in the (σ, α) plane the bifurcation values for the elasticity of substitution σ as a function of the firms' bargaining power α, given an empirically based calibration for the set of other parameters described below. Several interesting results can be drawn from the simple observation of this figure. First, the local dynamics of the model is indeterminate for a wide range of parameters values, including the empirically relevant ones. In particular, as proved in Dufourt *et al.* (forthcoming), the steady state is *always* indeterminate when the production function is Cobb-Douglas ($\sigma = 1$). When the elasticity of substitution between capital and labor is different from one, both flip and Hopf bifurcations may occur. These bifurcations arise for empirically plausible values of σ (not far away from one) as soon as the unions' bargaining power is strong enough.

In light of this analysis, it is clear that the two kinds of "sophisticated" dynamics resulting from the existence of sunspot equilibria with self-fulfilling changes in expectations, or from the existence of quasi periodic deterministic equilibria fluctuating along an invariant curve, are credible possibilities, since they occur for plausible values of the structural parameters.

In our view, both types of equilibria would have been worth studying. Unfortunately, the kind of strongly non-linear dynamics occurring around the invariant closed curve cannot always be easily recovered by standard solution techniques, including recent numerical ones.[15] For this reason, in order to study the empirical predictions of the model in terms of unemployment

[14] To be more precisely explained

[15] For example, we have attempted to recover the invariant closed curve using several numerical solvers provided by Matlab without success.

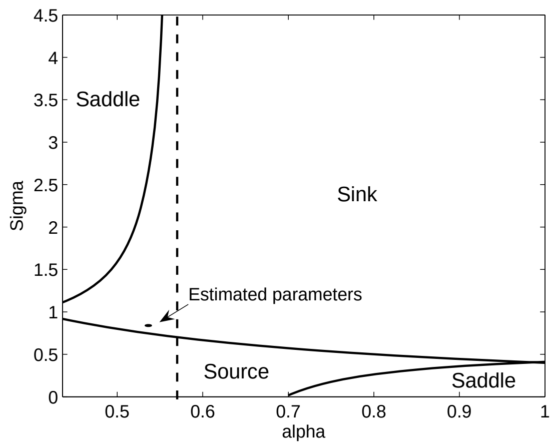

Figure 1: Local dynamics properties and bifurcations values in the space of parameters (α, σ), for given (calibrated) values of the set of parameters $\gamma_1 = (\beta,\, \delta,\, b,\, \varphi)$.

fluctuations and persistence, we have chosen to retain the more traditional approach pioneered by Benhabib and Farmer (1994) and Farmer and Guo (1994), consisting of generating fluctuations around the steady state due to self-fulfilling changes in expectations (sunspots), in an economy which is locally indeterminate.

4 Model evaluation

We now wish to investigate whether this model with self-fulfilling changes in expectations (sunspots shocks) can generate persistent, empirically consistent, fluctuations in the unemployment rate and output growth. In order to do so, an approximation of the solution to the dynamic system (16) and (17) is needed. Since we wish to consider the possibility that the economy be located near the points where the flip and Hopf bifurcations occur, it might be the case that the true dynamics of the model is too rich to be approximated sufficiently well by a standard linearization procedure. For this reason, we have followed instead the suggestion of Schmitt-Grohé and Uribe (2004) of approximating the solution using a second-order expansion of the policy function. This is likely to better capture the nonlinearities of the model, as we will show below.

When the steady state is a sink, sunspot equilibria driven by self-fulfilling changes in expectations exist, and a second order Taylor expansion of a solution satisfying (19) may be written as

$$\begin{aligned}\widehat{k}_{t+1} &\simeq J_k \widehat{z}_t + \frac{1}{2}\widehat{z}_t' H_k(\overline{z})\widehat{z}_t \\ \widehat{l}_{t+1} &\simeq J_l \widehat{z}_t + \frac{1}{2}\widehat{z}_t' H_l(\overline{z})\widehat{z}_t + \epsilon_{t+1}\end{aligned}$$

where $\widehat{z}_t \equiv (\widehat{k}_{t,}\widehat{l}_t)$ is the vector of endogenous variables expressed in percentage deviations from the steady-state, ϵ_{t+1} is a sunspot shock of bounded support with variance γ, $J_i \equiv \partial h_i(\overline{z})/\partial \overline{z}$ is the raw of the Jacobian matrix of $h(.)$ corresponding to variable and H_i is the Hessian matrix of $h(.)$ relative to variable $i \in (k, l)$, $H_i \equiv$ (complete)

