Markets with pairwise meetings : when frictions favour information revelation

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Abstract

We study information revelation in markets with pairwise meetings. We focus on the one-sided case and perform a dynamic analysis of a constant entry flow model. The same question has been studied in an identical framework in Serrano and Yosha (1993) but they limit their analysis to the stationary steady states. We show that there is a dramatical loss when restricting the analysis of a constant entry flow model to stationary steady states. We establish the existence of a limit cycle. Our second main result is that, in some non pathological cases, there exists an equilibrium such that information revelation is worse when frictions are weaker.
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Introduction

As proposed in Isaac (2008), we can illustrate the issue of this paper by a parallel with what we can observe in all the places of interest in Egypt.

In all these places, one observes bargaining between Egyptians and Tourists. The Egyptians try to sell a guided tour of the place. The Tourists are the potential buyers. There is neither a central institution nor a unique public price. The phase of bargaining happens after a matching between one seller and one buyer. When an Egyptian reaches an agreement with a Tourist, the two quit the market to effectuate the tour. In case of disagreement, the two separate and are matched anew with an agent of the opposite type.

The asymmetric information concerns the interest of the place. It is not obvious, for a Tourist, whether the place has a long history, whether there are many anecdotes about the site. Some Tourists can be uninformed about the interest while some others are informed, for instance, because they know some friends who previously visited the same place. Of course, all the Egyptians know the exact interest of the place.

The interest of the place has an influence on the value and the cost of the guided tour. It is more interesting to have a guide when there are a lot of things to say about the site. At the same time, it is more costly for an Egyptian to guide when the place is interesting, at least because it takes more time.

We can expect that the *good* price is higher when the place is of high interest. It is also natural that the uninformed Tourists try to extract information from their matches with different partners. This learning is expensive because there is a waste of time. Naturally, sellers try to exploit their informational advantage by misrepresenting. By misrepresenting, sellers incur also a cost for the same reason, i.e. the waste of time.

The main issue will be to determine if the trading process will imply an information revelation. Especially when the agents become infinitely patient, i.e. the market becomes approximately frictionless.

In market with pairwise meetings, the information revelation literature began with the seminal paper Wolinsky (1990) [1]. The model studied in this paper is more general than ours because there are also some uninformed sellers. In our egyptian story, it would mean that some Egyptians are not

[1] Concerning the market with pairwise meetings with perfect information, there is a significant literature studying following the seminal works of Gale, Rubinstein and Wolinsky. For a review, see Osborne and Rubinstein (2000).

aware of the interest of the place. The main result of Wolinsky (1990) is that some trades occur at a wrong price according to the state even when the market becomes approximately frictionless.

Gale (1989) conjectures the great importance of the assumption that uninformed agents are present in the two sides of the market because a noise is created if the learning cost decreases. Indeed, a decreasing learning cost causes the probability - of an uninformed agent to meet another uninformed agent - to increase. This requires, however, the information power of meeting to decrease when the learning cost declines.

Serrano and Yosha (1993) show that Gale's conjecture makes sens. They use the same model than Wolinsky (1990), but they assume that all sellers are informed. The noise disappears, since uninformed buyers always meet informed sellers. Finally, Serrano and Yosha (1993) establish that typically all transactions occur at the right price whenever the market becomes approximately frictionless.

Wolinsky (1990) and Serrano and Yosha (1993) use a constant entry flow model. At each period, a certain number of new agents enter the market. To simplify the analysis, these papers consider only the stationary steady states. In other words, they consider the situations where the number of agreements is exactly equal to the entry flow.

Blouin and Serrano (2001) study the same question of information revelation but in a one-time entry model[2]. At the first period, all the agents are present and nobody enters the market in the following periods. They obtain a dramatically different result in the one-sided case. They conclude, in this case, that some transactions occur at wrong prices even when the market is frictionless. The two-sided analysis provides results similar to Wolinsky (1990).

One could think that the difference of results is due to the restriction of the analysis in Serrano and Yosha (1993). Isaac (2008) uses the same model than Serrano and Yosha (1993) but introduces an initial period and does not assume a priori the stationarity of the equilibria. In this framework, it is established that for a discount factor high enough, there exists a steady state equilibrium with full information revelation. This equilibrium is unique when the uninformed buyers are suspicious enough, i.e. their prior belief that the place is interesting is relatively low.

Does it mean that the steady state assumption is relatively innocuous?

[2]For a discussion of these two hypothesis (constant entry flow and one time entry) in the perfect information case see Gale (1987).

In this paper, it will be shown that it is not the case. We reconsider the constant entry flow model in the *other* case, i.e. when uniformed buyers are not so suspicious. Another equilibrium exists when the market becomes approximately frictionless. This equilibrium tends in some situation towards a limit cycle of period two instead of a steady-state. Moreover, at this equilibrium, the weaker the frictions are, the worse the information revelation is.

This last result is the most disturbing. Therefore, we have to be careful when we consider the frictionless case as the mark-up. If we are unable to eliminate all the frictions, it could be better to conserve or even to enlarge the frictions.

One could argue that information revelation and efficiency are two different concepts and that the major concern is the efficiency rather than the information revelation. In this kind of models, actually, the two go together as established in Serrano and Yosha (1996).

In the related literature, one can place Janssen and Karamychev (2002) which is also dealing with a model of decentralized trade with constant entry of agents and cyclical equilibria. Clearly, the differences are huge since Janssen and Karamychev (2002) belongs to the *lemon market* literature. Nevertheless, it could be fruitful to check if the similarities in the results rise from common features in the model or have a complete different explanation.

In the first section, we present the model. The second section is devoted to preliminary results. The characterization and the existence of the equilibrium constitute the third section's matter. The fourth section is concerned with the dynamics of the equilibrium. We introduce and discuss the results about information revelation in the last section.

