Does entry improve welfare? A general equilibrium approach to competition policy

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Abstract

We consider a simple general equilibrium model with imperfect competition. Firms are price taker in the input market and compete à la Cournot in some or all of the product markets (their technology displays constant returns to scale). We show that an increase in the number of firms does not always improve welfare. We also provide a characterization in terms of mark-up rates of the sectors for which entry is welfare enhancing. Thus, this paper challenges the common idea that mergers with no cost synergy are not desirable for consumers.

Key words : Cournot competition, competition policy, general equilibrium and imperfect competition, efficiency.

JEL Classification: D50, L13, L40

1 Introduction

How does entry in oligopolistic markets affects welfare? This question which is central in the modern analysis of competition has been mainly considered from a partial equilibrium view point. The literature is huge and insightful (see e.g. Motta (2004)). Yet, it is well known that partial equilibrium analysis may be incomplete (see for instance François and Horn (2000) and Herendorf, Schmitz and Teixeira (2005)). Therefore,

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to test the robustness of the conclusions obtained in partial equilibrium, a general equilibrium approach to competition policy is called for.

The need for a general equilibrium approach is also clear when competition policy is explicitly considered as a tool for boosting agents' real incomes (Blanchard and Giavazzi (2001), Blanchard (2006)).

There has been several attempts to study the welfare effects of entry in general equilibrium economies with increasing returns to scale or fixed costs. In such economies, it has been shown that a marginal decrease in the number of firms from the free-entry equilibrium level may be welfare improving (Negishi (1962), Konishi *et al.*. (1990)).

It is not clear however that some pervasive unexploited increasing returns to scale exist in modern economies (see Posner (2001) for the case of traditional industries). As a result, considering the case of constant returns to scale may be relevant.

Competition policy in economies with constant returns to scale has been studied by Kelton and Rebelein (2003) as well as Neary (2002 (a,b) , 2003 (a,b)) who relies on a subjective demand approach to general equilibrium with imperfect competition (Negishi (1961))[1]. Kelton and Rebelein show the striking result that social welfare can be higher under monopoly than under perfect competition. However, their result requires the assumption that individual welfare can be aggregated (social welfare is higher when there are monopolies because the welfare of their owners is greater than at a competitive equilibrium and their weights in the social welfare function are important). Neary (2003b) obtains the surprising result that in an economy without heterogeneity across production sectors (featureless economy), increasing the number of firms and hence competition in each sector has no effect on welfare. In other words, Neary shows that competition policy does not affect welfare. As shown in Crettez and Fagart (2005), under the specific assumptions used by Neary, the general equilibrium is Pareto efficient and as a result, competition policy can have no positive effect on welfare. However, Neary's analysis demonstrates the rich potential of a general approach to competition policy.

Finally, Blanchard and Giavazzi (2001) propose a model to study deregulation in goods and labor markets. However, in this work, competition policy is only seen as a tool to increase the numbers of products as well as making products more substitutable.

[1]In addition to these papers, one should also mention Spear (2001) - who uses a market games approach to study electricity markets, and Ruffin (1999), who does not study competition policy per se but oligopoly and trade.

This paper considers an economy with a representative agent and constant returns to scale which slightly differs from Neary's model (but sticks to a subjective approach to general equilibrium with imperfect competition). In contrast to Neary, we assume that there is a finite number of production sectors. This allows us to investigate the general effect of a change in the number of oligopolistic firms within a particular sector which may be decided by a political or an administrative decider[2]. Our framework also allows us to consider the co-existence of competitive and non-competitive sectors. In addition, we work with general (albeit additively separable) utility functions[3].

We show that when the economy has both competitive and non-competitive sectors, all competitive sectors overproduce with respect to their efficiency level (this happens if there is at least one non-competitive market). Interestingly, some non-competitive sectors may have their production equal to their efficiency level. We also prove that the effect of competition policy differ across sectors. It may well be the case that decreasing the number of firms in a sector (or even turning a competitive sector into a non-competitive one) is welfare improving.

These results rest on the fact that, in equilibrium, some sectors over-produce compared to efficiency and some others under-produce. Increasing the number of firms in a given sector modifies the output of other sectors, enhancing welfare when a sector that previously over-produced (resp. under-produced) reduces (increases) its output. We show that there is a case where mergers should be prohibited in every non-competitive sectors. In that case, competition policy has a simple flavor. But otherwise, competition policy should be different depending on the sector considered. The point is that identifying these sectors could be difficult (though theoretically possible).

The plan of the paper is as follows. The next section presents the model and studies the equilibrium. Section 3 provides two examples illustrating the fact that increasing the number of firms in a sector is not always welfare enhancing. The first example is a two-sector economy with imperfect competition in both sectors. The second example is a multiple-sector economy with Cobb-Douglas preferences. Section 4 presents more general results on the effects of competition policy. All proofs are in the appendix.

[2]Contrary to Blanchard and Giavazzi (2001) we will assume that the number of products in the economy is constant and we shall not consider the impact of competition policy on the substitutability of goods.

[3]Following Neary, we assume that there are differentiated Ricardian technologies across production sectors.

2 The model

Consider a simple closed economy inhabited by a representative agent. There are N consumption goods, indexed by $k = 1, \ldots, N$. The agent's preferences are described by a utility function $U : \mathbb{R}^N_+ \to \mathbb{R}$, which satisfies the following assumption:

Assumption 1. $U : \mathbb{R}^N_+ \to \mathbb{R}$ *is:*

1. *increasing.*
2. *strictly concave.*
3. *smooth.*
4. *separable, i.e.* $\dfrac{\partial^2 U(.)}{\partial x_j \partial x_k} = 0$ *if* $j \neq k$.
5. *Furthermore, for all* k, $\lim_{x_k \to 0^+} \frac{\partial U}{\partial x_k} = +\infty$.

The agent supplies a fixed amount of labor L and considers a positive price vector $(p_1, \ldots, p_N)$ as given. The quantities he wishes to consume maximize his utility subject to the affordability constraint:

$$\max_x \; U(x_{1,} \ldots, x_N) \tag{1}$$

$$s.t \sum_k^N p_k x_k \leq R. \tag{2}$$

In the last equation R denotes the agent's income, which is equal to the sum of wages and profits.

We now turn to the production side. We assume that firms are identical within sectors and that the technology is Ricardian (labor is the sole input). Let w be the wage rate. The unit cost of firms in sector k is $\alpha_k w$, $k = 1, \ldots, N$, where α_k is a positive technical coefficient. We let n_k be the positive number of firms in a sector k. We assume that these numbers of firms are exogenous (there is no free entry).

There is a subset I of sectors where firms compete à la Cournot. Firms maximize profits taking both the wage and a subjective demand function for its market as given.

In the others sectors, $k \notin I$, there is perfect competition: firms are price takers. In the spirit of Neary (2003 b) - see also Crettez and Fagart (2005) - we will consider the following concept of general equilibrium with imperfect competition. This concept derives from Negishi (1961)'s approach and relies on subjective demand functions. It is intermediate between the objective demand approach of Nikaido (Nikaido (1975), Stahn (1999)) and the notion of "cost minimizing market equilibrium" of Brown and Wood (2004) where explicit monopoly pricing is discarded from the analysis.

