The Dynamics of the Legal System

Giuseppe Dari-Mattiacci,* Bruno Deffains† and Bruno Lovat‡

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Abstract

In his seminal book, Posner advances the hypothesis that legal systems may follow a cyclical pattern concerning the rate of litigation and the degree of uncertainty of the law. He postulated that litigation reduces the degree of uncertainty of the law as it produces precedents that have the beneficial effect of clarifying the law. In turn, when the law becomes more certain, parties expectations on the outcome of trials converge and settlement becomes more frequent; thus, the litigation rate decreases. When less cases are litigated, it is more likely

*Amsterdam Center for Law and Economics, Tinbergen Institute, and George Mason University School of Law. Email: gdarimat@uva.nl. The financial support by the NWO grant 016.075.332 is gratefully acknowledged.

†EconomiX, CNRS and Université Paris Ouest - Nanterre La Défense, Email: bdeffains@u-paris10.fr.

‡BETA, CNRS and Université Nancy 2. Email: bruno.lovat@univ-nancy2.fr.

that some uncertainty in the law emerges again due to technological and social changes to which the law does not adapt sufficiently fast. Hence, the degree of uncertainty rises again re-igniting the cycle. We present a dynamic model of the legal system identifying the conditions under which Posner's cyclical hypothesis is examined.

JEL classification: K10, K40, K41.

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1 Introduction

There is a growing body of economic literature focusing on the study of different legal systems. The main question this literature addresses concerns the *quality* of the rules produced by different legal systems and of their enforcement. In turn, such quality is measured according to various indicators, ranging from efficient financial markets to economic growth and stability (see e.g. La Porta, Lopez de Silanes and Shleifer, 2008).

Many authors emphasize adaptability as a determinant of success for a legal system. For instance, Beck and al. (2003) assert that common law regimes—where judicial opinions are a source of law—are better able to respond to changing circumstances than are civil codes based on statutes. The "adaptability" channel holds that (1) legal traditions differ in their ability to adjust to changing commercial circumstances, and (2) legal systems that adapt quickly to minimize the gap between the contracting needs of the economy and the legal system's capabilities will foster economic development more effectively than would more rigid legal traditions.

Authors working along those lines focus on the substantive content of legal rules: the provisions of a constitution, the elements of a corporations or antitrust statute, the law governing the enforcement of contracts or property rights. In contrast, relatively little attention has been paid to the process that brings about such rules.

Some writings in the comparative literature on the common law and the civil law suggest that these legal regimes differ in the extent to which procedural formalism is constraining (Dankov et al., 2003), to which courts are isolated from political control (Mahoney 2001, Glaeser and Shleifer, 2002). More recently, differences in the accumulation of legal (human) capital have been identified as an important determinant of the dynamic of common law and civil law systems (Hadfield, 2008). More precisely, the quality of legal adaptation depends on "the institutional determinants of judicial incentives and the capacity for a legal regime to generate investments in legal human capital that reduce legal error". However, courts are fed with cases through litigation. In turn, the decision of which cases to litigate is up to private parties. Thus, in order to gain a better grasp of the legal process, it is crucial to take into account the dynamic interaction between courts as producers of information about the law, on the one hand, and potential litigants as receivers of such information, on the other hand.

In this paper, we present a dynamic model in which the parties' decision whether to litigate depends on information produced by courts and, vice versa, the courts involvement in the lawmaking process depends on the cases proposed by the parties. Our aim is to provide insights into the process of adjudication and lawmaking by courts. Thus, we focus on the process of rule formation and not on the quality of the rules produced. In this sense, we depart from most of the previous literature on the topic. In sequel work, we plan to extend this approach to include also questions of efficiency.

Our argument can be sketched as follows. Technological and/or social changes generate important challenges for the legal systems. Social and technological events (labor law after social movements, products liability after complex innovations...) require legal evolution. Recent litterature in law and economics insists on the complementarities between legal incentives (the threat of being held liable for damages for instance) and normative incentives (the fear of social disapproval or stigma). Thus, changes in social behavioral norms may require legal changes. For instance, compliance with a social behavioral norm seems to be particularly relevant in litigation involving medical malpractice, professional liability or breach of contract (Deffains and Fluet, 2008). In his seminal book, Posner (2003, p. 554) advances the hypothesis that the legal system in common law jurisdictions exhibits a cyclical pattern concerning the rate of litigation and the degree of uncertainty of the law.