1 The model

We consider the model of Serrano and Yosha (1993) without modifying it but we study the outcomes without assuming an *a priori* stationarity of the equilibrium.

Time runs discretely from 0 to ∞[3]. Each period is identical. On one side, there are sellers who have one unit of indivisible good to sell. On the

[3]Serrano and Yosha 1993 consider that times runs from $-\infty$ to ∞. To make the steady state analysis, it is sufficient to assume that the initial conditions are the values of the steady state.

other side, there are buyers who want to buy one unit of this good. In each period, a continuum of measure M of new sellers and the same quantity of buyers enter on the market. The numbers of sellers and buyers who arrive on the market are equal. The agents quit the market when they have traded. Hence, the number of sellers is always equal to the number of buyers.

There exist two possible states of the world, which influence on the payoff of the agents. If the state is low (L), the cost of production (c_L) for the sellers but also the utility (u_L) of the buyers are low. If the state is high (H), the corresponding parameters (c_H and u_H) are high. The state remains identical during all the periods.

All sellers know the state of the world, whereas not all buyers are perfectly informed about it. Among the new comers, there is a part x_B of buyers which is perfectly informed. The remaining buyers are uninformed and possess a common prior belief $\alpha_H \in [0, 1]$ that the state is H and $(1 - \alpha_H)$ that the state is L. This belief can be updated as explained below.

At each period, all the agents are randomly matched[4] with an agent of the other type. At each meeting, the agents can announce one of two prices : p^H and p^L. If both agents announce the same price, trade occurs at this price. If a seller announces a lower price, trade occurs at an intermediate price p^M. If a seller announces a higher price, trade does not occur. The different parameters are assumed to be ordered such that :

$$c_L < p^L < u_L < p^M < c_H < p^H < u_H \tag{1}$$

Staying on the market implies a zero payoff. The instantaneous payoff when a trade occurs is the price minus the cost for a seller and the utility minus the price for a buyer. All agents discount the future by a constant factor δ.

In state H, we call p^H the *good* price because trade at other prices implies a loss for the sellers. Similarly, the price p^L is the *good* price in state L because trade at other prices involves loss for the buyers.

After each meeting with a seller who announces p^H, a buyer will actualize his belief α_H according to Bayes'rule. If an uninformed buyer meets a seller who announces p^L, he knows that state is L but it does not really matter since this buyer will trade and leave the market.

It is convenient to say that a seller (resp. a buyer) plays *soft* when he announces p^L (resp. p^H) and *tough* when he announces p^H (resp. p^L). When an agent plays *soft*, he is ensured to trade and to quit the market. So, to describe completely the strategy of an agent, it is sufficient to give the

[4]see Duffie and Sun 2007 for a rigorous proof of the existence of independent random matching between two continua. I am in debt to Walter Trockel for this reference.

number of periods in which he plays *tough*. The strategy of an agent might depend on the time of entry on the market. We note $n_{SH}(t)$ the number of periods during which a seller plays *tough* when he enters at time t on a market that is in state H. Similarly, we define $n_{SL}(t)$, $n_{BH}(t)$, $n_{BL}(t)$ and $n_B(t)$. Naturally, the strategy of an uninformed buyer $n_B(t)$ is independent of the state of the world.

We define now the proportion of agents who play *tough* when state is L. The proportion of the total number of buyers in the market who at period t announce p^L is called $B_L^l(t)$. Similarly, $S_L^h(t)$ is the proportion of sellers who at period t announce p^H. The meaning of $B_H^l(t)$ and $S_H^h(t)$ is obvious. These values are known to all agents.

An equilibrium is a profile of strategies where each agent is maximizing his expected payoff, given the strategies of the other agent. All parameters $(p^H, p^M, p^L, c_H, c_L, u_H, u_L, x_B, \delta, \alpha_H)$ are common knowledge.

2 Preliminary results

In this section, we establish the strategy of sellers and informed buyers in state H and strategy of informed buyers in state L. Then we give a condition that constrains uninformed buyer's strategy.

2.1 Trivial or constrained strategies

In the following claim, we characterize the equilibrium strategies of informed buyers and of sellers in state H.

Claim 1 *In any equilibrium* $n_{SH}(t) = \infty$*,* $n_{BL}(t) = \infty$ *and* $n_{BH}(t) = 0$ $\forall t$*.*

Proof An informed seller in state H knows that his payoff will be negative if he trades at an other price than p^H. Since the payoff of perpetual disagreement is 0, he will always prefer to play *tough* even if it implies a long delay before trading. The reasoning is identical for an informed buyer in state L. An informed buyer in state H will understand that $n_{SH}(t) = \infty$ and thus he will never trade while he plays *tough*. Playing *tough* only delays the payoff. So, it is better for this kind of buyer to play immediately *soft*.

Now, we would like to have a sufficient condition to ensure that $n_B(t) = 0$ is an optimal strategy. To establish the following claims, we define ΔV_B, which is the difference between the gain of playing *soft* tomorrow and the one of playing *soft* today for an uninformed buyer.

$$\begin{aligned}\Delta V_B &= \Delta V_B(S_L^h, S_L^h(+1)) \\ &= \alpha_H(u_H - p^H)\delta \\ &+ (1-\alpha_H)[(1-S_L^h)(u_L - p^L) + \delta S_L^h[(u_L - p^M) + S_L^h(+1)(p^M - p^H)]] \\ &- [\alpha_H(u_H - p^H) + (1-\alpha_H)[(u_L - p^M) + S_L^h(p^M - p^H)]] \qquad (2)\end{aligned}$$

The last line corresponds to the payoff involved by playing *soft* today. The payoff in state H, which is equal to $(u_H - p^H)$, is multiplied by the probability that the state is H. The term in brackets, which is multiplied by the probability that the state is L, is naturally the payoff in state L. This payoff can be written $(1 - S_L^h)(u_L - p^M)$ (i.e. the probability to meet a *soft* seller times the payoff involved by this meeting) plus $S_L^h(u_L - p^H)$ (i.e. the probability to meet a *tough* seller times the payoff involved). The two first lines correspond to playing *tough* today and *soft* tomorrow. The meaning of the first line is obvious. It is just important not to forget the discount factor δ. Indeed, if the state is H, a buyer who announces p^L does not trade. In the case where the state is L, there is a probability $(1 - S_L^h)$ that a buyer meets a *soft* seller and obtains today $(u_L - p^L)$. If a buyer does not have this luck, which happens with probability S_L^h, he will have tomorrow an expected payoff equal to the expression in brackets. Once again, we must not forget the discount factor.