Definition 1. *A general equilibrium with imperfect competition is a price system* $p \in \mathbb{R}_+^N$*, a wage rate* w*, and quantities* $x^e \in \mathbb{R}_+^N$ *such that:*

1. $x^e = X(p, R)$.
2. $(\forall x \notin I)$, $x_k^e \in \arg\max_{x_k} \{p_k x_k - w\alpha_k x_k \mid x_k \geq 0\}$.
3. $(\forall x \in I)$, $\frac{x_k^e}{n_k} \in \arg\max_{x_k^i} \left\{P_k(\frac{n_k-1}{n_k}x_k^e + x_k^i, x_{-k}^e, R)x_k^i - w\alpha_k x_k^i \mid x_k^i \geq 0\right\}$.
4. $\sum_{k=1}^N p_k x_k^e = R$.
5. $\sum_{k=1}^N \alpha_k x_k^e = L$

where: $X(p, R) = \arg\max \left\{U(x) \mid \sum_{k=1}^N p_k x_k \leq R\right\}$ *and* $P(x, R)$ *is its inverse,* R *being fixed*[4]*.*

In this definition, firms' perceived demands functions are equal to the inverse demand functions that arise in equilibrium. Moreover, firms do not take into account the effects of their choices on the agent's income (through the distribution of dividends and wages). This assumption rests on the idea that firms are large in their market but small in the economy as a whole.

Assumption 2. *Let* k*,* R *and* x_{-k} *be given. Then:*

$$2P_k'\left(\frac{n_k-1}{n_k}x_k + y, x_{-k}, R\right) + yP_k''\left(\frac{n_k-1}{n_k}x_k + y, x_{-k}, R\right) \leq 0 \text{ for all } y \geqslant 0 \quad (3)$$

whenever:

$$P_k(x_k, x_{-k}, R) - \alpha_k w + x_k^i P_k'(x_k, x_{-k}, R) = 0 \quad (4)$$

This assumption ensures that (4) characterizes firms' best responses in a Cournot equilibrium (every Cournot equilibria on market k are symmetrical). It allows us to characterize equilibria with positive productions of all goods.

[4]The inverse demand function for good k writes: $P_k(x, R) = RU_k'(x_k)/(\sum_{j=1}^N U_j'(x_j)x_j)$. This expression obtains using the optimality conditions for the agent's problem (namely $U_k'(x_k) = \mu p_k$, where μ is a Lagrange multiplier) and the budget constraint.

Proposition 1. *Assume that assumption 2 holds true. Let x be a production vector with positive coordinates. Then x is a production vector for a general equilibrium with imperfect competition if and only if there exists $\lambda > 0$ such that:*

$$(\forall k \notin I),\ U_k'(x_k) = \lambda \alpha_k \tag{5}$$

$$(\forall k \in I),\ U_k'(x_k) = \lambda \frac{n_k}{n_k + \sigma_k(x)} \alpha_k \tag{6}$$

$$\sum_{k=1}^{N} \alpha_k x_k = L \tag{7}$$

where $\sigma_k(x) = \frac{\partial P_k(x,R)}{\partial x_k} \frac{x_k}{P_k(x,R)}$ is the elasticity of the inverse demand function.

Finally, the Pareto optimum of the economy, denoted by $x_k^*,\ k = 1, \ldots, N$, is the solution of following problem:

$$\max_x \ U(x_1, \ldots, x_N) \tag{8}$$

$$s.t \sum_k \alpha_k x_k \leq L. \tag{9}$$

This solution is characterized by the next equations:

$$\frac{U_k'(x_k^*)}{\alpha_k} = \frac{\sum_j U_j'(x_j^*) x_j^*}{L},\ k = 1, \ldots, N. \tag{10}$$

Let us now define:

$$\beta_k(x) \equiv 1 \text{ if } k \notin I \tag{11}$$

$$\beta_k(x) \equiv \frac{n_k(x)}{n_k + \sigma_k(x)} \text{ if } k \in I \tag{12}$$

Thus, from Proposition 1, a general equilibrium with imperfect competition is Pareto-optimal if and only if:

$$\beta_k(x) = \beta_j(x),\ \forall j, k. \tag{13}$$

When the above condition is satisfied, imperfect competition acts very much like a tax on final-good consumption (whose lump-sum redistribution of the receipts). In that case, all relatives prices remain the same and there is no efficiency loss from the tax.

3 Two examples

In this section, we illustrate the notion of general equilibrium with imperfect competition presented above. We also point out that a competition policy favoring supply in a particular sector is not always welfare increasing.

3.1 Example 1 - A two sectors economy

Consider an economy with two goods, good 1 and good 2, and let us represent the equilibrium of the economy in a plane (x_1, x_2) (see figure 1 at the end of the paper).

Let D be the production frontier (defined by $\alpha_1 x_1 + \alpha_2 x_2 = L$) and C an indifference curve. The Pareto optimum is located at point (x_1^*, x_2^*), where C is tangent to D. The equilibrium productions lie on the production frontier at point E and satisfy:

$$\frac{U_1}{U_2} = \frac{\alpha_1}{\alpha_2} \frac{\beta_1}{\beta_2}. \tag{14}$$

The equilibrium is thus efficient if and only if $\beta_1 = \beta_2$, or, equivalently, if and only if the mark-up rates are equal across sectors. This is obviously the case when competition is perfect in the whole economy, since $\beta_1 = \beta_2 = 1$. But this is also possible when the two sectors compete à la Cournot. Note that whatever the utility functions may be, efficiency of equilibrium occurs if the ratio of the numbers of firms in the two sectors equates the ratio of elasticities evaluated at the efficient productions, that is if:

$$\frac{n_1}{n_2} = \frac{\sigma_1(x_1^*, x_2^*)}{\sigma_2(x_1^*, x_2^*)}.$$

This formalizes an old statement by Lerner (Lerner (1934), page 172): "If the "social" degree of monopoly is the same for *all* final products (including leisure) there is no monopolistic alteration from the optimum at all. The absolute height of "social" degrees of monopoly becomes completely unimportant" (see also Ruffin (1999), Proposition 1, for the Cobb-Douglas case).

Assume now that the equilibrium is not efficient. If the equilibrium (located at point E in figure 1) is on the left of the optimal point (x_1^*, x_2^*), the mark-up in market 1 is higher than in market 2 ($\beta_1 > \beta_2$) and sector 2 over-produces while sector 1 under-produces compared to their efficiency levels. As a consequence, increasing production in sector 1 (hence reducing that of sector 2) improves welfare. Indeed, differentiating welfare with respect to quantities and using equilibrium conditions (5), (6) and (7) leads to:

$$dW = U_1' dx_1 + U_2' dx_2 = \lambda\{\beta_1 - \beta_2\}\alpha_1 dx_1.$$

Therefore, if for instance sector 1 is characterized by the highest mark-up, (so under-produces in equilibrium), then sector 2 over-produces. Any policy which either favors production of sector 1 or discourages that of sector 2 enhances welfare. Any policy working in the opposite direction, leading sector 2 to increase production and sector 1 to reduce it, would make the agent worse off.