> "If [legal uncertainty] is great, there will be much litigation [...]. But since litigation [...] generates precedents, the surge in litigation will lead to a reduction in legal uncertainty, causing the amount of litigation to fall in the next period. Eventually, with

few new precedents being created, legal uncertainty will rise, as the old precedents depreciate (because they are less informative in a changed environment), and this uncertainty will evoke a new burst of litigation and hence an increased output of precedents."

Evolutionary theory has been already invoked in the legal literature (Hathaway, 2001). Indeed, the language used to describe the common law process often draws on evolutionary metaphors. The Supreme Court has written that the "flexibility and capacity for growth and adaptation is the peculiar boast and excellence of the common law" and that "the common law is not immutable but flexible, and by its own principles adapts itself to varying conditions". This language reflects an underlying reality. In a common law system, the decision in each new case draws on the stock of existing precedent, and that new case forms the foundation of precedent on which future cases are based. As Justice Cardozo once explained:

> "The implications of a decision may in the beginning be equivocal. New cases by commentary and exposition extract the essence. At last there emerges a rule or principle which becomes a datum, a point of departure, from which new lines will be run, from which new courses will be measured. Sometimes the rule or principle is found to have been formulated too narrowly or broadly, and has

> to be reframed. Sometimes it is accepted as a postulate of later reasoning, its origins are forgotten, it becomes a new stock of descent, its issue unite with other strains, and persisting permeate the law."

Hathaway also remarks that evolutionary theory enjoys a long lineage in jurisprudence. It first emerged in legal literature in the nineteenth century German historical school of jurisprudence, which was founded in the nineteeth century by von Hugo and continued by Freidrich Karl von Savigny. Scholars took it up with renewed vigor in the wake of the publication of Darwin's On The Origin of Species. These early works were followed by the scholarship of Holmes, who took legal evolutionary theory to the level of legal doctrine. In perhaps the most famous statement of his views on the topic, Holmes wrote:

> "The life of the law has not been logic: it has been experience. The felt necessities of time, the prevalent moral and political theories, intuitions of public policy, avowed or unconscious, even the prejudices which judges share with their fellow-men, have had a good deal more to do than the syllogism in determining the rules by which men should be governed."

In essence, the model of the common law adopted by many scholars is a legal version of the Darwinian paradigm. As noted by Hathaway, the adaptive rate of historical processes may proceed more slowly than changes in the environment, leading to a perpetual lag and, therefore, perpetual disparity between the institution or rule and its environment. Institutions are often resistant to change: They embed routines in a structure, develop their own criteria of appropriateness and success, and socialize existing arrangements. She adds:

> "The same phenomenon occurs in the law. When the rate of change in the legal environment outpaces the adaptation of legal rules, equilibrium is unlikely to be achieved before the environment changes. Because the legal environment is no more stable or static than the biological or political one, there will often be some degree of mismatch between the legal rule and the environment in which it is applied. This mismatch between the rate of adaptation and the rate of change in the environment is perhaps even more pronounced in legal institutions than in any other institution. The rule of stare decisis and the life tenure of most judges are, in fact, designed to embed resistance to change in the common law system. Indeed, many legal scholars celebrate this very quality of the legal system... The law's failure to adapt quickly

> may lead to inefficiency, but it also protects against instability and immediate surrender to momentary and ill-conceived whims of the public. This is where the biological and legal analogies diverge perhaps most significantly: Animals are largely powerless to preserve themselves in the face of marked changes in their environment, but judges, litigants, and institutions are capable of perpetuating legal rules long after the conditions that gave rise to them disappear."