4.1 Calibration and estimation procedure

In order to simulate the model and evaluate its capacity to match empirical regularities, a sensible parameterization is needed. The model contains, besides the scale parameter A, six structural parameters: β, δ, σ, α, b and φ. Our general strategy is to partition these parameters into two groups: those for which there exists relatively common and rather uncontroversial estimates in the literature, or for which we can match balanced growth path values with observed averages ; and those for which such estimates are not available or are more controversial. The first set of parameters is calibrated, while the second set is chosen so as to minimize a measure of the distance between some preselected moments characterizing our data set and their model-implied counterparts.

The first set of parameters is $\gamma_1 = (\beta,\ \delta,\ b,\ \varphi)$. As we define the time period to be a quarter, we set $\beta = 1.03^{-0.25}$, which implies a steady state annualized real interest rate of 3 percent. We set $\delta = 0.025$, which implies an annual depreciation rate on capital of 10 percent. We calibrate the real amount of unemployment compensation b and the (unobserved) technological parameter φ so as to match the long-run labor share of output in France over the period 1978:1 to 2007:2, $s_L = 0.6$, and the long-run level of unemployment over this period: $u = 9.6\%$.[16]

[16] Write note on the steady-state capital-labor ratio.

The second set or parameters includes the firms' bargaining power α and the elasticity of substitution between capital and labor σ, $\gamma_2 = (\sigma, \alpha)$. As these parameters are hardly observed or estimated, we follow Rotemberg and Woodford (1997), Christiano *et al.* (2005), and others, by estimating these parameters so as to match as closely as possible the preselected set of empirical moments using a Minimum Distance Estimation (MDE) procedure. To be more precise, let Ψ_T^e be a set of empirical moments characterizing our data set of length T, and let $\Psi(\gamma_2)$ be the mapping from the (non calibrated) structural parameters to the corresponding theoretical set of moments. The Minimum Distance Estimator of γ_2, denoted $\widehat{\gamma}_2$, is given by

$$\widehat{\gamma}_2 = \arg\min_{\gamma_2 \in \Gamma} \left(\Psi_T^e - \Psi(\gamma_2)\right)' W \left(\Psi_T^e - \Psi(\gamma_2)\right)$$

where W is a positive definite weighting matrix.

A problem that may arise in practice is that, given the relatively small number of observations in our data set ($T = 118$), the model-generated sample equivalent of Ψ_T^e may be quite different from the theoretical one, $\Psi(\gamma_2)$. For this reason, we relied instead on a standard Method of Simulated Moments, where a short sample equivalent of $\Psi(\gamma_2)$, denoted $\widehat{\Psi}_T(\gamma_2)$, was determined by repeatedly generating from the model artificial data sets of length T and then averaging the sample estimates. These repeated simulations were also used to compute an estimate $\widehat{\Sigma}$ of the variance-covariance matrix of $\widehat{\Psi}_T(\gamma_2)$, which served as a basis for the confidence bounds below. Following Christiano et al. (2005), we chose as weighting matrix a diagonal matrix containing along the diagonal the inverse of the sample variances of $\widehat{\Psi}_T(\gamma_2)$, i.e. the inverse of the diagonal elements of $\widehat{\Sigma}$. With this choice, the vector or parameters γ_2 is chosen so that the empirical moments Ψ_T^e lie as much as possible in these confidence bounds.

Finally, some discussion is required about the set of moments that we aimed to match. As the main issue of our paper is on unemployment persistence, and persistence in general, we have chosen to match the two statistical measures which emphasize the most strongly this dimension on the data. Namely, we have chosen to match the autocorrelation functions of the (HP-filtered) unemployment rate and of output growth of the French economy. In addition, a choice had to be made about the number of lags in the ACFs to consider. As the ACF function of output growth essentially vanishes after 6 lags, we chose as a benchmark to retain the first 6 lags of these autocorrelation functions. Results were not substantially altered, however, when we