3.2 Example 2 - Cobb-Douglas Economy

Assume now that the agent's preferences are represented by a log-linear Cobb-Douglas function:

$$U(x_1, \ldots, x_N) = \sum_{k=1}^{N} \gamma_k \log x_k \text{ with } \gamma_k > 0 \text{ and } \sum_{k=1}^{N} \gamma_k = 1.$$

Assume further that the number of firms is higher than 1 in every non-competitive sectors. A straightforward computation shows that all demands have the same elasticity, equal to -1. Equations (5), (6), (7), (11) and (12) allow us to determine the equilibrium allocations as follows:

$$\begin{aligned} x_k &= \frac{\gamma_k}{\alpha_k} \frac{\delta_k L}{\sum_j \gamma_j \delta_j} \text{ with } \delta_j \equiv \frac{1}{\beta_j} \qquad (15) \\ \text{that is } \delta_j &= \frac{n_j - 1}{n_j} \text{ if } k \in I \text{ and } \delta_j = 1 \text{ otherwise.} \end{aligned}$$

The equilibrium value of welfare is may be expressed in terms of the vector δ of the variables δ_k:

$$W(\delta) = \sum_k \gamma_k \log\left(\frac{\gamma_k L}{\alpha_k}\right) + \sum_k \gamma_k \log \delta_k - \log\left(\sum_k \gamma_k \delta_k\right). \qquad (16)$$

Finally, note that Pareto efficiency requires that:

$$x_k^* = \frac{\gamma_k}{\alpha_k} L. \tag{17}$$

- *Production efficiency*

Comparing (15) and (17) we can express the difference between equilibrium and efficient productions in sector k:

$$x_k - x_k^* = \frac{\gamma_k L}{\alpha_k} \left(\frac{\delta_k - \sum_j \gamma_j \delta_j}{\sum_j \gamma_j \delta_j} \right).$$

Thus, as long as δ_k is constant across sectors, equilibrium productions are efficient. This property occurs not only when no imperfect competition exists, but also when competition is imperfect in all sectors and the number of firms is the same in each market. In this case, all mark-up ratios are identical, that is $\delta_k = \sum_j \gamma_j \delta_j = \delta_m$ for all k.

When none of these conditions hold true, the equilibrium allocation is inefficient, implying that some sectors produce too much in equilibrium (compared to efficiency), while the output of others is too low. More precisely, whenever $\delta_k > \sum_j \gamma_j \delta_j \equiv \delta_m$, sector k over-produces. Note that all competitive sectors over-produce, but it may occur that some of the imperfect competitive sectors either produce efficiently (when $\delta_k = \delta_m$), or over-produce.

- *Shall one encourage or discourage mergers ?*

Assume that a regulator can favor entry in market for good h or conversely favor mergers. Increasing the number or firms increases the inverse mark-up rate δ_h and improves welfare if and only if:

$$\frac{\partial W}{\partial \delta_h} = \frac{\gamma_h}{\delta_h} - \frac{\gamma_h}{\sum_k \gamma_k \delta_k} > 0, \text{ that is if } \delta_h < \frac{\sum_{k \neq h} \gamma_k \delta_k}{\sum_{k \neq h} \gamma_k}, \tag{18}$$

which is equivalent to $\delta_h < \delta_m$. Thus an increase in the number of firms n_h improves welfare only if sector h under-produces in equilibrium, compared to efficiency. Indeed, more firms in sector h always implies a higher output in that sector and a smaller one in other sectors. This is because one has:

$$\frac{\partial x_h}{\partial \delta_h} = \frac{\gamma_h L}{\alpha_h}\left(\frac{\sum_{j\neq h}\gamma_j\delta_j}{(\sum_j \gamma_j\delta_j)^2}\right) > 0 \text{ and } \frac{\partial x_k}{\partial \delta_h} = -\frac{\gamma_k}{\alpha_k}\frac{\gamma_h\delta_k L}{(\sum_j \gamma_j\delta_j)^2} < 0.$$

When sector h under-produces in equilibrium, it is efficient to stimulate the production of good h, even if this implies a decrease in production everywhere else. A good policy then consists in discouraging mergers in sector h.

Therefore, the decision of the regulator with respect to mergers should depend on the market in which they occur. When perfect competition dominates, δ_m is high: increasing competition (or controlling mergers to avoid a decrease in the number of firms) in almost all markets characterized by imperfect competition makes sense. On the contrary, when imperfect competition sectors dominate, δ_m is low: competition policy should favor mergers in markets where $\delta_h > \delta_m$ and control it otherwise.

- *Is perfect competition desirable for consumer ?*

Consider an economy where the set I has at least two elements (there is imperfect competition in at least two sectors). Then imagine that an imperfect competition sector, say h, can be turned into a perfect competition one. Is such a modification always desirable for the consumer ? In technical terms, this amounts to compare welfare evaluated at $(\delta_1, \ldots, \delta_{h-1}, \delta_h, \delta_{h+1}, \ldots, \delta_N)$ with welfare evaluated at $(\delta_1, \ldots, \delta_{h-1}, 1, \delta_{h+1}, \ldots, \delta_N)$. It is easy to verify that $W(.)$ is first increasing and then decreasing with respect to δ_h, goes to $-\infty$ when δ_h goes to 0 and reaches a maximum at $\delta_h = \delta^* = \dfrac{\sum_{k\neq h}\gamma_k\delta_k}{\sum_{k\neq h}\gamma_k}$ (recall that this last condition is equivalent to $\delta_h = \delta_m$).

Consequently, when I has at least two elements, there exists $\widehat{\delta}$ such that $0 < \widehat{\delta} < \delta^*$ and such that:

$$W_{|\delta_h=\widehat{\delta}} = W_{|\delta_h=1} \text{ so that } W_{|\delta_h} < W_{|\delta_h=1} \Leftrightarrow \delta_h < \widehat{\delta}. \tag{19}$$

Perfect competition in market h is thus desirable only if the inverse of the mark-up is small enough[5]. Note that a mark-up rate "near the mean" δ_m is always preferred by the consumer to perfect competition[6].

The reason for this last results is as follows. When condition (19) is satisfied, production h is low with respect to its efficiency level. Turning sector h into a competitive

[5] The condition that I has at least two elements is important and plays a crucial role here. Indeed, if h is the only non-competitive sector $\delta^* = 1$ and turning h into a competitive sector is always the best policy.

[6] As a result, we see also that turning a competitive sector into a non-competitive one can also be welfare improving.

turns underproduction into overproduction. The condition used above ensures that an agent prefers overproduction in sector h with perfect competition over underproduction with imperfect competition.