In the following, we build on these ideas to present a dynamic model of legal evolution. We consider that technological and social changes fuel a physiological degree of uncertainty in the law, which tends to increase over time.[1] Uncertainty makes it more difficult for private individuals to foresee the outcome of trials and, thus, it is more likely that their expectations over the adjudication will diverge. When the parties expectations diverge—and they do typically so towards self-serving optimism (Loewenstein, Issacharoff, Camerer, and Babcock, 1993; Bar-Gill, 2006)—settlement attempts may more easily fail and a substantial portion of the cases go to trial (Landes, 1971; Posner, 1972, 1973; Gould, 1973). As uncertainty increases due to the depreciation of existing precedents, so does the litigation

[1]See Dari-Mattiacci and Deffains (2007) for a survey of the literature and a formal analysis in a static setting.

rate (period 1).

The fact that more cases go to trial give judges the possibility to examine them (Fiss, 1984). Hence, litigation reduces uncertainty because it eventually leads to the formation of new precedents that have the beneficial effect of clarifying the law (period 2). In turn, when the law becomes less uncertain, parties expectations over the outcome of the trial converge and they tend to settle rather than litigate; thus, the litigation rate decreases (period 3). When fewer cases are litigated, the pace at which the law adapts to ever-changing social and technological conditions also slows down. It is thus likely that over time uncertainty in the law increases again (period 4) giving raise to a new cycle.

2 Model

In the model, we consider litigation over non-contractual damages between two risk-neutral[2] strangers. With a slight abuse of terminology, the plaintiff could be a victim of an accident, a copyright holder or a landowner; respectively, the defendant could be an injurer, a copyright infringer or a trespasser. The plaintiff seeks compensation from the defendant for an amount D (the harm). The parties can either settle for an amount $S \geq 0$ or go to trial; in

[2] See Clermont and Eisenberg (2007) on the effects of risk aversion on the choice whether to settle or litigate.

the latter case, a court will assess the merit of the case and award the plaitiff damages equal to pD, with $p \in [0,1]$.

In order to decide whether to settle or go to trial, the parties need to make an estimate of the damages award. We postulate that the legal rules according to which damages should be apportioned between the plaintiff and the defendant are subject to a certain degree of uncertainty; that is, legal uncertainty makes it difficult for the parties to predict what level of p the court will apply.

To capture this feature, we assume that each party randomly and independently derives an idiosyncratic estimate of p—p_Π for the plaintiff and p_Δ for the defendant—from a unimodal distribution with parametric density function $f(p;\bar{p},\sigma)$ and cumulative distribution function $F(p;\bar{p},\sigma)$.[3] The mean μ of the distribution indicates the merit of case, that is, the expected value of p. Note that, although the parties' estimates are typically wrong, they are unbiased. In turn, the variance σ^2 indicates the uncertainty of the case. We assume that the single crossing property applies when comparing two distributions with different variances.[4]

When deciding whether to settle or to litigate, the parties will compare the

[3] That is, p_Π and p_Δ are *iid.* As it is common, we assume that $f(0;\bar{p},\sigma)$ is positive and differentiable on $[0,1]$, $F(0;\bar{p},\sigma)=0$ and $F(1;\bar{p},\sigma)=1$. Note that the distribution is not necessarily symmetric, thus mode and mean need not coincide.

[4] More precisely, for $\sigma_2 > \sigma_1$, there exists a $\hat{p}$ such that $F(p;\bar{p},\sigma_2) - F(p;\bar{p},\sigma_1) \leq (\geq) 0$ for $p \geq (\leq) \hat{p}$.

expected outcome from litigation with the settlement amount S. Normalizing the settlement costs to zero, let c_Π and c_Δ be the positive litigation costs borne by the parties under the American rule.[5]

To keep the analysis simple, we assume that settlement will occur if and only if, given the parties' estimates, there exists a settlement amount such that no party prefers litigation. Thus, the parties litigate if the following two conditions are simultaneously satisfied; otherwise, they settle:

$$\begin{cases} p_\Pi D - c_\Pi > S \\ -p_\Delta D - c_\Delta > -S \end{cases}$$

Summing up and rearranging, we obtain the necessary and sufficient condition for litigation: $p_\Pi - p_\Delta > \frac{c_\Pi + c_\Delta}{D} \equiv r$. The ex post probability of litigation can be estimated from the beliefs distribution as follows:

$$\begin{aligned} L &\equiv P\left(p_\Pi - p_\Delta > r\right) \\ &= \int_r^1 F\left(p - r\right) f\left(p\right) dp \end{aligned}$$