Claim 2 *If* $\forall t$ $\Delta V_B(S_L^h(t), S_L^h(t+1)) < 0$ *then* $n_b(t) = 0$ *is a best reply for uniformed buyers* $\forall t$*.*

Proof The gain of playing *tough* during T periods compared to playing immediately soft is given by

$$\sum_{i=t}^{t+T} \delta^{i-t} \Delta V_B(S_L^h(i), S_L^h(i+1)) \qquad (3)$$

Clearly, this sum is always negative by assumption. Hence, $n_B(t) = 0$ is the best reply.

2.2 Dynamic of the market

In what follows, we will assume that $n_B(t) = 0$ is a best reply for uninformed buyers in all periods. We will check later that it is effectively the case at the equilibrium presented in this paper. The market is then at a stationary steady-state when the state of the world is H

$$K^H = M \tag{4}$$
$$B^l_H = 0 \tag{5}$$
$$S^h_H = 1 \tag{6}$$

Since $n_{SH}(t) = \infty \quad \forall t$, all sellers play *tough* in each period. So, the proportion of sellers who in state H announce p^H is equal to one. The proportion of buyers who announce p^L is always equal to 0. Indeed, $n_{BH}(t) = 0$ and by assumption $n_B(t) = 0$. All agents announce the same price p^H, which implies that all matches involve a trade and that all the agents quit the market. The number of agents on the market is thus equal to the number that has just entered the market.

If the state of the world is L, the variables evolve according to the following rules

$$K^L(+1) = K^L B^l_L S^h_L + M \tag{7}$$
$$B^l_L(+1) = \frac{K^L B^l_L S^h_L + x_B M}{K^L B^l_L S^h_L + M} \tag{8}$$

$B^l_L(0) = x_B$. $S^h_L(t)$ is chosen such that the payoffs of sellers are maximized. By claim 1, $n_{BL}(t) = \infty \quad \forall t$ which implies $B^l_L \neq 0$. If $S^h_L \neq 0$, there will be in each period $K^L B^l_L S^h_L$ matches that will end up on a disagreement. The concerned agents will remain on the market. The total number of agents in the market will thus be equal to the sum of agents who did not reach an agreement in the previous period and of agents who freshly entered the market. Buyers who did not reach an agreement in the previous period are obligatorily informed since by assumption $n_B(t) = 0$. Considering $n_{BL}(t) = \infty$, all these buyers will continue to play *tough*. The informed buyers who arrive in the market will also announce p^L. Hence, the total number of *tough* buyers is effectively equal to the numerator of expression (8).

We can show that for each B^l_L there is only one possible K^L. Indeed, the number of buyers who play soft is by definition $K^L(1-B^l_L)$. As we have seen only uninformed buyers play soft, therefore this number is also equal to $(1 - x_B)M$.

$$K^L = \frac{1 - x_B}{1 - B^l_L} M \tag{9}$$

We can rewrite (8) as

$$B_L^l(+1) = \frac{(1-x_B)B_L^l S_L^h + x_B(1-B_L^l)}{(1-x_B)B_L^l S_L^h + (1-B_L^l)} \tag{10}$$

Obviously, the right term is increasing in S_L^h. It implies that if $S_L^h \in [0,1]$ then

$$x_B \leq B_L^l(+1) \leq \frac{x_B + (1-2x_B)B_L^l}{1-x_B B_L^l} \equiv p(B_L^l) \tag{11}$$

So, $B_L^l(+1) \in [x_B, p(B_L^l)]$.

3 Equilibrium

Our approach does not consist in studying all the dynamic cases, we retrict our analysis to the construction of an equilibrium where uninformed buyers always play *soft*.

In the first subsection, we characterize partially $S_L^h(t)$ at the equilibrium. From this characterization, we derive the one of $B_L^l(t)$ at the equilibrium if all the uninformed buyers play *soft*. Actually, our characterization of $B_L^l(t)$ gives us an iterative rule to build from an initial condition $B_L^l(0)$ the unique sequence $B_L^l(t)$ compatible with optimal behaviour of sellers when uninformed buyers always play *soft*. This second characterization is presented in the second subsection. The last subsection proves that playing immediately *soft* is indeed the optimal strategy of uninformed buyers at this equilibrium. Hence, the existence of the equilibrium is established.

The next step is the characterization of the evolution of the market at equilibrium. Especially, we characterize the sequences $S_L^h(t)$ and $B_L^l(t)$ at equilibrium.