4 More General Results

4.1 Competition Policy and Efficiency

We now present more general results. First of all, recall from the preceding section that a general equilibrium with imperfect competition may be Pareto efficient. Indeed, whenever all sectors have the same mark-up, that is when all the β_k are identical, equilibrium productions satisfy the conditions of efficiency (10). This will arise, however, only if imperfect competition prevail in *all* sectors, and if the numbers of firms are such that

$$\frac{\sigma_k(x^*)}{n_k^*} = \frac{\sigma_j(x^*)}{n_j^*} \text{ for all } k \neq j. \tag{20}$$

In particular, notice that in every symmetrical economy, the equilibrium is always efficient. Hence the existence of different mark-up rates is the reason for inefficiency. As a consequence, when some competitive sectors exist, the equilibrium is never efficient. The features of inefficiency, however, differ across sectors, as stated in the next result.

Proposition 2. - *Assume that a general equilibrium with imperfect competition is Pareto inefficient. Then there exists a number $\widehat{\beta}$ such that $\max_k \beta_k > \widehat{\beta} > \min_k \beta_k$. Sectors with mark-up β_k higher than $\widehat{\beta}$ under-produce compared to their efficiency levels and sectors with mark-up lower than $\widehat{\beta}$ over-produce.*

The idea that Cournot competition in a given sector reduces production compared to its efficiency level is no longer valid in a general equilibrium context. Indeed, assume that no competitive sector exists. In our economy, a low production implies a low employment level, so when some sectors under-produce in equilibrium compared to their efficiency levels, the equilibrium of the labor market requires that some others must over-produce. Under-production occurs in markets in which the mark-up rates are relatively high.

Moreover, if competitive sectors coexist with non-competitive ones, their mark-up rates are all equal to the smallest feasible mark-up. Hence every competitive sector (strictly) over-produce and at least one sector with imperfect competition under-produces. The fact that competition is imperfect in at least one market implies that all competitive markets produce inefficiently. Surprisingly, some sectors may produce in an efficient way in equilibrium, but these efficient sectors are characterized by imperfect competition.

4.2 Competition Policy and Welfare

To analyze the welfare effects of a competition policy, it is convenient to to notice that:

$$dW = \sum_{k=1}^{N} U_k'(x_k)dx_k = \sum_{k=1}^{N} \left\{ \frac{U_k'(x_k)}{\alpha_k} - \rho \right\} \alpha_k dx_k \text{ for all } \rho. \tag{21}$$

Setting $\rho = \dfrac{U_k'(x_k^*)}{\alpha_k}$, which is constant across k (see equation (10)), in the above equation, we obtain:

$$dW = \sum_{k=1}^{N} \left\{ \frac{U_k'(x_k)}{\alpha_k} - \frac{U_k'(x_k^*)}{\alpha_k} \right\} \alpha_k dx_k.$$

Welfare is enhanced if both over-productions and under-productions are reduced, which means decreasing (resp. increasing) production in sectors with small (resp. high) mark-up rates (see equations (5) and (6)).

Consider a policy which favors production in sector h, for instance a policy prohibiting mergers. In a partial equilibrium framework, we would generally conclude that this policy improves welfare. In a general equilibrium setting, what happens in other markets is crucial to reach a conclusion. Indeed, setting $\rho = \lambda\beta_h$ in (21) (recall equations (5) and 6), we obtain the following expression for the change in welfare:

$$dW = \lambda \sum_{k=1}^{N} \{\beta_k - \beta_h\} \alpha_k dx_k.$$

Assume that an increase in the production of sector h implies that firms decrease their production in all other markets (because, for instance, the wage rate increases). This generates a positive effect on welfare in all markets where $\beta_k < \beta_h$, and a negative one in all markets where $\beta_k > \beta_h$. A priori, the net effect is indeterminate except when the mark-up rate of sector h is either the smallest or the highest one. In particular, favoring production in the sector with the smallest mark-up (when no competitive sectors exists) would be welfare reducing, which contradicts the partial equilibrium common view.

These conclusions, however, rest on the fact that changes in productions levels behave in a "friendly" way, that is: more firms in sector h would stimulate production of good h and reduce that of other goods - this case arises when the consumer preferences can be represented by a Cobb-Douglas function as in the previous section. In a more general setting, cross effects affect the elasticity of the inverse demand functions, making general conclusions more difficult to obtain. However, Proposition 2 below confirms the main conclusions of the Cobb-Douglas example.

Proposition 3. *i) Assume that a general equilibrium with imperfect competition is inefficient and that there are no competitive sectors. Then there exist two (non empty) subsets of production sectors, such that increasing the number of firms in any sector belonging to the first (resp. second) subset improves (resp. decreases) welfare.*

ii) Assume that a general equilibrium with imperfect competition is inefficient and that the elasticity of marginal utilities r_k are identical across sectors. Then, there exists a set of positive weights η_k with $\sum_{k=1}^{N} \eta_k = 1$, such that increasing the number of firms in market j improves welfare if an only if the mark-up β_j satisfies $\beta_j > \sum_{k=1}^{N} \eta_k \beta_k$.

Point *i*) confirms the intuitions given above. When all sectors are concerned by imperfect competition, entry must be favored in some markets and discouraged in others, hence entry has opposite effects on welfare depending on the concerned market. More competition is not always desirable. The key point is that a sector in which competition is imperfect may over-produce with respect to the efficient level. As a consequence mergers would be desirable in such sectors since they reduce firms' supply.

Point *ii*) states that the sectors in which entry is desirable (respectively not desirable) are those with relatively high (resp. low) mark-up rates and that this conclusion holds even when perfect competition prevails in some sectors.

What are the consequences of Proposition 3 with regard to competition policy?

There is a case where competition policy has a simple flavor. Considering point ii) in the Proposition above, assume that the weighted average $\sum_k \eta_k \beta_k$ is just a little bit above 1. In fact, it could fall below the mark-up of *every* non-competitive sectors. In this case, our result implies that increasing the numbers of firms in market j improves welfare if and only if the market j is non-competitive. Thus, horizontal merger control should be exercised in *every* non-competitive sectors and there are *no* sectors in which industrial policy should favor concentrations or collective practices.

Things are more intricate in the other cases. Competition policy should favor mergers in sectors with low mark-up rates and prevent mergers in sectors with high mark-up rates. However, identifying the two groups of sectors may be difficult (though in principle all the elements in the construction of the weights η_k are "observable" (see the end of the section)).

Our last point is that competition policy needs coordination. Indeed, prohibiting mergers in sectors with high mark-up rates may increase welfare whereas prohibiting mergers in sectors with low mark-up rates may decrease it. As these decisions play in opposite directions, they could neutralize each others, and finally have no positive impact on welfare. Moreover, because of this last point, more coordination could reduce the volume of adjudication.

Let us briefly discuss some of the assumptions used in the Proposition above. The separability of $U(.)$ is a key assumption to obtain Proposition 2. It allows us to differentiate relatively easily equilibrium conditions (5) and (6) and (7) with respect to the numbers of firms in sectors with imperfect competition.