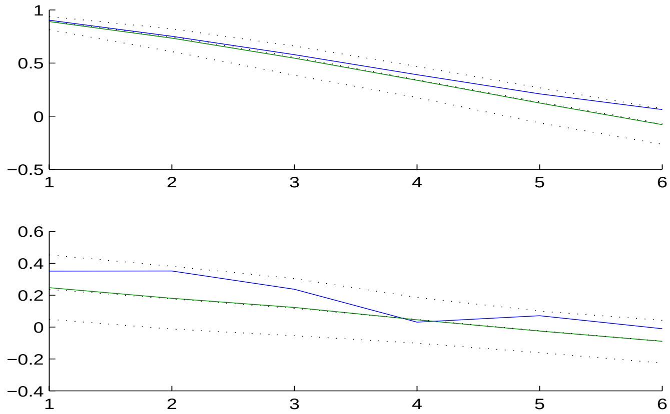

Figure 2: Empirical (blue line) and theoretical (green line) autocorrelation functions of unemployment rate (upper graph) and output growth (lower graph). French economy over the period 19xx-200x. The dotted lines represent the 95% confidence bounds.

experimented with different numbers of lags.

4.2 Estimation results

The estimated vector of parameters was obtained based on the previously described minimization procedure using 600 simulations of data sets including 144 observations. Table 1 reports the estimated values for $\widehat{\gamma}_2 = (\widehat{\sigma}, \widehat{\alpha})$[17], while Figure 2 reports the empirical autocorrelation function together with their theoretical (sample average) counterpart. Overall, the match appears very good, with the empirical and theoretical autocorrelation functions being very close from each other, and the two empirical autocorrelation functions lying entirely within the simulated 95% confidence bounds.[18]

Hence, the simulated version of the model appears to be able to replicate the large amount of persistence in unemployment fluctuations and output growth which characterizes the French economy (and many similar countries).

[17] Table 1 to be inserted. Compute standard deviations.

[18] To be more precise, 95% of the observed point estimates of the model-implied autocorrelation functions lie within the two dotted lines in figure 3.

Note that these strongly persistent effects of shocks occur while, by definition, sunspot shocks are restricted to be *i.i.d* stochastic processes. There is therefore no doubt that this large amount of persistence is *endogenous* to the model, resulting entirely from internal propagation mechanisms and not from an exogenous source of persistence introduced through the stochastic driving processes. This is an important point, because Cogley and Nason (1995) and Rotemberg and Woodford (1996) strongly pointed out to the difficulties of many DSGE models to replicate the autocorrelation function of output growth without introducing an exogenous source of persistence[19].

Another easy way to emphasize this strong endogenous persistence is to look at the estimated values for the set of parameters γ_2. Observe in this respect that the estimated vector $\widehat{\gamma}_2 = (\widehat{\sigma}, \widehat{\alpha})$ falls very close to the locus characterizing the Hopf bifurcation values for σ (see Figure 1). It is clear in this case that the model will display large endogenous persistence. In fact, in the context of our model, it is almost tautological to say that the model displays strong endogenous persistence or to emphasize that it falls close to a Hopf bifurcation in the parameters space. Indeed, persistence in a dynamic model can be analyzed by referring to the roots (eigenvalues) of its characteristic polynomials. As our (reduced) dynamic model is two-dimensional, in a sink configuration, persistence requires that both roots have their modulus close to one. But this is precisely what is occurring when the model is in the neighborhood of a Hopf bifurcation, since in this case two complex conjugates eigenvalues of the dynamic system cross the unit circle under a small parameter change.

The important point to emphasize is that the Hopf bifurcation occurs for *realistic* values of all structural parameters. For example, the estimated value for the elasticity of substitution between capital and labor, $\widehat{\sigma} = .74$, is very close to the value of $\sigma = 0.7$ used in Pissarides (1997), Maffezzoli (2001), Cheron and Langot (2004), and others. Similarly, the estimated value for the firms bargaining power ($\widehat{\alpha} = 0.54$) is close to the standard value of 0.5 usually considered in the Labor Economics literature. It is also close to the value of 0.6 considered in the Real Business Cycle literature with wage bargaining

[19]Of course, since the time of publication of these two papers, several extensions to the standard model have since been considered to try to improve these deficiencies of the original model. Relevant mechanisms include, without exhaustibility, factor hoarding (Burnside et al., 1996), etc... - include other references-. See the survey by King and Rebelo (1999) for more discussion on this issue.. As far as we know, however, very few papers can replicate these observations using only i.i.d. sunspot shock.

