The demand driven and the supply-sided Input-Output models. Notes for the debate

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Abstract: Input-Output is a production model; in the original Leontief's open version final demand determines the level of activity, assuming technology as given. Ghosh's formulation, on the contrary, assumes that value added determines output in a supply-driven account; producers must induce sales in order to achieve a desired level of income. The supply–driven model has been criticised on the bases of its implausibility and its difficult interpretation. This paper argues that, even though the theoretical foundations of the supply-side model are somewhat unusual, it fits perfectly with the standard demand-driven model and Leontief's arguments.

Keywords: Input-Output model, demand-driven model, supply-driven model, Leontief, Ghosh, balanced growth

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Introduction

Wassily Leontief's major contribution to economics is -no doubt- the formulation of the Input-Output (IO) model, for which he was awarded the Nobel Prize in 1973. The best well-known version is a workable open multisector account that determines production, as a function of final demand, given the technology that each sector uses; the model should yield optimal results, even if it does not discuss optimality explicitly -but there are no reasons to expect that producers would choose their technologies otherwise. Under the standard assumptions, each industry produces one homogeneous commodity, using one homogeneous technology, which also determines the proportions of inputs employed to produce; those technologies are also decisive in the way sectors interrelate and -in turn- condition the shape of the economic structure. Leontief first presented a closed model to study the interdependence between sectors in 1937: this is the keystone to reach the two alternative (independent) solutions that Leontief offers in that article, one for quantities and one for prices. Later on, in 1944 Leontief published an open version of the model and used it to study the impacts that final demand changes may have on production. That would prove to be the most extended empirical use of the model for decades.

In 1958 Ambica Ghosh, from the Department of Applied Economics, at the University of Cambridge, published the article "Input-Output Approach in an Allocation System" (*Economica, New Series*, Vol. 25, No. 97, pp. 58-64), a publication that has been quoted and discussed extensively. Here the author presents an IO model solved on the allocation of output, where value added is the exogenous variable; the solution can be associated to a supply-sided economy, as coefficients are calculated on the revenues that each sector derives from sales to its intermediate and final customers. According to Ghosh (1958) the model would be useful to analyse centrally planned economies, as well as systems dominated by monopolisitic markets and economies constrained by scarce resources (as opposed to Keynesian frameworks, limited by final demand). Under those circumstances, allocation of outputs would be a more complex task, which could be understood by this alternative model.

Nevertheless, Ghosh's proposal does not seem to have received a warm welcome and there was not significant discussion around it, until María Augustinovics presented an analysis of actual economic structures from such viewpoint in 1970. Later on forward linkages[2] have been often calculated from the perspective of the allocation of outputs and the supply side of the IO model (Jones, 1976; Bulmer-Thomas, 1982), although earlier applications used the coefficients matrix to determine those indices, from the demand perspective (e.g. Chenery and Watanabe, 1958; Hazari, 1970; Laumas, 1976). However, some contemporary authors disputed the rationality of measures derived from the supply model, since it is a matter of debate in itself (e.g., McGilvray, 1977).

Ghosh's version of the IO model has also received attention in regional analysis and energy models (e.g., Giarratani, 1976 and 1980). After 1981 there has been a long discussion that re-emerges every now and then, on the applications and meaning of the supply sided model and even its plausibility; a few academics involved are Bon (1986, 1988), Dietzenbacher (1997), de Mesnard (e.g. 2007, 2009), Guerra and Sancho (2011) Oosterhaven (1988). Most authors reject the model and question its rationality that does not seem to comply with reality, which -presumably- would be closer to the demand-driven logic; in such a case, however, arguments against should not be limited to Ghosh's contribution, since there are many more supply-sided economists. It should be acknowledged, however that in the referred article, Ghosh does not seem to subscribe that line of theory, but he was rather concerned with some developing economies would encounter while growing and modernising.

The purpose of this paper is to analyse a few issues concerning Ghosh's supply-sided model and to assess some ideas that have been at the bases of that discussion. This paper argues that despite debatable logic and general disapproval, Ghosh's model fits as a similar development (in the mathematical sense) of the standard demand-driven Leontief model and can be seen as an

2 Forward linkages measure the capacity of each sector to induce the use of its output as input by other producers (Bulmer-Thomas, 1982).

extension to it. Maybe it would be necessary to appeal to authors such as Say, in order to understand its meaning and certainly, such interpretation takes the IO model away from more accepted perspectives, based on demand-driven economics that modern economics takes for granted. On the other hand, accepting the supply-side model reinforces the notion that the IO framework is useful to study a variety of empirical problems from various theoretical perspectives, including those opposed to demand-sided economics. The remaining of the paper is organised as follows: section 1 discusses the IO model; section 2 presents the solution by Ghosh; section 3 presents the main ideas in the debate about the supply side model and its interpretation. Finally a few remarks are discussed in the last section.