Furthermore, note that even if the condition that the coefficients r_k are identical across sectors seems restrictive, it is satisfied for a large class of utility functions, i.e. the CRRA functions, which are frequently used i.e.:

$$U(x) = \sum_{k=1}^{N} \gamma_k \frac{(x_k)^\rho}{\rho}, \text{ with } \rho < 1. \tag{22}$$

Finally, note that all terms in η_k are "observable" in principle, so our threshold could be empirically measured. Indeed,

$$\begin{aligned}
\eta_k &\equiv \frac{\frac{\alpha_k x_k}{A_k}}{\sum_{j=1}^{N} \frac{\alpha_j x_j}{A_j}} \\
\text{with } A_k &\equiv r_k + \frac{[(1-\theta_k) r'_k x_k - (1+r_k)^2 \theta_k]}{n_k + \sigma_k} \text{ if } k \in I \text{ and } A_k \equiv r_k \text{ otherwise.}
\end{aligned}$$

The variables in these expressions are the technical coefficients (α_k), the production levels (x_k), the elasticity of the marginal utility of good k (as well as the derivative of the latter with respect to x_k, which requires an evaluation of the third derivative of the original utility function), the budget coefficients (θ_k), and the elasticity of the "perceived" demand function.

5 Conclusion

This paper is an attempt to evaluate competition policy within a general equilibrium framework. To do so, we use a simple general equilibrium model with imperfect competition, based on that used by Negishi (1961) and Neary (2003 b). Even if our model extends that of Neary (we use more general utility functions), our assumptions that firms are symmetrical within markets, and that consumer's preferences are separable limit the validity of our results. Despite these weaknesses, our results show that the insights of a general equilibrium approach differ deeply from that of partial equilibrium analysis, and call into question the usual conclusions drawn from industrial economics.

We have challenged the view that increasing supply in a given sector is welfare enhancing. We also have provided some conditions which could help identifying sectors where stimulating supply (or discouraging mergers) is welfare enhancing. Our results contradict the juridical view according to which all sectors should be treated in the same way, and suggest that horizontal merger control should limit its action in sectors with high mark-up rates and disregard others. Moreover, there could be a case for an industrial policy that favors concentrations or collusion practices among firms in sectors with low mark-up rates. Finally, our general equilibrium approach shows that what happens in other markets is crucial to measure how welfare changes in response to an horizontal merger in a given market. We have also challenged the idea that anticompetitive effects of a merger could be measured on a relevant market or area, and we claim that, as long as the price of the input reacts to the merger, attention should be extended to the whole economy.

Finally, there are two assumptions on which the analysis of this paper rests and which should be relaxed in further research. We have assumed the existence of a representative agent and that firms use the same and unique input. With several inputs and several agents, it would be possible and interesting to study the redistributive effects of competition policy.

References

[1] Blanchard, O. (2006), "Is there a viable European social and economic model ?", W.P 06-21, MIT.

[2] Blanchard, O. and F. Giavazzi (2001), "Macroeconomics effects of regulation and deregulation in goods and labor markets".

[3] Brown, D. and G.A. Wood (2004), "The social cost of monopoly power", Cowles Foundation Discussion Paper 1466.

[4] Crettez, B. and M. C. Fagart (2005), "A note on the Pareto efficiency of general oligopolistic equilibria", Working Paper Crest.

[5] François J. and H. Horn (2000): "Competition policy in an open economy", Working Paper.

[6] Herendorf, B., Schmitz, J.A. and A.M.F. Teixeira (2005), "Are the plain-vanilla costs of monopoly small ?", W. P.

[7] Kelton, C.M.L and R.P. Rebelein (2003), "A static general-equilibrium model in which monopoly is superior to competition", Working Paper.

[8] Konishi, H. M. Okuno-Fujiwara and K. Suzumura (1990), "Oligopolistic competition and economic welfare", *Journal of Public Economics*, 42, 67-88.

[9] Lerner, A.P. (1934), "The concept of monopoly and the measurement of monopoly power", *The Review of Economic Studies*, 1, 3, 157-175.

[10] Motta, M. (2004), *Competition Policy, Theory and Practice*, Cambridge University Press.

[11] Neary, P. (2002a), "Competition, trade and wages", in D. Greenaway, R. Upward and K. Wakelin (eds) : *Trade, Migration and Labour Market Adjustment*, London: Macmillan, 28-45.

[12] Neary, P. (2002b), "Foreign competition and wage inequality", *Review of International Economics*, 10, 680-93.

[13] Neary, P. (2003,a), "Competitive versus comparative advantage", *The World Economy*, 26, 4, April , 457-470.

[14] Neary, P. (2003,b), "The road less travelled: olipoly and competition policy in general equilibrium", in R. Arnott, B. Greenwald, R. Kanbur, B. Nabeluff. Cambridge (eds): *Imperfect Economics: Essays in Honor of Joseph Stiglitz*, Mass, MIT Press, 485-500.

[15] Negishi, T. (1961), "Monopolistic competition and general equilibrium", *Review of Economic Studies*, 28, 196-201.

[16] Negishi, T. (1962), "Entry and the optimal number of firms", *Metroeconomica*, XIV, 86-96.

[17] Nikaido, H. (1975), *Monopolistic competition and effective demand*, Princeton University Press.

[18] Posner R. (2001), "Antitrust and the New Economy", *Antitrust Law Journal*, 68, 925.

[19] Spear, S. E. (2001), "The electricity market game", Working Paper, Carnegie Mellon University.

[20] Ruffin, R.J. (1999), "Oligopoly and trade: what, how much, and for whom ?", Working Paper.

[21] Stahn, H. (1999), "Monopolistic behavior and general equilibrium: a generalization of Nikaido's work", *Journal of mathematical economics*, 32, 87-112.

APPENDIX

Proof of Proposition 1

Proof. As for the only if part, assume that we are given a general equilibrium satisfying the assumptions. In particular, $x = X(p, R)$, and by the Kuhn and Tucker's Theorem, there exists a positive scalar ρ such that:

$$U'_k(x_k) = \rho p_h, \ \forall h. \tag{23}$$

Using the budget constraint, we have:

$$U'_k(x_k) = p_h \left(\frac{\sum_{j=1}^{N} x_j U'_j(x_j)}{R} \right) \tag{24}$$

$$U'_k(x_k) = p_h \left(\frac{\sum_{j=1}^{N} x_j U'_j(x_j)}{\sum_{j=1}^{N} p_j x_j} \right) \tag{25}$$

In the competitive sectors, profit maximization implies that:

$$\forall k \notin I, \ p_k = \alpha_k w \tag{26}$$

In the competitive sectors, the same maximization leads to:

$$\forall k \in I, \ P'_k(x_k, x_{-k}, R)\frac{x_k}{n_k} + P_k(x_k, x_{-k}, R) = \alpha_k w. \tag{27}$$

Using the definition of the elasticity, one gets:

$$w\alpha_k = P_k(x_k, x_{-k}, R)\left(1 + \frac{\sigma_k(x)}{n_k}\right) \tag{28}$$

Hence the first-order condition conditions for the consumptions write now:

$$\forall k \notin I,\ U_k'(x_k) = \left(\frac{\displaystyle\sum_{j=1}^{N} x_j U_j'(x_j)}{\displaystyle\sum_{j=1}^{N} p_j x_j} \right) \alpha_k w \tag{29}$$

$$\forall k \in I,\ U_k'(x_k) = \left(\frac{\displaystyle\sum_{j=1}^{N} x_j U_j'(x_j)}{\displaystyle\sum_{j=1}^{N} p_j x_j} \right) \left(\frac{\alpha_k w}{1 + \frac{\sigma_k(x)}{n_k}} \right) \tag{30}$$

Defining:

$$\lambda = w \ \frac{\displaystyle\sum_{j=1}^{N} x_j U_j'(x_j)}{\displaystyle\sum_{j=1}^{N} p_j x_j} \tag{31}$$

we obtain equations (5) and (6).