(see e.g. Andolfatto (1996), Cheron and Langot, 2004).

This aspect is, we believe, one important contribution of our model with respect to the literature. In fact, in formal terms, the endogenous persistence resulting from our model occurs for similar reason to other papers in the literature. It results from the fact that the parameters values are such that the economy is close to the point where the Hopf bifurcation occurs. However, in many papers, bifurcations and indeterminacy can only occur under rather controversial calibrations of parameters. This includes strong enough increasing returns to scale in production (see e.g. Benhabib and Farmer (1994), Farmer and Guo (1994), Barinci and Chéron (2001), Wen (1998), include others), distortive taxation (insert reference) or a high share of public spending in production (Schmitt-Grohe and Uribe (1997).[20] In this model, by contrast, indeterminacy typically prevails under constant returns to scale and an *arbitrary* (positive) size of public redistribution.[21] Furthermore, provided that the unions' bargaining power is strong enough, the Hopf bifurcation arises for plausible values for the capital-labor elasticity of substitution .

Note also that the model offers an explanation to the high persistence of unemployment fluctuations which is relatively new in the literature. As mentioned in the introduction, early explanations for this feature have typically relied on *hysteresis* models with multiple equilibria, such as the "insiders-outsiders" types of models, in which the preferences of unions are implicitly assumed to exclude previously fired or unemployed workers.[22] In this tradition of models, persistent unemployment fluctuations occur because transitory shocks affect permanently the long run (or natural) level or the unemployment rate. While this type of explanations has received a great attention in the literature, the empirical evidence trying to assert it was at best mitigated. In fact, in many countries, different statistical tests applied to different economies often led to a rejection of a unit root in the unemployment series, suggesting a rather stable natural rate of unemployment (see e.g. Evans, 1989)

[20]Wen (1998) is an interesting example of a model where the Hopf bifuraction arises for reasonnable degrees of increasing returns to scale (higher than 10%). Benhabib and Wen (2004) have shown that this model could explain many features of the US business cycles. We discuss how our paper compares to this model in the next section.

[21]See Dufourt et al. (2005) for a more in depth discussion as well as an explanation for why indeterminacy occurs.

[22]See in particular Blanchard and Summers (1986).

More recently, dynamic general equilibrium models in the RBC tradition have also attempted to account for the persistence in unemployment fluctuations without giving up the assumption of a unique (or stable) natural unemployment rate. Various frictions on the labor market have been considered. For example, Merz (1995), Andolfatto (1996), Gomes *et al.* (2001) considered frictions in the matching process between workers and firms. Maffezzoli (2001), Cheron and Langot (2004) introduced wage bargaining. In general, these papers showed that standard Dynamic Stochastic General Equilibrium (DSGE) models could reproduce the amount of unemployment persistence found in the data as long as persistent exogenous shocks (in particular technological innovations) were introduced as driving processes. They further showed that labor markets frictions were able to magnify the effects of these persistent exogenous shocks.[23]

Two important features differentiate our model to those of this literature. First, it is clear that persistent unemployment fluctuations can occur in our model *even in the absence of any shocks on fundamentals.* Rather, pessimistic or optimistic expectations of consumers and firms may help explain transitory but persistent fluctuations in the unemployment rate. In fact, business cycles practitioners often pay a lot of attention to the "confidence indices" of firms and consumers, such as those provided by the University of Michigan, because they know that these indices are reliable leading indicators of the business cycle. Our model is fully consistent with this view, as persistent fluctuations may be the result of autonomous changes in expectations. To our knowledge, this model is the first to account for persistent unemployment fluctuations resulting entirely from self-fulfilling changes in expectations.[24]

However, although we emphasize sunspots shocks as a potential source of unemployment fluctuations, it should be clear that such long lasting variations in the unemployment rate would result in this model from *any kind* of shocks (whether on fundamentals or on expectations) and *whatever the degree of persistence of these shocks.* Using white noise sunspot shocks is clearly an eloquent way to emphasize this dimension. We, of course, do not claim that sunspot shocks are the only source or fluctuations. But they may explain why similar countries with roughly similar economic conditions (as is the case for many European countries) have often experienced drastically

[23]Describe more?