It is easy to verify that L increases in σ and decreases in r while it is constant in $\bar{p}$.[6] In turn, since r increases in c_Π and c_Δ and decreases in D,

[5] Under the American rule each party pays his own costs irrespective of who wins in court

[6] For a formal proof, see Dari-Mattiacci (2007).

we have that L decreases in c_{Π} and c_{Δ} and increases in D. For the time being, let us focus on the effect of σ on L and keep all other parameters constant. We can hence write the litigation rate at time t as a function of the divergence in the parties' expectations:

$$L(t) = g^{-1}(\sigma(t)) \tag{1}$$

where g^{-1} is a strictly increasing function. A natural process of obsolescence—due to social, economic or technological changes—makes legal rules progressively less in tune with the underlying characteristics of accidents. This process of absolescence makes the divergence in the parties' expectations $\sigma(t)$ increase over time at a constant rate $b > 1$, as it becomes more difficult to predict the court decision. A countering force is offered by the production of information by the court, which reduces the divergence of expectations at a rate $\frac{1}{U(t)}$. The net effect will depend on whether $\frac{1}{U(t)} > (<)\, b$. Keeping the rate of obsolescence constant, we can write:

$$\frac{d\sigma(t)}{dt} = f(U(t)) \tag{2}$$

where f is a strictly increasing function and $U(t)$ is an index of the uncertainty of law at time t. Thus, we now turn to the production of information by the court. A natural way to formalize the production of information is simply to look at the number of precendents $n(t)$ issued at time t, with the understanding that more precedents produce more information. Thus, we can write:

$$U(t) = G^{-1}(n(t)) \tag{3}$$

where G^{-1} is a strictly decreasing function. In turn, the number of precedents produced at time t varies depending on the number of cases filed at that time. This can be understood by thinking of the incoming cases as putting pressure on the court to speed up decisions. Denoting by N the number of disputes arising, we have that $NL(t)$ cases will be filed at time t. The rate of precedents production increases or decreases depending on whether the number of filed cases is greater or less than a threshold a. Keeping N constant,[7] we can write:

$$\frac{dn(t)}{dt} = F(L(t)) \tag{4}$$

3 The dynamics of uncertainty and litigation

From the expressions above, we have:

[7] We assume that N is a constant and thus abstract from considerations about deterrence at this stage. RELAX LATER___

$$\begin{cases} \frac{d\sigma(t)}{dt} = \frac{dg(L(t))}{dt} = \frac{\partial g}{\partial L} L'(t) = f(U(t)) \\ \frac{dn(t)}{dt} = \frac{dG(U(t))}{dt} = \frac{\partial G}{\partial U} U'(t) = F(L(t)) \end{cases}$$

Since $g'G' \neq 0$, we have the following differential equation system governing the rate of litigation and the degree of uncertainty:

$$\begin{cases} L'(t) = \frac{f[U(t)]}{g'[L(t)]} = \frac{\frac{d\sigma(t)}{dt}}{\frac{d\sigma(t)}{dL}} \\ U'(t) = \frac{F[L(t)]}{G'[U(t)]} = \frac{\frac{dn(t)}{dt}}{\frac{dn(t)}{dU}} \end{cases} \tag{5}$$

The solution to (5) gives the stationary point of the legal system. Given the legal context, it is characterized by a litigation rate ($L^* = F^{-1}(0)$) and a certain level of uncert
At this point the rate of change is zero for both variables ($\frac{dL(t)}{dt} = \frac{dU(t)}{dt} = 0$). We can define this situation has a "natural" equilibrium of the legal system[9].

Considering this point, we can imagine two different situations.

First, if the initial conditions of the legal system are such that $L(0) = L^*$ and $U(0) = U^*$, neither litigation nor uncertainty change over time. In other

[8] We have $u^* = f^{-1}(0)$ because $L'(t) = 0$ implies $f[U(t)] = 0$, and $l^* = F^{-1}(0)$ since $U'(t) = 0$ implies $F[L(t)] = 0$.