3.1 Characterization of $S_L^h(t)$ at equilibrium

We define $\Delta V_{SL}(B_L^l(t), B_L^l(t+1))$ which is the gain difference between playing *soft* tomorrow and playing *soft* today for an informed seller in state L. This difference depends on time because $B_L^l(t)$ may be non-stationary. Remark that $\Delta V_{SL}(B_L^l(t), B_L^l(t+1) < 0$ does not imply that the best solution is to stop in t.

$$\begin{aligned}
\Delta V_{SL}(B^l_L, B^l_L(+1)) &= (1-B^l_L)(p^H - c_L)... \\
&... +B^l_L\delta[((1-B^l_L(+1))(p^M - c_L) + B^l_L(+1)(p^L - c_L)]... \\
&... -[((1-B^l_L)(p^M - c_L) + B^l_L(p^L - c_L)] \\
&= B^l_L\bigg[(-p^H + p^M - p^L + c_L) + \delta(p^M - c_L) + ... \\
&... +\delta B^l_L(+1)(p^L - p^M)\bigg] + (p^H - p^M) \\
&\equiv B^l_L[X - B^l_L(+1)Y] + Z \qquad (12)
\end{aligned}$$

Clearly, Y and Z are positive. The sign of X is undetermined. In the first equality, the two first lines correspond to playing *tough* today and *soft* tomorrow while the third one corresponds to playing *soft* today.[5]

Assume that a seller stops playing *tough* today. ΔV_{SL} is a measure of gain for a seller if he decides to play *tough* one period more. The measure of gain for a seller if he decides to play *tough* T more periods is given by the sum of successive ΔV_{SL} balanced in order to take into account of the discount factor δ. If there exists a T such that this sum is positive, then playing *tough* T more periods gives a higher expected payoff than playing *soft* today. If this sum is negative for all T, then the maximum expected payoff is reached by playing *soft* today. If the sum is null for some T, then the seller is indifferent between playing *soft* today or playing *tough* T more periods.

Proposition 1 *Optimal strategies are such that the sequence* $S^h_L(t) \in [0,1]$

[5]If a seller plays *soft* today, he has a probability $(1 - B^l_L)$ to meet a *soft* buyer and consequently to obtain a payoff $(p^M - c_L)$, otherwise (i.e. with probability B^l_L) he will get $(p^L - c_L)$ due to a meeting with a *tough* buyer. If a seller announces p^H, he will reach an agreement only if he is matched with a *soft* buyer. It occurs with a probability $(1 - B^l_L)$ and the payoff is then $(p^H - c_L)$. Otherwise, with a probability B^l_L, he will remain in the market. In the next period, if he plays *soft*, he has an expected payoff equal to the expression between brackets which must be multiplied by the discount factor δ because trade occurs one period later.

satisfies

$$S_L^h(t) = 1 \implies \exists\, T\ s.t. \sum_{i=0}^{T} \delta^i \Delta V_{SL}(t+i) \geq 0 \quad (13)$$

$$S_L^h(t) < 1 \implies \sum_{i=0}^{T} \delta^i \Delta V_{SL}(t+i) \leq 0 \quad \forall\, T \quad (14)$$

$$\exists\, T\ s.t. \sum_{i=0}^{T} \delta^i \Delta V_{SL}(t+i) > 0 \implies S_L^h(t) = 1 \quad (15)$$

$$\sum_{i=0}^{T} \delta^i \Delta V_{SL}(t+i) < 0 \quad \forall\, T \implies S_L^h(t) = 0 \quad (16)$$

3.2 Characterization of $B_L^l(t)$ at equilibrium

Unusually, in the first part of this subsection, the main message will be stated using some sets and functions that are not defined yet. These definitions constitute the second part of the subsection. The reason to proceed this way is that the proposition is true by construction. So, we believe it is easier to understand the goal of the different definitions knowing the proposition. Figure 1 is helpful to understand the intuition of this subsection.

3.2.1 The proposition

In the Egyptian story, guides in a place of low interest can have at the start an incentive to misrepresent because there exists a possibility to meet an uninformed Tourists who accepts to pay the high price. Consequently, the proportion of informed Tourists on the market increases since they refuse to trade at the high price. The increase reduces the incentive to misrepresent. Then, there is a period in which some sellers tell the truth and the market is partially cleared.

We will define some sets: a set O, which B_L^l can never reach because there exist some informed buyers; a set P where ΔV_{SL} is positive or null even when all the sellers play *tough*. When $B_L^l \in P$, $p(B_L^l)$ is in P or in another set A. This last set includes β, which is a steady-state. The set A is divided in two subsets A_1 of elements lower or equal to β and A_2 of elements higher than β.

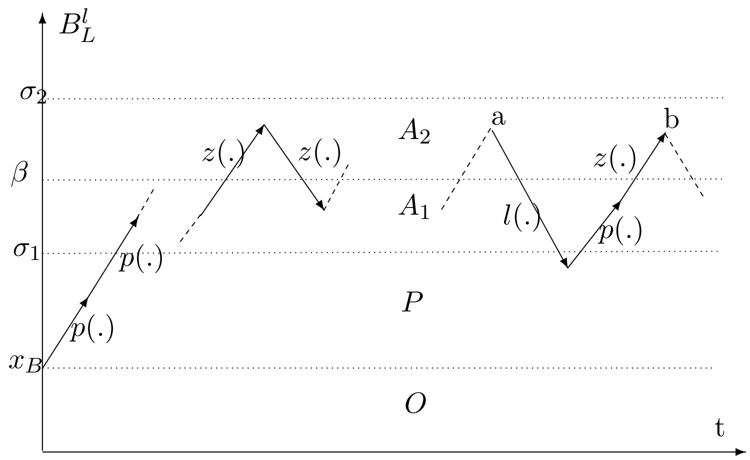

Figure 1: In P, the numbers of *soft* buyers is high. So, misrepresenting is the best action for all the sellers. In $A = A_1 \cup A_2$, the proportion of buyers who play *soft* is smaller, and misrepresenting is not always the best strategy for sellers. The function $z(.)$ is such that the payoff gain of misrepresenting compared to truth-telling is null if all the following $B_L^l(t)$ are in A. So, individually, sellers are indifferent between these two actions. Otherwise, $l(.)$ ensures that the payoff implied by telling the truth in a is equal to the payoff of telling the truth in b. Hence, sellers in a are individually indifferent between misrepresenting and truth telling.