[24]Note also that, in contrast to these former models, the lack of a full insurance mechanism available to workers implies that being in unemployment is costly in terms of welfare.

different episodes of unemployment fluctuations.[25]

4.3 Other business cycles features

The ability of our model to account for the kind of strongly persistent fluctuations in the unemployment rate and output growth would be undermined if the model failed importantly in other dimensions of the business cycles. For this reason, we now turn to the evaluation of this model with respect to other standard features of the business cycle. This is an important step, because Schmitt-Grohe (2000) emphasized the difficulties of many DSGE models with sunspot-driven fluctuations to account for many stylized fact of actual economies. In particular, she pointed out... (to be completed)

In a recent paper, Benhabib and Wen (2004) showed that many of these deficiencies could be alleviated by considering a version of Wen's (1998) model with variable utilization rate, moderate increasing returns to scale and exogenous (serially-correlated) variations in aggregate demand, resulting from preferences of government spending shocks. When their model is calibrated, as in our case, so as to fall close to the point where the Hopf bifurcation occurs, they show, that it can display the correct amount of persistence in output growth, as measured by the different but related statistics emphasized by Cogley and Nason (1995) and Rotemberg and Woodford (1996). What is clear from their analysis is that these successes rely very much on the capacity of their model to generate a *hump-shaped* response of output to transitory/demand shocks.

It is therefore important to relate our paper to this one in order to emphasize the similarities and differences.[26] The impulse response functions (IRFs) of the main variables to a sunspot shock obtained with the second-order approximation of the solution are displayed in Figure 3.[27] For comparison pur-

[25] Insert examples.

[26] It should be noted that we will mostly refer to the stylized facts emphasized by Cogley and Nason (1995), regarding the autocorrelation of output growth and the hump shaped response of output to demand shocks, and not to the "forecastable movements" in the variables emphasized by Rotemberg and Woodford (1996), because the latter measures mostly make sense in the presence of persistent technological shocks affecting the long run level of output.

[27] For clarity of the figure, we have plotted the impulse response function of the employment rate, instead of that of the unemployment rate, because the latter series is by definition more volatile and would lead to a compression of the IRFs of the other variables. Given the definition of $u = 1 - l$, it is clear however that both variables are directly related

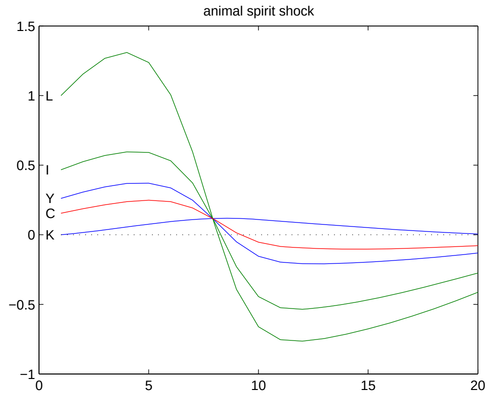

Figure 3: Impulse Response Functions with second-order approximation.

poses, Figure 4 displays similar IRFs when a simple linearization (first-order approximation) was used. As expected, we can observe from Figure 3 that sunspot shocks generate highly persistent periods of booms and recessions affecting simultaneously all the variables. In particular, after a one percent positive sunspot shock on the employment rate, output, total consumption, investment and hours all increase simultaneously for several periods, reaching a peak after 5 periods, then decreasing towards negative values for a few periods, and eventually reverting back slowly to the initial steady state. Observe that deviations from the steady state are still significant even after more than 20 periods.

Another important feature to notice is that the impulse response functions in Figure 3 display the typical hump-shaped response of output to transitory shocks characterizing many real economies. Note that this hump-shaped pattern occurs even in response to purely *white noise* sunspot shock. This is important to emphasize because Benhabib and Wen (2004) stress that, in their model, sunspot shocks alone cannot account for this feature. What is required is rather the combination of a calibration close to the Hopf bifurcation *and* serially correlated (real) demand shocks, such as variations in preferences or government spending. By contrast, our model succeeds in this dimension even with *i.i.d.* sunspots. Clearly, from a technical point of view, the fact that, near a Hopf bifurcation, the two eigenvalues of the

in the business cycle.