Let us now turn to the if part. Let us then assume that there exists a positive λ such that (5) and (6) are both satisfied.

Now let us define $w = \lambda$. Define a set of positive prices p_k such that:

$$\sum_{j=1}^{N} p_j x_j = \sum_{j=1}^{N} x_j U_j'(x_j) \tag{32}$$

As a result, we get:

$$\forall k \notin I,\ p_k = \frac{\sum_j p_j x_j}{\sum_j U_j'(x_j) x_j} U_k'(x_k) = \alpha_k w \tag{33}$$

$$\forall k \in I,\ p_k = \frac{\sum_j p_j x_j}{\sum_j U_j'(x_j) x_j} U_k'(x_k) = \frac{\alpha_k w}{1 + \dfrac{\sigma_k(x)}{n_k}}. \tag{34}$$

From Assumption 2, these conditions ensure that firms maximizes profits by choosing x_k/n_k in each sector $k \in I$. As there are free entry in the other sectors, profit maximization is compatible with firms choosing x_k in aggregate in each $k \notin I$.

Using the definition of the prices given above in (5) and (6), the fact that $U(.)$ is concave implies that x_k maximizes the agent's preferences under the constraint $R \equiv \sum_j p_j x_j$. $\square$

Proof of Proposition 2

Note that $U_k'(x_k) \geqslant$ (respectively $\leq$) $U_k'(x_k^*)$ for all k in equilibrium implies that $x_k \leq$ (resp. $\geq$) x_k^*, $\forall k$. This is infeasible if the equilibrium is not efficient. Indeed, the labour market equilibrium implies that $\sum_k \alpha_k x_k = \sum_k \alpha_k x_k^*$. As a consequence, there exist two goods $h \neq j$ such that $x_h < x_h^*$ and $x_j > x_j^*$, so using Proposition 1:

$$\lambda \beta_h = \frac{U_h'(x_h)}{\alpha_h} > \frac{U_h'(x_h^*)}{\alpha_h} = \frac{U_k'(x_k^*)}{\alpha_k} > \lambda \beta_j = \frac{U_j'(x_j)}{\alpha_j}.$$

Therefore, there exists a threshold $\widehat{\beta} \equiv \dfrac{U_k'(x_k^*)}{\lambda \alpha_k}$ (for all k) such that $\max_k \beta_k > \widehat{\beta} > \min_k \beta_k$ and:

$$x_k \geqslant x_k^* \text{ if and only if } \beta_k \leq \widehat{\beta}. \tag{35}$$

When competitive sectors exist, the mark-up in these sectors equals 1, so $\min \beta_k = 1$, and the production of any competitive sector is higher than its efficiency level. $\square$

Proof of Proposition 3

We prove Proposition 3 in three steps. The first step is the most difficult and is cumbersome. We use the equilibrium conditions to study how welfare varies when the number of firms changes in one sector. In the second and the third steps, we prove the two points of Proposition 3.

First step

• Recall that the inverse demand functions $P_k(x_1, x_2, \ldots, x_N, R)$, $k = 1, \ldots, N$ are given by

$$P_k(x, R) = \frac{RU'_k(x_k)}{\sum_j U'_j(x_j)x_j}. \tag{36}$$

A direct calculation gives the elasticity of the inverse demand function of good k:

$$\sigma_k(x) = \frac{\partial P_k(x,R)}{\partial x_k}\frac{x_k}{P_k(x,R)} = \frac{x_k U''_{kk}(x_k)}{U'_k(x_k)} - \frac{x_k\{U'_k(x_k) + x_k U''_{kk}(x_k)\}}{\sum_j x_j U'_j(x_j)} \tag{37}$$

where $U''_{kk}(x_k) \equiv \dfrac{\partial^2 U(x)}{\partial x_k^2}$. In what follows, we denote:

$$r_k(x_k) \equiv \frac{x_k U''_{kk}(x_k)}{U'_k(x_k)} \text{ and } \theta_k \equiv \frac{x_k U'_k(x_k)}{\sum_j x_j U'_j(x_j)}, \tag{38}$$

where θ_k is the share of good k in total expenditures. The elasticity of the inverse demand function of good k can be expressed more easily as

$$\sigma_k = r_k - (1 + r_k)\theta_k. \tag{39}$$

Now let us consider a technical detail. Differentiating θ_k with respect to x_k gives:

$$\frac{\partial \theta_k}{\partial x_k} = \frac{(1+r_k)\theta_k(1-\theta_k)}{x_k} \text{ and } \frac{\partial \theta_k}{\partial x_j} = -\frac{(1+r_j)\theta_j\theta_k}{x_j}. \tag{40}$$

Hence derivating the elasticity σ_k with respect to x_k yields:

$$\frac{\partial \sigma_k}{\partial x_k} = (1-\theta_k)r'_k - \frac{(1+r_k)^2\theta_k(1-\theta_k)}{x_k} \text{ and } \frac{\partial \sigma_k}{\partial x_j} = \frac{(1+r_k)(1+r_j)\theta_j\theta_k}{x_j} \text{ for } j \neq k. \tag{41}$$

As a consequence, we obtain that

$$d\sigma_k = \sum_j \frac{\partial \sigma_k}{\partial x_j}dx_j = \Big((1-\theta_k)r'_k x_k - (1+r_k)^2\theta_k\Big)\frac{dx_k}{x_k} + \theta_k(1+r_k)\sum_{j=1}^{N}\theta_j(1+r_j)\frac{dx_j}{x_j}. \tag{42}$$

• Let us now turn to the system of equations defining the equilibrium, that is (5), (6) and (7). Let us rewrite (6) in the following way:

$$\begin{aligned} \log(n_k + \psi_k\sigma_k) + \log U'_k &= \log\alpha_k + \log\lambda + \log n_k \qquad (43)\\ \text{with } \psi_k &= 1 \text{ if k} \in I \text{ and } \psi_k = 0 \text{ otherwise.} \end{aligned}$$

Differentiating (43) and (7) gives:

$$\frac{\psi_k d\sigma_k}{n_k + \psi_k\sigma_k} + r_k\frac{dx_k}{x_k} = \frac{d\lambda}{\lambda} + (1-\beta_k)\frac{dn_k}{n_k}, \text{ and} \tag{44}$$

$$\sum_k \alpha_k x_k \frac{dx_k}{x_k} = 0. \tag{45}$$

Finally, using (42), equation (44) can be written as:

$$A_k\frac{dx_k}{x_k} + B_k C = \frac{d\lambda}{\lambda} + (1-\beta_k)\frac{dn_k}{n_k}, \;\; k = 1,\ldots,N, \tag{46}$$

$$\begin{aligned}
\text{with } A_k &\equiv r_k + \frac{\psi_k\Big((1-\theta_k)r_k' x_k - (1+r_k)^2\theta_k\Big)}{n_k + \psi_k\sigma_k} \\
B_k &\equiv \frac{\psi_k\theta_k(1+r_k)}{n_k+\psi_k\sigma_k} \\
C &\equiv \sum_j \theta_j(1+r_j)\frac{dx_j}{x_j}.
\end{aligned}$$

We want to solve the system of equations (46) and (45) with respect to the relative quantities $\frac{dx_k}{x_k}, k = 1,\ldots,N$ to determine how welfare changes when the number of firms in a sector increases. Note that under our concavity assumption with regard to the utility function, $r_k' \leq 0$ ensures that $A_k < 0$.