[9] In the same vein we find a model proposed by Rasmusen (1994) in which judicial legitimacy is seen as a repeated game. This influence model assumes a game of perfect information with symmetric players. Rasmusen discusses the possibility of moving to a better equilibrium rather than abandoning hope of a responsible judiciary. He draws on game theory to identify the ways in which judicial expectations are formed: an equilibrium that is Pareto optimal, that uses simple strategies, that has been played out in the past, and that is publicly announced to be the equilibrium becomes a "focal point", i.e. an equilibrium attractive for psychological reasons (Rasmusen 1994: 78).

words, the system in completely stable:

$$\begin{aligned} L(t) &= L(0) = L^* \\ U(t) &= U(0) = U^*. \end{aligned}$$

However, there is no reason to admit that the initial situation correspond the "natural" equilibrium. So, the second case appears when the initial conditions depart from the stationary point. As a result, litigation and uncertainty will typically follow a motion that can be described by transforming (5) in an algebraic equation. Let:

$$\begin{aligned} h(x) &= \int^{x} F(S)\, g'(S)\, dS \\ H(x) &= \int^{x} f(S)\, G'(S)\, dS \end{aligned}$$

By totally differentiating $h(L(t)) - H(U(t))$ we have:

$$\begin{aligned} \frac{d}{dt}\left[h(L(t)) - H(U(t))\right] &= F[L(t)]\, g'(L(t))\, L'(t) - f[U(t)]\, G'(U(t))\, U'(t) \\ &= F[L(t)]\, g'(L(t))\, \frac{f[U(t)]}{g'[L(t)]} - f[U(t)]\, G'(U(t))\, \frac{F[L(t)]}{G'[U(t)]} \\ &= 0 \end{aligned}$$

yielding the phase curve:

$$h(L(t)) - H(U(t)) = constant \tag{6}$$

If the phase curve in (6) is closed, the analysis above shows that the point $[L(t), U(t)]$ moves periodically in a cycle. The mathematical conditions to observe such a cycle imply the existence and the unicity of $f^{-1}(0)$ and $F^{-1}(0)$. Metaphorically, the stationary point plays the same role in the legal system as the sun in the solar system. Like a planet goes along an orbit around the sun, the legal evolution can be described as a closed trajectory around a natural equilibrium. Of course, considering the legal origins (e.g. common law or civil law traditions) the position of the equilbria and the characteristic of the trajectories are certainty different. A key insight here is that evolution is directly constrained by the history of the legal system. The possibility for today and tomorrow are determined by the past. For instance, judges in Continental law are quite different from judges in Anglo-American law. The legal systems are different and the legal traditions diverge. However, even if the legal cultures that have grown out of the different legal evolution processes, we propose a unifying model that will apply for both sides of the Atlantic.

Proof: see appendix.[10]

Using this result, we can go back to the opinion of Posner about the evolution of the legal system. According to him, the legal system in common law jurisdictions exhibits a cyclical pattern concerning the rate of litigation and the degree of uncertainty of the law. Precisely, if legal uncertainty is great, there will be much litigation. But since litigation generates precedents, the surge in litigation will lead to a reduction in legal uncertainty, causing the amount of litigation to fall in the next period. Eventually, with few new precedents being created, legal uncertainty will rise, as the old precedents depreciate (because they are less informative in a changed environment), and this uncertainty will evoke a new burst of litigation and hence an increased output of precedents.

We propose to refer to Figure 1 to illustrate such an evolution:

The figure describes the legal cycle. It shows that uncertainty and litigation revolve around the stationary point $(L^*; U^*)$. In the traditional eco-

[10] One can remark that this explanation of the evolution of legal systems could also be developed with a predatory-prey approach. For instance, we could analyze the following situation:

$$\begin{aligned} \frac{dL}{dt} &= \frac{f(U)}{g'(L)} = L\left[b_2 U - r_2\right] \\ \frac{dU}{dt} &= \frac{F(L)}{G'(U)} = U\left[r_1 - b_1 L\right] \end{aligned}$$

However, our perspective is more general because we don not have to put restrictions on the functions. The prey-predator approach is only a special case.