Proposition 2 *If $n_B(t) = 0$, $\forall t$, sellers' optimal strategies are such that $B_L^l(t) \in [x_B, 1]$ evolves according to the following rules :*

$$\begin{array}{rcl} \text{If } B_L^l \in P \Longrightarrow B_L^l(+1) & = & p(B_L^l) \\ \text{If } B_L^l \in A_1 \Longrightarrow B_L^l(+1) & = & z(B_L^l) \\ \text{If } B_L^l \in A_2 \Longrightarrow B_L^l(+1) & = & z(B_L^l) \quad \text{if} \quad z(B_L^l) \in A \\ B_L^l(+1) & = & l(B_L^l) \quad \text{if} \quad l(B_L^l) \in P \\ B_L^l(+1) & = & x_B \quad \text{if} \quad l(B_L^l) \in O \end{array}$$

Let's note the following properties if $n_B(t) = 0$ is indeed the best reply for uninformed buyers[6]. The sequence $B_L^l(t)$ described is compatible by construction with an equilibrium. The rule is well defined i.e. by iteration no point outside the domain of the rule is reached. Clearly, it is possible to find a profile of strategies such that $B_L^l(t)$ evolves according to the proposition. It is beyond the scope of this text but proposition 2 describes the unique sequence possible as an equilibrium.

3.2.2 The definitions

The first set O is simply all the value below x_B. It is impossible that $B_L^l < x_B$ since there is a part x_B of informed buyers among the newcomers in each period.

The second set P is such that ΔV_{SL} is positive or null even when all the sellers play *tough*. According to proposition 1, all the sellers will then play *tough*. In this situation, $B_L^l(+1) = p(B_L^l)$ with the following definition for $p(B_L^l)$

$$p(B_L^l) = \frac{x_B + (1 - 2x_B)B_L^l}{1 - x_B B_L^l} \tag{17}$$

This definition is obtained by putting $S_L^h = 1$ in (10).

We will now establish that P is a convex set. One corollary of the convexity is that there exists a threshold value σ_1 $\Delta V_{SL}(B_L^l, p(B_L^l)) \geq 0$ (resp. $\Delta V_{SL}(B_L^l, p(B_L^l)) \leq 0$) for all $B_L^l \leq \sigma_1$ (resp. $B_L^l \geq \sigma_1$). Without this property, some of the results presented here below would be quite challenging to prove.

Claim 3 $P \equiv \{B_L^l \in [x_B, 1] | \Delta V_{SL}(B_L^l, p(B_L^l)) \geq 0\} = [x_B, \sigma_1]$ *with* σ_1 *defined by* $\Delta V_{SL}(\sigma_1, p(\sigma_1)) = 0$.

[6]Formal proofs of these properties are given in Isaac (2006).

Proof By substitution, one can write:

$$\Delta V_{SL}(B_L^l, p(B_L^l)) \quad = \quad B_L^l[X - Y\frac{x_B + (1-2x_B)B_L^l}{1 - x_B B_L^l}] + Z$$

Let's observe that $\Delta V_{SL}(x_B, p(x_B)) > 0$ and $\Delta V_{SL}(1, p(1)) > 0$ $\forall \delta < 1$. Hence, there is between x_B and 1 an odd number of roots for $\Delta V_{SL}(B_L^l, p(B_L^l))$ which is a polynomial of degree 2. Consequently, there is a unique root that we will note σ_1 and the claim is established.

Let's now establish some properties of $p(B_L^l)$ which will be useful later. First, it is clear that $p(B_L^l) > B_L^l$. The function $p(B_L^l)$ is therefore increasing in its argument. In other words,

Claim 4 *If* $\gamma > \lambda$ *then* $p(\gamma) > p(\lambda)$.

And the last useful property : if we define a sequence $B_L^l(t)$ by $B_L^l(t+1) = p(B_L^l(t))$, we observe $\lim_{t\to\infty} B_L^l(t) = 1$. In other words, $\forall \gamma \in [x_B, 1[$ and $\forall B_L^l(0) \in [x_B, 1]$ there exists a $\bar{t}$ such that $B_L^l(t) > \gamma \forall t \geq \bar{t}$.

To fulfill proposition 1, above σ_1, S_L^h can not be equal to 1. Let's define $z(B_L^l)$ such that $\Delta V_{SL}(B_L^l, z(B_L^l)) = 0$. It is easy to establish :

Claim 5 *If* $\gamma > \lambda$ *then* $z(\gamma) < z(\lambda)$.

So, if we define $\sigma_2 = p(\sigma_1) = z(\sigma_1)$, we are sure that, if for all $B_L^l \in [\sigma_1, \sigma_2]$, we apply a rule such that $B_L^l(+1) \leq z(B_L^l)$, then the sequence B_L^l will never reach a value larger than σ_2.

Imagine that the rule is

$$B_L^l(+1) \quad = \quad p(B_L^l) \quad \text{if} \quad B_L^l \in P \tag{18}$$

$$B_L^l(+1) \quad = \quad z(B_L^l) \quad \text{if} \quad B_L^l \in A = [\sigma_1, \sigma_2] \tag{19}$$

This rule would be consistent with an equilibrium (according to proposition 1) if the sequence of $B_L^l(t)$ does not exit set A once this set is reached. But, we cannot ensure that once the sequence enters in A, the sequence will remain in this set for all the subsequent periods. The unique way to escape the set A is to return in the set P. If it happens, then it is possible to find a triplet $B_L^l(t)$, $B_L^l(t+1)$, $B_L^l(t+2)$ such that $B_L^l(t+1) = z(B_L^l(t))$ and $B_L^l(t+2) = p(B_L^l(t+1))$. Clearly, if such a triplet exists, we have $\Delta V_{SL}(B_L^l(t), B_L^l(t+1))) + \Delta V_{SL}(B_L^l(t+1), B_L^l(t+2))) > 0$. Then for all sellers in period t, as established in our first proposition, it would be optimal to play *tough*. So, the proposed rule can not be the good one. To avoid this kind of phenomena, we define $l(B_L^l)$, which will replace $z(B_L^l)$ when the sequence defined by the rule here above exits A.