1. The Input-Output Model

An economic system is defined as a set of interrelated industries, each one identified by a productive process, consuming produced commodities as inputs in given proportions, in order to produce one particular homogeneous good, by means of a technological relation. Disregarding non-produced merchandises in the system, it would be possible to find a productive process for each good: consumption and investment are also economic activities that demand inputs to produce outputs, also useful in the productive processes, such as factors, using some technology. The economy is closed (Cassel, 1918; Leontief, 1937; von Neumann, 1936; Sraffa, 1960; Walras, 1874). On the contrary, if non-produced goods exist and they are available for productive and consumptive activities, the system is open (Leontief, 1944; Marx, 1885) and exogenous variables determine the level of activity.

The IO model defines an n-dimensional space, of the n produced goods, demanded both as inputs or final demand goods, which are linearly transformed into n produced goods, by the n industries that define the economic system; those industries employ n productive techniques, observing strict constant returns to scale with zero rates of substitution between inputs (it is a short term scheme and isoquants are L- shaped). It can be postulated that agents use the most efficient technologies within the set of all possible ones, as the model omits any explicit discussion on the choice of technology. Thence, the system remains in equilibrium, as long as prices persist. As a result, technical coefficients are constant. The system transforms a given set of goods (inputs) into a similar set of goods (outputs) through a given technological relation. In order to complete the circular flow of the economy, production is transformed into revenue for all agents, which changes once again into demands of all kinds (Aroche, 1993).

Industries are numbered 1, 2, ..., *i, j*, ..., *n*; those sectors exchange goods valued $z_{ij} = p_i q_{ij} \geq 0$, in amounts determined by the consuming sector, at given prices. Therefore, ordering those transactions conveniently, a square matrix can be arranged, $\mathbf{Z} = [z_{ij}]$ (Leontief, 1936). Adding up over the columns of **Z,** results in a row vector of the value of the inputs that each industry requires in production; conversely, summing up on the rows, one gets the value of the goods that each industry (*i*) offers to the rest of the producers, both during a stated period of time.

In an open model, **Z** is a square matrix showing the exchange of produced goods between industries; demand for non-produced goods (factors) appears in a (second) rectangular array of the n+1, ..., $n+f$ different types of factors employed by the n industries in the system. Adding up over the columns of the latter matrix, yields a row vector of value added (**v**'). Besides, the various types of agents (1, ..., *g*) that own those primary inputs, consume the *n* produced goods outside the productive processes, as final demand, which can be arranged in a (third) rectangular matrix of the *n* sectors and the *m* types of agents. Summing up over the rows of this array results in a column vector of final demand (**d**).

Adding up the sum of the supplies of goods to other producers and final demand agents yields the revenues of each industry; conversely, the demand for produced inputs plus primary inputs for each sector results in a row vector of industry expenditures. Revenues per branch equal expenditures and the value of sectoral supply equals that of sectoral demand, i.e. no activity makes profits and each factor receives equilibrium income[3].

Following Leontief's reasoning two equations represent the model in its open version, although as stated above, Leontief (1944, 1986) concentrated his attention on the first one:

$$(1)\ \mathbf{Z}\iota + \mathbf{d} = \mathbf{x}$$

$$(2)\ \iota'\mathbf{Z} + \mathbf{v}' = \mathbf{x}'$$

ι is the sum vector, **x** is the (column) vector of outputs accounted by sectoral revenue and **x**' is the (row) vector of outputs accounted by sectoral expenditures. Those equations represent two sides of the same phenomenon, that of production. On the one hand, the IO model is demand-driven, one can assume that output is infinitely elastic to demand and there are no scarce factors or sticky prices that may impede instantaneous adjustments; on the other, the model is supply-sided and revenues would be explained by the generation of value added (factors' incomes). Output is infinitely elastic to factor revenues and consumers (of all sorts) absorb as much output as it is offered[4]. Both equations are independent, but can be linked when output becomes factor incomes and, conversely, when value added is transformed into final demand.

However, both value added and final demand are exogenous in the IO model, therefore those transformations are beyond the scope of equations (1) and (2), which means that those metamorphosis are exogenous as well; for that reason, those variables cannot be determined simultaneously as it happens in a general equilibrium schemes (Debreu, 1959). Therefore, the above equations cannot be solved simultaneously (Schummann, 1990) and consequently, they

3 Alternatively, if individual sectors are not in equilibrium (and transfers between them are allowed), still the system as a whole must comply with that condition. For example, empirical national accounts often show that sectors do not always comply with equilibrium conditions, but so does the economy as a whole.
4 In order to understand that reasoning, perhaps it would be useful to appeal to the widely debated Say's law.

are not dual one another (again, as it is the case in a general equilibrium model). Moreover, as said above, Leontief (1944) discusses equation (1) only, making the model demand-driven; supply and prices are beyond the interest of the 1944 paper and later developments, despite the fact that in the 1937 closed version Leontief offered a solution to prices in the first place, which was neither simultaneous nor dual to that of quantities, explained in the second place. So it happens that in fact one can choose either solution (prices or quantities), leaving the other one undetermined.