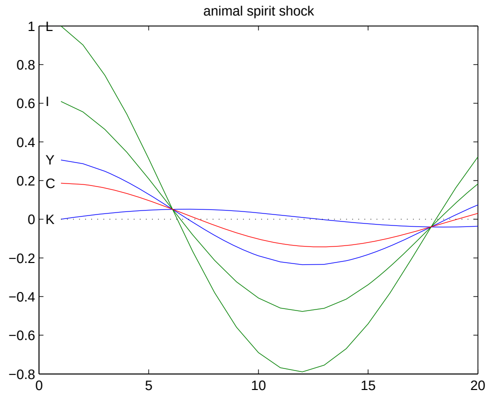

Figure 4: Impulse Response Functions with first-order approximation

Jacobian matrix are *complex conjugates* with modulus close to one explains the nature of this non-monotonous convergence to the steady-state.

As this is also the case in the Benhabib and Wen (2004) model, the different performances in this dimension may appear surprising. However, Figure 4 emphasizes an important potential caveat which may be at stake. It shows that the hump in output fluctuations predicted by the model would have been artificially eliminated if we had considered a simple first-order approximation of the solution (19). Hence, a proper account of the true dynamics of the model implied by transitory shocks seems to require an approximation of the true data-generating process which is sufficiently precise to take into account the nonlinearities of the model sufficiently well. In fact, it would be interesting to assert whether the inability of the Benhabib and Wen (2004) model to generate a hump in output after a transitory sunspot shock is inherent to their model or is simply due to the fact that they used a simple linear approximation to compute the solution of their model. We suspect the second case.

Finally, we discuss briefly the performance of our model in terms of the standard business cycle statistics emphasized in the Real Business Cycle literature. Table 2 summarizes the main statistics in terms of cross-correlations, relative standard deviations and autocorrelations for the French economy and those implied by the model when submitted to sunspot disturbances of

Table 2 - Business Cycle Statistics

	Relative standard deviations with output $\frac{\sigma_x}{\sigma_y}$					
Variable (x)	y	c	c^w	i	l	w
Data	1					
Model	1	0.61	0.71	2.00	3.03	2.32

	Cross correlations with output $Corr(y, x)$					
Variable (x)	y	c	c^w	i	l	w
Data	1					
Model	1	1.00	1.00	1.00	1.00	-0.99

	AR1 Coefficients					
Variable (x)	y	c	c^w	i	l	w
Data						
Model	0.85					

arbitrary size.[28]

Several important quantitative successes are worth emphasizing. (TO BE WRITTEN)

The model generates simultaneous procyclical movements of aggregate consumption, investment and output. This is worth stressing, as standard business cycle models naturally tend to generate countercyclical movements of consumption or investment in response to demand or sunspots shocks, unless large increasing returns to scale are introduced.[29] The main reason of this failure is that an increase in employment tends to decrease etc... etc.. In the present model, by contrast, the procyclical behavior of aggregate consumption stems from the fact that workers are financially constrained. Indeed, consumption purchases of workers are determined, in each period, by their wage bill. For an elasticity of substitution between capital and labor which is higher than the capital share, this wage bill is increasing with

[28] Of course, the size of the sunspots must be small enough to ensure that the dynamics remain in the basin of attraction of the steady state.

[29] This issue is discussed in detail in the survey by Benhabib and Farmer (1999) and in Schmitt-Grohe (2000). One solution to mitigate this problem is to combine increasing returns with variable capacity utilization, as in Benhabib and Wen (2004), or endogenous countercyclical markups, as in Dos Santos Ferreira and Dufourt (2006).

employment (despite the fact that the marginal productivity of labor – and therefore real wages – decrease with employment).[30]

The same reason also explains why the model performs better than standard Real Business Cycle models in terms of excessive smoothness of consumption. Indeed, it is a well-known feature that in real economies, the sensitivity of consumption to current income (and production) is much higher than what is predicted by standard dynamic models of consumption. In the present model, as workers spend all their available income on consumption, the overall sensitivity of aggregate consumption to current production is stronger: the relative volatility of workers consumption relative to output is 0.71, and that of total consumption (which includes consumption of capitalists) is 0.61, compared to a value of z.zz for the French economy.

etc. etc.

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