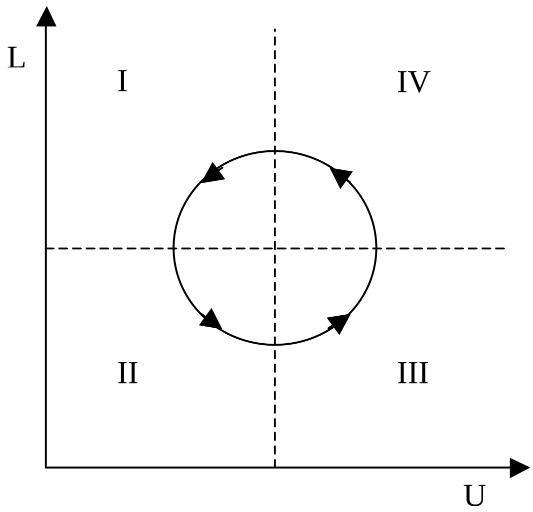

Figure 1: The legal cycle

nomic approach of litigation, the litigation rate should respond immediately to changes in the predictability of the outcome of trials, and hence to uncertainty of law. Instead, here we consider a framework in which responses are somewhat delayed. There are many reasons for the existence of delays. Most of them refer to congestion effects and rigidities inherent to the functioning of legal institutions. Some authors have already studied the consequences of such rigidities on the evolution of law. They generally develop an approach in terms of to path dependency: Most legal choices allow for an evolutionary adaptation of legal policy. Over time, the practices of courts become a source of path dependence; that is, sequential legal choice within a relatively stable or narrowing range of alternatives (Thelen 2003). An important conclusion of this reasoning is that "legal change is unpredictable ex ante and non ergotic, and early outcomes may become locked in. The law evoluate gradually over time, drawing on an existing stock of precedents, punctuated with periods of rapid adaption" (Hathaway, 2001).

In our model, we agree on the importance of rigidities, but we consider that the legal evolution is, at least, partially predictible. Consider for instance, that the legal system is initially characterized by a low uncertainty($U_0 < U^*$) and a high litigation rate ($L_0 > L^*$). The resulting dynamic is given by: $\frac{dU}{dt} < 0$ and $\frac{dL}{dt} < 0$. Thus, in stage 1 both uncertainty and litigation are expected to decrease. When the rate of litigation arrives at L^*, in stage 2, there are too few litigated cases to provide for the needed precedents update. Thus uncertainty begins to increase up to the level U^*, whereas litigation still decreases: $\frac{dU}{dt} > 0$ and $\frac{dL}{dt} < 0$. At the following stage 3, uncertainty has reached a point that triggers an increase in litigation. Thus, while uncertainty still increases, litigation begins to increase: $\frac{dU}{dt} > 0$ and $\frac{dL}{dt} > 0$. In the last stage, litigation continues to increase but uncertainty will decrease: $\frac{dU}{dt} < 0$; $\frac{dL}{dt} > 0$. When the legal system attains stage 4, the conditions are verified to start a new cycle.[11]

[11] **EXPAND THIS PART**

4 The dynamic effects of "shocks"

What happens if a "shock" affects the legal system? In this section, we analyze the consequences of an exogeneous event that could imply a legal change. The origins of such events are numerous. For instance, the state can decide to modify the cost of trial, cut-backs in the number of judges, introduce improvements in the judicial selection process, promote ADR methods to alleviate the judicial caseload, or better means of monitoring judges for possible misconduct or change procedural rules, etc. As a result, the natural evolution of the legal system will be modified.

To explain the nature of legal change, we propose to focus on the average values of U and L.

4.1 Average values of uncertainty and litigation

In the continuous time case, the average values can be defined in the following way.[12] Take μ a monotonic and continuous function and f a continuous

[12] This definition is a generalization of the classical definition of an average value: consider a monotonic function μ. We call $\mu-$ average of two positive numbers a and b , the number m that verifies :

$$\mu(m) = \frac{\mu(a) + \mu(b)}{2}$$

function on $[a,b]$. We call μ-average of f on $[a,b]$ the number m that satisfies:

$$\mu(m) = \frac{1}{b-a}\int_a^b \mu(f(x))dx$$

or:

$$m = \mu^{-1}(\frac{1}{b-a}\int_a^b \mu(f(x))dx)$$

For example, if $\mu(x) = x$, the mean value is:

$$m = \frac{1}{b-a}\int_a^b f(x)dx$$

This value can be used to analyze the interaction between uncertainty and litigation.