On the contrary, finding the solution to any modern version of the general equilibrium model, from von Neumann (1937) to Arrow and Debreu (1954) and beyond, means determining two vectors at the same time, one for prices and one for quantities, which are dual one another: no other possibility exists. Von Neumann (1937) suggested using Brower's fixed point theorem for the task and determined also a uniform rate of growth for all sectors, together with prices and quantities, while the latter and more modern variations use Kakutani's simplified fixed point theorem and do not discuss balanced growth. Leontief solves his open demand model (and one could add the supply equation) following different roads: as a first step, rewriting both equations above in proportions (or coefficients):

$$(3) \qquad \mathbf{Ax} + \mathbf{d} = \mathbf{x}$$

$$(4) \qquad \mathbf{x'E} + \mathbf{v'} = \mathbf{x'}$$

As usual, $\mathbf{A} = \{a_{ij}\} = \{z_{ij}/x_j\}$ is the technical coefficients matrix, i.e., the proportions that each industry *j* uses of each good *i* in its bill of inputs. Matrix $\mathbf{E} = \{\mathbf{e_{ij}}\} = \{z_{ij}/x_i\}$ contains the proportions of the total product that industry *i* provides to each industry *j*. Coefficients a_{ij} and e_{ij} are proportions of the sectoral expenditure (x_j) and revenue (x_i), respectively; both measures are possible because z_{ij}, x_j and x_i are valued in the same units (p_iq_j); despite basic assumptions seldom (if ever) discussed and postulated repeatedly by Leontief himself (e.g., 1944) and some other authors, the model is not expressed in physical terms, otherwise it would be impossible to make any simple mathematical operation such as an addition of different inputs, necessarily measured in different units (e.g., grams, metres and litres). **A** is technically determined, whereas **E** is not: from the viewpoint of the producer it is reasonable to say that the technology determines the list and proportions of inputs, whereas there is no explanation of the amounts or proportions that suppliers sell to each consumer, it is also unimportant for the seller whether her product is used as an input or as a final demand good.

As it is well known, matrices **A** and **E** are square, semipositive and non-singular; besides, they share the associated eigenvalues and eigenvectors (in short, matrices **A** and **E** are similar), because the models are closely related in a way that it could be said that they are similar as well. Both models transform linearly the space of produced goods into a space of produced goods by different means: consumption and distribution of inputs. Moreover, the sum of each column of **A** and each row of **E** are less than unity, because each industry use goods as inputs in lesser value than that produced and –on the other hand- the value of total supply of each produced good is larger than the value of the goods provided to other producers to use as inputs. As a result, the maximum eigenvalue associated to both models is no larger than one: the economic system produces surplus (Aroche, 1993; Nikaido, 1970).

The solutions to the above equations are:

$$(5) \qquad (\mathbf{I} - \mathbf{A})^{-1}\,\mathbf{d} = \mathbf{L}\,\mathbf{d} = \mathbf{x}$$

$$(6) \qquad \mathbf{v}'\,(\mathbf{I} - \mathbf{E})^{-1} = \mathbf{v}'\,\mathbf{H} = \mathbf{x}'$$

These expressions determine, first, the level of total production necessary to satisfy final demand, **f** and, second, the level of output necessary to generate the desired level of value added. **L** is called Leontief or the multipliers matrix; its entries show the direct and indirect (total) requirements of inputs produced by *i* per unit of output produced by industry *j*. Analogously, the entries of matrix **H** show the direct and indirect sales that sector *j* must encourage in each sector *i* so that **v**' is attainable. These models imply also that the productive process follow a circular logic, when a proportion of outputs return to the productive sphere as inputs required to produce new products to satisfy demand. Parallel to that, a proportion of outputs are sold as inputs (and return to the productive sphere) in order to support value added.

2. *An Allocation System: A. Ghosh's Model*

Ambica Ghosh´s (1958) model is similar to that shown by equations (2), (4) and (6); as already said, Ghosh suggests that model (1), Leontief's original 1944 model or his alternative formulation (2) are similarly valid under different institutional conditions, concerning the competitive regime of the economy and the availability of resources. One curious point of the paper is the surmise that it is possible to find non-optimal resource allocations that (nevertheless) maximise welfare, by maximising the employment of labour, regardless of its productivity (p.59). Perhaps the problem could be rephrased saying that the central planner has the goal of maximising labour employment, regardless of any other consideration (perhaps lowering wages); alternatively the model could be built assuming different rationality conditions.