• It is however not necessary to solve all the system to see how welfare changes with the number of firms. Indeed, the change in welfare with respect to quantities is:

$$\begin{aligned}
dW &= \sum_k U_k'(x_k)dx_k \\
&= \sum_k (x_k U_k'(x_k) - \rho\alpha_k x_k)\frac{dx_k}{x_k} \text{ for all } \rho \text{ as (45) holds.}
\end{aligned} \tag{47}$$

Let us introduce the following weights:

$$\eta_k \equiv \frac{\frac{\alpha_k x_k}{A_k}}{\sum_j \frac{\alpha_j x_j}{A_j}} \text{ and } \widehat{\eta}_k \equiv \frac{\frac{x_k U_k'}{A_k}}{\sum_j \frac{x_j U_j'}{A_j}}$$

and note that $\eta_k > 0$, $\sum_k \eta_k = 1$, $\widehat{\eta}_k > 0$ and $\sum_k \widehat{\eta}_k = 1$. Moreover,

$$\widehat{\eta}_k = \frac{\eta_k \beta_k}{\sum_j \eta_j \beta_j} = \frac{\eta_k \beta_k}{\beta_m} \text{ where } \beta_m \equiv \sum_j \eta_j \beta_j. \tag{48}$$

Using the weights η_k and $\widehat{\eta}_k$ the expression of the change in welfare can be simplified as follows:

$$\begin{aligned} dW &= \sum_k (\widehat{\eta}_k \sum_j \frac{x_j U_j'}{A_j} - \eta_k \rho \sum_j \frac{\alpha_j x_j}{A_j}) A_k \frac{dx_k}{x_k} \\ &= (\sum_j \frac{x_j U_j'}{A_j}) \sum_k (\widehat{\eta}_k - \eta_k) A_k \frac{dx_k}{x_k} \text{ for } \rho = \frac{\sum_j \frac{x_j U_j'}{A_j}}{\sum_j \frac{\alpha_j x_j}{A_j}}. \end{aligned} \tag{49}$$

Multiplying (46) by $(\widehat{\eta}_k - \eta_k)$ and summing with respect to k gives:

$$\sum_k (\widehat{\eta}_k - \eta_k) A_k \frac{dx_k}{x_k} = -C \sum_k (\widehat{\eta}_k - \eta_k) B_k + \sum_k (\widehat{\eta}_k - \eta_k)(1 - \beta_k) \frac{dn_k}{n_k}. \tag{50}$$

In order to evaluate the effect of the increase in the number of firms we shall compute C from (46) and (45).

- Multiplying (46) by $\dfrac{\theta_k(1+r_k)}{A_k}$ and summing over k gives:

$$C[1 + \sum_k \frac{\theta_k (1+r_k) B_k}{A_k}] = \frac{dw}{w} \sum_k \frac{\theta_k(1+r_k)}{A_k} + \sum_k \frac{(1-\beta_k)\theta_k(1+r_k)}{A_k} \frac{dn_k}{n_k} \tag{51}$$

or

$$C\left(\sum_j \widehat{\eta}_j A_j + \sum_k \widehat{\eta}_k (1+r_k) B_k \right) - \frac{d\lambda}{\lambda} \sum_k \widehat{\eta}_k (1+r_k) = \sum_k (1-\beta_k) \widehat{\eta}_k (1+r_k) \frac{dn_k}{n_k}. \tag{52}$$

In the same manner, multiplying (46) by $\alpha_k x_k / A_k$, summing over k and taking into account (45) leads to:

$$C \sum_k \frac{\alpha_k x_k B_k}{A_k} = \frac{d\lambda}{\lambda} \sum_k \frac{\alpha_k x_k}{A_k} + \sum_k \frac{\alpha_k x_k (1-\beta_k)}{A_k} \frac{dn_k}{n_k}; \tag{53}$$

that is:

$$C \sum_k \eta_k B_k - \frac{d\lambda}{\lambda} = \sum_k \eta_k (1-\beta_k) \frac{dn_k}{n_k}. \tag{54}$$

Now, we have to solve (52) and (54) for C.

• Taking into account (52) and(54) leads to express C as:

$$C\Big(\sum_k \widehat{\eta}_k[A_k + (1+r_k)B_k] - (\sum_j \eta_j B_j)\sum_k \widehat{\eta}_k(1+r_k)\Big) \quad (55)$$
$$= \sum_k (1-\beta_k)\Big(\widehat{\eta}_k(1+r_k) - \eta_k(\sum_j \widehat{\eta}_j(1+r_j))\Big)\frac{dn_k}{n_k}.$$

Let us show that $\sum_k \widehat{\eta}_k[A_k + (1+r_k)B_k] - (\sum_j \eta_j B_j)\sum_k \widehat{\eta}_k(1+r_k) \neq 0$. Assume the converse is true. Thus, the right term of (55) equates 0, whatever the market in which the number of firms is modified. As a consequence, we would have

$$\widehat{\eta}_k(1+r_k) = \eta_k \sum_j \widehat{\eta}_j(1+r_j) \text{ for all } k \text{ such that } \beta_k \neq 1, \quad (56)$$

but this leads to

$$\sum_k \widehat{\eta}_k[A_k + (1+r_k)B_k] - (\sum_j \eta_j B_j)[\sum_k \widehat{\eta}_k(1+r_k)] = \sum_k \widehat{\eta}_k A_k \neq 0. \quad (57)$$

which contradicts our assumption.

• Finally, taking into account the value of C given in (55) in the definition of the variation of welfare given by (49) leads to:

$$dW = \Big(\sum_j \frac{x_j U'_j}{A_j}\Big)\sum_k (1-\beta_k)\Gamma_k \frac{dn_k}{n_k} \quad (58)$$
$$\text{with } \Gamma_k = \widehat{\eta}_k - \eta_k - \frac{\Big(\widehat{\eta}_k(1+r_k) - \eta_k \sum_j \widehat{\eta}_j(1+r_j)\Big)\big(\sum_j(\widehat{\eta}_j - \eta_j)B_j\big)}{\sum_j \widehat{\eta}_j[A_j + (1+r_j)B_j] - \sum_j \eta_j B_j \sum_j \widehat{\eta}_j(1+r_j)}. \quad (59)$$

Hence, an increase in the number of firms in the sector with imperfect competition k enhances welfare if and only if $\Gamma_k > 0$.