$$B_L^l[X - Yl(B_L^l)] + Z = -\max_T(\sum_{i=1}^{T} \delta^i \Delta V_{SL}(B_L^l(i), p(B_L^l(i))) \tag{20}$$

with $B_L^l(1) = l(B_L^l)$ and $B_L^l(i+1) = p(B_L^l(i))$ for $i > 1$. The left-hand term is the instantaneous ΔV_{SL} if we go from B_L^l to $l(B_L^l)$. The right-hand term, is the discounted sum of ΔV_{SL} when all sellers continue to play *tough* (i.e. $S_L^h = 1$) in the T periods following $l(B_L^l)$. T is chosen to maximize this sum. Remark that the left-hand term is a decreasing continuous function of $l(B_L^l)$ while the right-hand term is an increasing one. So, if there exists one $l(B_L^l)$, it is unique. Remark that

Claim 6 *If* $\gamma > \lambda$ *then* $l(\gamma) < l(\lambda)$.

β[7] is still to be defined such that $\Delta V_{SL}(\beta, \beta) = \beta[X - \beta Y] + Z = 0$. Due to claim 5, we know that $z(B_L^l(t))$ will never escape from A if $B_L^l(t) \in A_1 = [\sigma_1, \beta]$. So, we have to be careful with the possibility of an exit of A only for $B_L^l(t) \in A_2 =]\beta, \sigma_2]$.

To sum up, the sets are formally defined as follows

$$\begin{aligned} O &= [0, x_B] \\ P &=]x_B, \sigma_1[\\ A_1 &= [\sigma_1, \beta] \\ A_2 &=]\beta, \sigma_2] \\ A &= A_1 \cup A_2 \end{aligned}$$

3.3 Existence of the equilibrium

Until now, we did not prove that $n_B(t)$ is indeed the best strategy for uninformed buyers in the equilibrium that we build. We would like to prove that the condition for claim 2 is satisfied. We proceed in two claims. First, if the number of *tough* sellers is high and uninformed buyers are not too suspicious then $\Delta V_B(S_L^h(t), S_L^h(t+1)) < 0$. Secondly, if $n_B(t) = 0$ and δ is high enough, then the number of *tough* sellers is indeed high enough.

[7] Actually, in one steady state of the model, B_L^l takes this value. This steady state is called E3 in Serrano and Yosha (1993) and is not the steady-state considered by Isaac (2008).

Claim 7 *If* $\alpha_H > \frac{p^H - u_L}{u_H - u_L}$ *then there exists* $\bar{S}$ *such that* $\forall S_L^h > \bar{S}$

$$\Delta V_B(S_L^h, 1) < 0 \quad (21)$$
$$\Delta V_B(1, S_L^h) < 0 \quad (22)$$
$$\Delta V_B(S_L^h, S_L^h) < 0 \quad (23)$$

Proof $\Delta V_B(1,1) < 0$ if α_H satisfies the condition here above. Let's remark, that $\Delta V_B(S_L^h(t), S_L^h(t+1))$ is continuous in its two arguments. So, by continuity we obtain the three inequalities of the claim.

Claim 8 *For all equilibria following the rule given in proposition 2,* $lim_{\delta \to 1} S_L^h(t) = 1 \; \forall t$.

Proof The formal proof is given in the appendix. Intuitively, the result is obvious. If uninformed buyers play in any case *soft*, the unique consequence of reducing the frictions (i.e. of increasing δ) is to reduce the cost of misrepresentation. Hence, it is not surprising that more and more sellers misrepresent.

4 A limit cycle of period 2

In the first subsection, we show that the steady state is not stable when $X < 0$. When X is positive, the steady state is attractive. The proof of this last property is similar to the proof of the case of X negative. In the second subsection, we prove the existence of a limit cycle when $X < 0$. Considering this result, it is obvious that some phenomena are ignored by a steady-state approach.

4.1 Divergence from the steady-state

According to the comment after the definition of $p(B_L^l)$ and the characterization of $B_L^l(t)$ at equilibrium, we know that at least one element of $B_L^l(t) \in A$. The following claim states that the sequence $B_L^l(t)$ moves away from the stationary steady state when the sequence is in A.

Claim 9 *If* $X < 0$ *and* $x_B < \beta$, *we define* $B_L^l(+2)$ *as* $z(z(B_L^l))$ *then* $\forall B_L^l \in A$

$$B_L^l < \beta \iff B_L^l(+2) < B_L^l < \beta \quad (24)$$
$$B_L^l > \beta \iff B_L^l(+2) > B_L^l > \beta \quad (25)$$

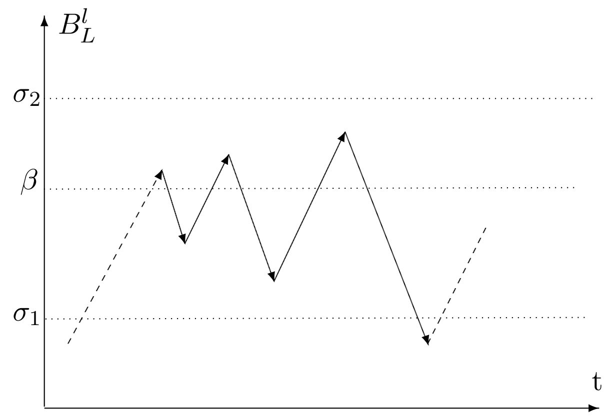

Figure 2: Divergence from steady state

Proof

$$B_L^l(+2) = \frac{\frac{ZY}{\frac{Z}{B_L^l}+X} + X}{Y} \tag{26}$$

If $X < 0$, $B_L^l(+2) > B_L^l$ is equivalent to

$$0 \quad > \quad B_L^l[X - YB_L^l] + Z$$

By definition of β, this inequality is true if and only if $\beta < B_L^l$.