Denote T the period of time necessary to complete a cycle of $L(t)$ and $U(t)$. As we know, $n(t)$ depends on the level of uncertainty $U(t)$. This correspond to a periodical function:[13]

$$n(T) = n(0)$$

[13] When $t = T$, we have $J(T) = J(0)$

This implies:

$$\frac{1}{T}\int_0^T \frac{dn(t)}{dt}dt = \frac{n(T)-n(0)}{T} = 0$$

As $\frac{dn(t)}{dt} = F\left(L\left(t\right)\right)$, we have:

$$\frac{1}{T}\int_0^T F\left(L\left(t\right)\right)dt = 0.$$

Consider L^* the function F-average of $L(t)$ on $[0, T]$:

$$L^* = F^{-1}\left(\frac{1}{T}\int_0^T F\left(L\left(t\right)\right)dt\right) = F^{-1}(0).$$

The same reasoning applies to the litigation rate and the divergence in the parties' estimates. Thus, we obtain the function f-average of $U(t)$ on $[0, T]$:

$$U^* = f^{-1}(0).$$

These values are useful to predict the consequences of shocks on the evolution of the legal system. More precisely, variations in the evolution of new precedents produced by the courts and/or in the evolution of divergence between the parties' expectations will modified the average values.

4.2 Changes in legal evolution

Suppose for instance that the legal system enters in a phase of expansion. This could be the result of the recruitment of new judges by the State on one side (facilitating the production of new precedents) and from the development of mediation on the other side (improving the convergence between's parties expectations). Consequently, the impact of U and L on the allocation of the damages award between the defendant and the plaintiff is modified. The impact on the productivity of old precedents is modified too.

In this case, we verify that[14]:

- $f(x)$ becomes $f_\varepsilon(x) = f(x) + \varepsilon_1$

- $F(x)$ becomes $F_\varepsilon(x) = F(x) + \varepsilon_2$ with $\varepsilon_i > 0$.

Consequently, the new average rate of litigation is:

$$\begin{aligned} L^{**} &= F_\varepsilon^{-1}(0) = F^{-1}(-\varepsilon_1) \\ \text{because } F_\varepsilon\left[F^{-1}(-\varepsilon_1)\right] &= F\left[F^{-1}(-\varepsilon_1)\right] + \varepsilon_1 = -\varepsilon_1 + \varepsilon_1 = 0. \end{aligned}$$

Moreover, we know that F^{-1} is increasing, so we have:

$$L^{**} = F^{-1}(-\varepsilon_1) < F^{-1}(0) = L^*$$

[14] ε can be interpreted as a shock affecting the organization of the legal framework.

This means an increase in the average litigation rate.

We can develop the same analysis for the degree of uncertainty that becomes:

$$
\begin{aligned}
U^{**} &= f_{\varepsilon}^{-1}(0) = f^{-1}(-\varepsilon_2) \\
\text{because } f_{\varepsilon}\left[f^{-1}(-\varepsilon_2)\right] &= f\left[f^{-1}(-\varepsilon_2)\right] + \varepsilon_2 = -\varepsilon_2 + \varepsilon_2 = 0.
\end{aligned}
$$

As f^{-1} is increasing, we have:

$$U^{**} = f^{-1}(-\varepsilon_2) < f^{-1}(0) = U^*$$

As a result, the legal system will be characterized (on average) by a decrease in uncertainty and a decrease in litigation.

More generally, we can analyze all the possible transformations of legal evolution due to "shocks". They correspond to changes in σ and/or n that modifies the stationary point given by (U^*, L^*).

The demonstration above shows that such a shock modifies simultaneously:

- the center of the trajectory,

- the stationary solution of the dynamic system,

- the average values of the rate of litigation and of the degree of uncertainty.

The following table resumes all the possible cases. There are two kinds of changes affecting σ and n: expansion or contraction. For all the combinations, the evolution of the litigation rate and degree of uncertainty is predictible:

	Contraction of n	Expansion of n
Expansion of σ	uncertainty decreases litigation increases	uncertainty decreases litigation decreases
Contraction of σ	uncertainty increases litigation increases	uncertainty increases litigation decreases

to be continued...