Second step : proof of point i)

It is easy to check that:

$$\sum_k \Gamma_k = 0 \quad (60)$$

$$\text{and } \sum_k \Gamma_k B_k = \frac{\big(\sum_j \widehat{\eta}_j A_j\big)\big(\sum_k (\widehat{\eta}_k - \eta_k) B_k\big)}{\sum_j \widehat{\eta}_j A_j + \sum_h \widehat{\eta}_h (1+r_h)(B_h - \sum_j \eta_j B_j)}. \tag{61}$$

So, another expression of Γ_k is:

$$\Gamma_k = \widehat{\eta}_k - \eta_k - \frac{\Big(\widehat{\eta}_k(1+r_k) - \eta_k \sum_j \widehat{\eta}_j (1+r_j)\Big) \sum_k \Gamma_k B_k}{\sum_j \widehat{\eta}_j A_j}.$$

Assume that no competitive sectors exist. If all $\Gamma_k = 0$ for $k \in I$, we obtain that $\sum_k \Gamma_k B_k = 0$, thus $\Gamma_k = \widehat{\eta}_k - \eta_k = 0$. Consequently, $\widehat{\eta}_k = \eta_k$ in all sectors and efficiency is ensured, which contradicts our assumption. As a consequence, there exists two non empty subsets of sectors, such that $\Gamma_k > 0$ in one of the subsets, and $\Gamma_k < 0$ in the other one. This proves Point i) of Proposition 3.

Third step : proof of point ii)

Assume r_k is constant across sectors. Taking into account the definition of B_k given by (46) and the fact that $r_k = r$ is identical across sectors implies that:

$$\begin{aligned}
\Gamma_k &= (\widehat{\eta}_k - \eta_k)\left(1 - \frac{(1+r)^2 \sum_j (\widehat{\eta}_j - \eta_j) \dfrac{\psi_j \theta_j}{n_j + \psi_j \sigma_j}}{\sum_j \widehat{\eta}_j A_j + (1+r)^2 \sum_j (\widehat{\eta}_j - \eta_j) \dfrac{\psi_j \theta_j}{n_j + \psi_j \sigma_j}}\right) &(62)\\
&= (\widehat{\eta}_k - \eta_k)\left(\frac{\sum_j \widehat{\eta}_j A_j}{\sum_j \widehat{\eta}_j A_j + (1+r)^2 \sum_j (\widehat{\eta}_j - \eta_j) \dfrac{\psi_j \theta_j}{n_j + \psi_j \sigma_j}}\right). &(63)
\end{aligned}$$

As

$$A_j + (1+r)^2 \frac{\psi_j \theta_j}{n_j + \psi_j \sigma_j} = r + \frac{\psi_j (1-\theta_j) x_j r_j'}{n_j + \psi_j \sigma_j} < 0, \tag{64}$$

the expression into brackets in (62) is positive. This implies that Γ_k has the sign of $(\widehat{\eta}_k - \eta_k)$ which proves point ii) of Proposition 3:

$$\Gamma_k \geqslant 0 \Leftrightarrow \beta_k \geqslant \beta_m = \sum_j \eta_j \beta_j. \ \square \tag{65}$$

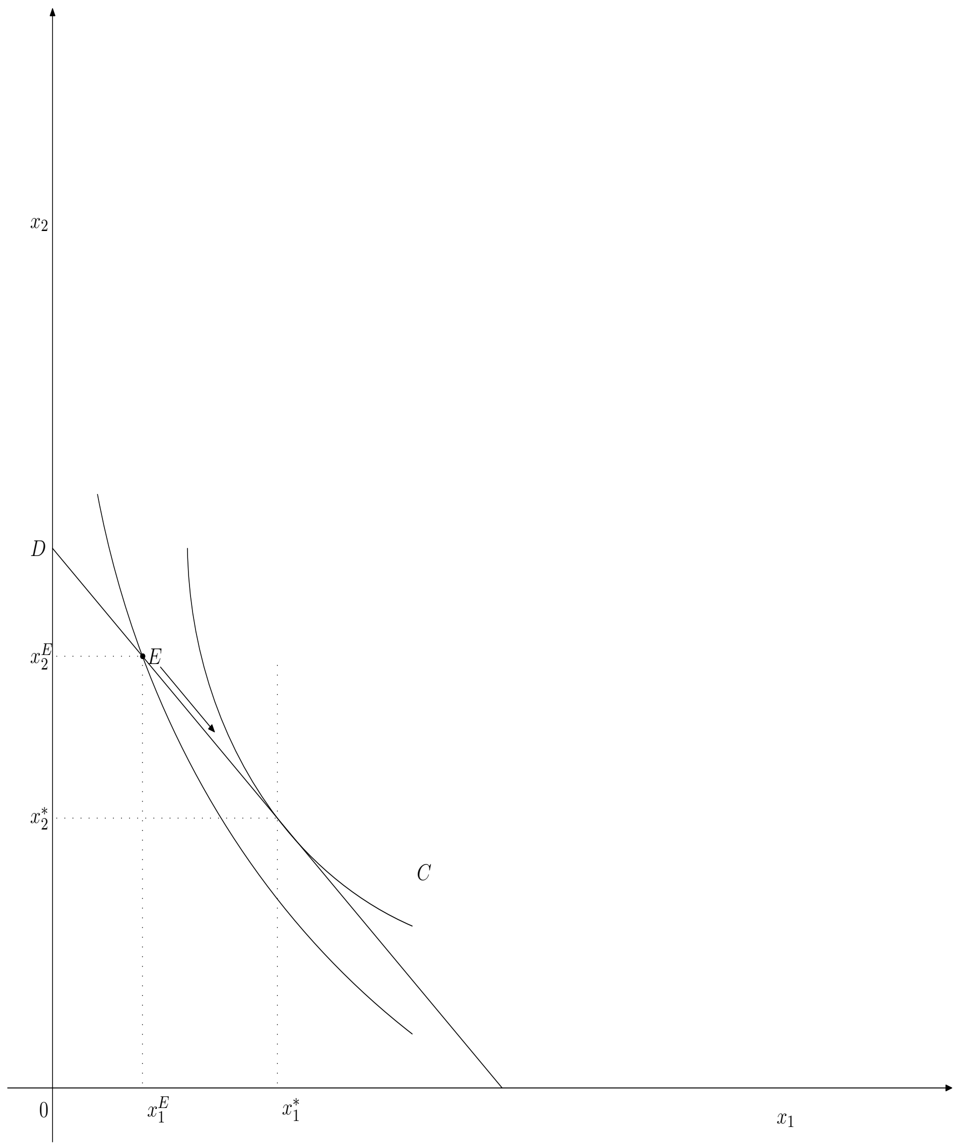

Figure 1