4.2 Limit cycle

In this subsection, we prove the existence of a limit cycle of period two. We start by a conjecture that seems quite challenging to prove analytically due to tractability problem. This conjecture is deduced from our numerical simulation. We did not find any configuration of parameters such that the conjecture is false. In any case, if this conjecture is false, quite interesting results are also obtained. Indeed, one can proof the existence of chaos in the sens of Li and York thanks to Marotto theorem if (27) is not satisfied and thanks to Li and York theorem if (28) does not hold.

Conjecture 1

$$\begin{aligned} p(l(\sigma_2)) &> \beta & (27)\\ l(p(l(\sigma_2))) &< \sigma_1 & (28)\end{aligned}$$

In what follows, we will assume that this conjecture is true. One can state the main proposition of this section:

Proposition 3 *There exists a limit cycle of period 2 when $X < 0$.*

The two following claims establish the proof of this proposition. In the first claim, one proves the existence of the cycle. Then, the convergence towards this cycle is proved.

Claim 10 *There exists a cycle of period 2 when $X < 0$.*

Proof Let's remark that $p(.)$ and $l(.)$ are continuous and that the derivative of their conjunction $p(l(.))$ is negative. By continuity and the first inequality of the conjecture, there exists a point ω_2 in A_2 such that $p(l(\omega_2)) = \omega_2$. Let's note $\omega_1 = l(\omega_2)$.

Claim 11 *All the trajectories converge towards the cycle of the previous claim when $X < 0$.*

Proof One will proof that a sequence starting in $B_L^l(0) = \sigma_2$ converges towards the cycle (*step 1*). Then, the convergence for all the sequences with an element in $[\omega_2, \sigma_2]$ is proved (*step 2*). Let's finally remark that all the sequences with $B_L^l(0) \in [x_B, \sigma_2]$ have such an element.

Step 1. $B_L^l(1) = l(\sigma_2) < l(\omega_2) = \omega_1$ by claim 6 since $\sigma_2 > \omega_2$.
$p(B_L^l(1) = B_L^l(2) < \omega_2 = p(\omega_1)$ by claim 4 and $B_L^l(1) < \omega_1$.
By claim 6 and the conjecture, $\sigma_1 > B_L^l(3) > \omega_1$.
By claim 4, $\sigma_2 > B_L^l(4) > \omega_2$.
Using once again claim 6, $\omega_1 > B_L^l(5) > B_L^l(1)$.
By iteration of the same arguments, $\lim_{t\to\infty} B_L^l(2n) = \omega_2$ and $\lim_{t\to\infty} B_L^l(2n+1) = \omega_1$.

Step 2. If $B_L^l(\bar{t}) > \tilde{B}_L^l(\bar{t}) > \omega_2$ then $\tilde{B}_L^l(t) \in [\omega_2, B_L^l(t)]\ \forall t = \bar{t} + 4n$. It is a consequence of claims 4 and 6. So, all the sequences with an element in $[\omega_2, \sigma_2]$ converge towards the cycle.

5 Information revelation

For each period, the fraction of transacting uninformed buyers who, in state L, end up trading at the wrong price is

$$f_B(t) = S_L^h(t) \tag{29}$$

The following proposition is then simply a corollary of claim 8

Proposition 4 *For all equilibria following the rule given in proposition 2,* $\lim_{\delta \to 1} f_B(t) = 1$

Since $f_B(t) < 1$, it implies that for some $\hat{\delta}$, $f_B(t)$ is increasing in δ when $\delta > \hat{\delta}$. In other words, at this equilibrium the best information revelation is not reached with the highest δ. Frictions have a positive effect on information revelation.

Let's observe that this result does not depend on $X < 0$. So, this proposition is also valid for situations where the steady state is stable. It may appear surprising and in contradiction with Serrano and Yosha (1993).

Serrano and Yosha (1993) remark that they can not exclude equilibria such that, in the limit, trade takes place at the wrong price, although they have not found any example of such a sequence. They add that at such an equilibrium the limiting fraction of wrong price trades is a precise number which is typically different from one.

This last observation contradicts proposition 4. Actually, as explained in the appendix, the statement of Serrano and Yosha (1993) is valid only for cases where $n_B(t) \neq 0$. So, once we are well aware that a part of the analysis of Serrano and Yosha (1993) is restricted to interior solutions while the equilibrium established in the previous sections is a corner solution, the contradiction disappears.

One can sum up our knowledge of constant entry flow model in a dynamic analysis in the following proposition.

Proposition 5 *For all economies such that (1) is satisfied, when frictions disappear (i.e.* $\delta \to 1$*)*

1. *there exists an equilibrium* ER *with complete information revelation*
2. *if* $\alpha < \frac{p^H - u_L}{u_H - u_L}$*, this equilibrium is unique*
3. *if* $\alpha > \frac{p^H - u_L}{u_H - u_L}$*, an other equilibrium* EM *exists. At this last equilibrium information revelation is a negative function of* δ

Part 1. and 2. were established in Isaac(2008). Our contribution is the third part. At this stage, open issues are
- Are there other equilibria ?
- In case of multiplicity of equilibria, is there a focus point which could help to solve the coordination problem ?
- How could a policy maker improve the situation ?