5 Conclusion

Holmes (1881, p.1) observed “In order to know what [the law] is, we must know what is has been, and what it tends to become. We must alternatively consult history and existing theories of legislation. But the most difficult labor will be to understand the combination of the two into new products at every stage”. We have attempted to describe the mutual interaction between the "historical" development of the law through the judicial process and the effects of judicial intervention through the lens of a modern theory of lawmaking. Our scope has necessarily been modest and limited for want of many extensions, which we will address in the following.

Although the law is necessarily incomplete ex ante, gaps may be filled ex post by the courts. This analysis starts with exploring two ways in which uncertainty and the litigation process may be connected, which were only analyzed separately in previous literature. We looked ex post adjudication as a way to reduce the incompleteness of the law and argued that the rate of litigation in a legal system and its degree of incompleteness are connected with each other. By using a dynamic model first, we interpret the completeness of law as a good that can be sought by the citizens through the judicial process of dispute resolution. In this context, our model of the legal system identifies the conditions under which Posner's cyclical hypothesis holds.

Far from providing definite answers to the practical problems lawmakers daily face in the creation and interpretation of the law, our theory brings attention to several issues that could be investigated empirically. First of all, in the face of the divide between civil law and common law countries, it seems to emerge from our theory that, given the broader reliance on judge-made law in common law jurisdiction, one may expect systematically higher litigation rates in such countries than in civil law countries, due to the fact that less completeness of law is provided by legislature in the former group of countries.

Another point that our theory raises concerns the modern waves of litigation in certain areas of the law. We have stressed that policies aimed at reducing the litigation rate also affect the degree of the law's incompleteness and may, under certain conditions, suffer from a feedback effect due to the fact that reduced litigation triggers incompleteness, which in turn tends to raise the litigation rate. Understanding the way in which litigation and incompleteness of law interact may help policy makers and scholars to better comprehend the effects of policies targeting one or the other problem.

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[1] **__to be completed__**

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6 Appendix 1

Theorem : Solutions to the dynamic system (3):

$$\begin{cases} L'(t) = \frac{f[U(t)]}{g'[L(t)]} \\ U'(t) = \frac{F[L(t)]}{G'[U(t)]} \end{cases}$$

are periodic.

Proof :

Closed orbits without fixed points in a phase portrait correspond to solutions with periodic behaviour.

Here the orbits of (3) correspond to level sets (Γ): $V(x,y) = h(x) - H(y) = c$.

Using Morse theory, we observe that these level sets are closed curves:

Here $(F^{-1}(0), f^{-1}(0))$ is the rest point:

$$\begin{aligned}
\frac{\partial V}{\partial x}(F^{-1}(0), f^{-1}(0)) &= h'(F^{-1}(0)) = F\left(F^{-1}(0)\right) g'\left(F^{-1}(0)\right) = 0 \\
\frac{\partial V}{\partial y}(F^{-1}(0), f^{-1}(0)) &= -H'(f^{-1}(0)) = -f\left(f^{-1}(0)\right) G'\left(f^{-1}(0)\right) = 0
\end{aligned}$$

and the hessian matrix of V at this rest point:

$$D^2V(F^{-1}(0), f^{-1}(0)) = \begin{pmatrix} F'\left(F^{-1}(0)\right) g'\left(F^{-1}(0)\right) & 0 \\ 0 & -f'\left(f^{-1}(0)\right) G'\left(f^{-1}(0)\right) \end{pmatrix}$$

is nondegenerate and has negative eigenvalues:

$$\begin{cases} \lambda_1 = F'\left(F^{-1}(0)\right) g'\left(F^{-1}(0)\right) < 0 \\ \text{since } F \text{ is a decreasing function and } g \text{ is an increasing function,} \\ \lambda_2 = -f'\left(f^{-1}(0)\right) G'\left(f^{-1}(0)\right) < 0 \\ \text{since } f \text{ and } G \text{ are increasing functions.} \end{cases}$$

According to Morse theory, the nondegenerate level curves $V(x, y) = c$ are homeomorphic to ellipses and form a nested family surrounding the rest point. They correspond to periodic solutions.