On this last point, we observed that increasing δ when it is possible (let's imagine that the policy maker has an influence on the time between two matchings) is not always the best thing to do. To ensure, that the economy is on the *good* equilibrium, one can try to fulfill the condition on α_H to obtain the uniqueness. One way to proceed could be to impose a tax t when the price is p_H. In the condition, p_H would be replaced by $p_H + t$ and effectively, for a given α_H, one can always reach a situation such that the condition is satisfied. According to the way the product of the taxation is redistributed, the incentives could change and a more cautious study is necessary to ensure that we can indeed, by taxation, improve the situation. It is beyond the scope of this paper but it seems to be an interesting question.

A Additional Proofs

Proof of claim 4 From (17), we compute

$$\frac{\partial p(B_L^l)}{\partial B_L^l} = \frac{(1-x_B)^2}{1-x_B B_L^l} > 0 \tag{30}$$

Proof of claim 5 By definition,

$$\begin{aligned} \gamma[X - Yz(\gamma)] + Z &= 0 \\ \lambda[X - Yz(\gamma)] + Z &> 0 = \lambda[X - Yz(\lambda)] + Z \\ z(\gamma) &< z(\lambda) \end{aligned}$$

Proof of claim 6 As $\gamma > \lambda$ and $[X - Yl(\lambda)] < 0$, we can write the first line. The second line is obtained by the definition of $l(.)$.

$$\begin{aligned} \gamma[X - Yl(\lambda)] &< \lambda[X - Yl(\lambda)] \\ \gamma[X - Yl(\lambda)] &< -\max_T \Big(\sum_{i=1}^{T} \delta^i \Delta V_{SL}(B_L^l(i), p(B_L^l(i))) \Big) \\ & \quad \text{with } B_L^l(1) = l(\lambda) \text{ and } B_L^l(i+1) = p(B_L^l(i)) \forall i > 1 \end{aligned}$$

The left term is decreasing in $l(.)$ while the right term is increasing. So, to find an equality, $l(\gamma)$ must be lower than $l(\lambda)$.

Proof of claim 8 This proof assumes that $X < 0$ and that the conjecture introduced in section 4 is true. It is not difficult to change this proof such that it is also valid for the other cases.

If $B_L^l(t+1) = p(B_L^l(t))$, $S_L^h(t) = 1$.

If $B_L^l(t+1) = z(B_L^l(t))$ or $B_L^l(t+1) = l(B_L^l(t))$, it is less obvious.

It is well established that $\lim_{\delta\to 1}\beta = 1$. It implies obviously $\lim_{\delta\to 1}\sigma_2 = 1$ since $\sigma_2 > \beta$. It is easy to check $\lim_{\delta\to 1} p(B_L^l) = 1 \Leftrightarrow \lim_{\delta\to 1} B_L^l = 1$. So, $\lim_{\delta\to 1}\sigma_1 = 1$. By definition, $z(\sigma_1) = p(\sigma_1)$. It implies that if $B_L^l(t) = \sigma_1$, $S_L^h(t) = 1$. Again by definition, $z(\beta) = \beta$. By replacing $B_L^l(t)$ and $B_L^l(t+1)$ by β in the following equation obtained from equations (8) and (9)

$$S_L^h(t) = \frac{(x_B - B_L^l(t+1))(1 - B_L^l(t))}{(B_L^l(t+1) - 1)B_L^l(t)(1 - x_B)} \tag{31}$$

one can check that if $B_L^l(t) = \beta$, $\lim_{\delta\to 1} z(B_L^l) = 1$. Since $z(.)$ is monotonic in A_1, we have proved that if $B_L^l(t) \in A_2$, $\lim_{\delta\to 1} z(B_L^l) = 1$. Also due to a monotonicity property in A_2, it remains to prove that $\lim_{\delta\to 1} S_L^h(t) = 1$ if $B_L^l(t) = \sigma_2$. With $B_L^l(t) = p(B_L^l(t+1))$, $\lim_{B_L^l(t)\to 1} S_L^h(t) = 1$. So, $\lim_{\delta\to 1} S_L^h(t) = 1$ if $B_L^l(t) = \sigma_2$ and $B_L^l(t+1) = \sigma_1$ (clearly this $B_L^l(t+1)$ is not the one given by the rule). Let's observe that $\lim_{\delta\to 1} l(\sigma_2) = \sigma_1$. So, we have proven that $\lim_{\delta\to 1} S_L^h(t) = 1$ if $B_L^l(t) = \sigma_2$.

Restricted analysis in Serrano and Yosha (1993) To prove their proposition 2, they assume that the solution of the uninformed buyer's problem is interior. Indeed, on page 493, they use a first order condition ($\frac{\partial}{\partial x} V_B(x; \alpha_H, 1, S_L^h) = 0$) to derive expression (30). So, this expression is not valid for situation with $n_B = 0$. Their proposition 2 is nevertheless correct because minor extensions of the proof are sufficient.

Unfortunately, it is not the case for the corollary part (a) on page 493. A correct statement would be *If* $\lim_{\delta\to 1} S_L^h > 0$ *then either* $\lim_{\delta\to 1} n_B = \infty$ *or* $\lim_{\delta\to 1} n_B = 0$.

Their proposition 3 suffers also from the fact that corner solutions are not considered. The non existence of equilibria E3 such that $\lim_{\delta\to 1} S_L^h = 1$ is not well established in the shaded area. The equilibrium that we have constructed is a counter example.

The weakness of the proofs does not affect the conclusion that the presence of uninformed agents on the two sides is a key feature to obtain the result about information revelation in Wolinsky (1990). This result states that, at all equilibria, even when the market becomes approximately frictionless, some trades occur at a wrong price. The existence of equilibrium E1 with complete information revelation in Serrano and Yosha (1993) is not affected by the weakness since this last one concerns only E3 equilibrium.

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