Further, Ghosh postulates that in economies with factor surplus, technical coefficients ($\mathbf{a}_{ij}$) might be unstable, whereas distribution proportions ($\mathbf{e}_{ij}$) are not. That amounts to saying that one can find continuous technical change in the economy, while the allocation of outputs remains, breaking the similarity between matrices **A** and **E**. On the contrary, according to Leontief (1944), in the short run, when matrix **A** is fixed, it is possible to perform experiments assuming changes in final demand while technology is given; alternatively, within Ghosh's assumptions **v** could change, but **E** should be fixed. Thus, it is not possible that allocation coefficients change on their own, keeping demand coefficients (or *vice versa*), unless the model does not comply with the principle of proportionality, on which Leontief bases the whole IO model in his 1937 paper. According to that principle, when one coefficient changes, relative quantities and prices change as well, but that can only happen in the long run, when there is substitution between factors and inputs in the production line. The 1944 paper does not mention that principle, because it deals with the demand side only, but the logic of the construction of the model allows one to expect that it remains valid. No coefficient in any matrix (**A** or **E)** can change independently, unless the whole economic system changes as well.

3. *The Dutch Connection*

J. Oosterhaven (1988) claims that within the logic of Ghosh's model it is feasible to increase output in some sectors while keeping value added static. "… The Ghoshian model takes demand for granted, i.e., demand is supposed to be perfectly elastic (…) local consumption or investment reacts perfectly to any change in supply, and that purchases are made, e.g., of cars without gas (*sic.*) and factories without machines …" (p. 207). The author concludes that the model is thus implausible. Further, Dietzenbacher (1997) explains that when production grows in one sector, no other industry needs increasing value added and Gruver (1989) argues that in the supply driven model no input is essential, so every input can be substituted by some other one, although one of the main criticisms to the IO model is that coefficients are fixed and indeed in the IO table no input is more important to any other. Dietzenbacher in his quoted 1997 paper argues ahead that the model by Ghosh is similar to the standard IO price model; such a conclusion, he claims, attends Oosterhaven's critique. According to that author, much of the confusion regarding the supply-driven model derives from its understanding as determining quantities (p. 631).

Louis de Mesnard (2009) re-examines the consistency of the supply-driven model, and concludes (correctly) that this is not the dual to Leontief's scheme, while it provides poor and uninteresting solutions; de Mesnard finds that it is unreasonable to assume that buyers are forced to buy as much as a producer decides to offer -in order to get the desired revenue. Surely, similar commentary can be raised about Say's law and other supply-sided hypothesis. However, de Mesnard is ready to accept Ghosh's as a price model, but, once again according to him it would be redundant, since Leontief's price model is much simpler. Guerra and Sancho (2011) present interesting considerations on Ghosh's model and show alternative closure possibilities, in order to analyse the plausibility problem, but they do not accept the principles on which the model is built.

Probably based on the dual solutions general equilibrium model, one for quantities and one for prices, Leontief's production model has been identified as a quantities model, lacking a price dual formulation (despite the 1937 paper). However, as it has been stated above, the IO model determines outputs as sectoral expenditures and revenues, rather than quantities or prices. The price model that has been accepted for long is (Miller and Blair, 2009):

$$(7)\ \mathbf{p} = (\mathbf{I} - \mathbf{A}')^{-1}\,\mathbf{v}$$

The founding assumption of equation (7) is that the price level in each sector depends on the level of direct plus indirect costs of primary inputs, given the technology used in the system as a whole. Once again, that equation can be solved independently from the productions equation (5). There is no duality in the IO model. Returning to Dietzenbacher interpretation of equation (6) as equivalent to Leontief price model[5], it would imply that matrix **E** is "equivalent" to matrix **A**'. If the term "equivalent" means "equal", it should be noted that performing numerical exercises with random matrices, those arrays are in general unequal (see above); in the particular case that **Z** is symmetrical, **A** = **E**'.

What happens when final demand changes in one sector in Leontief's demand-driven model? The immediate reply is that output changes in that

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[5] "… the equivalence of the supply-driven input-output model and the Leontief price model can also be shown in another, surprisingly simple manner … Post multiplying both sides of Equation (9) with $\hat{x}_0$ and using $B_0 = \hat{x}_0 A_0 \hat{x}_0$ yields $x_1' = v_1' (I - B_0)^{-1}$, which is exactly the supply-driven input-model in Equation (6)." (Dietzenbacher, 1997, p. 634).

sector proportionately and thus, provokes changes in the demand for inputs of that sector as well, causing changes in the production of the supplying industries to the initial activity, as demand grows (or contracts). In turn, it is expected that output changes in every sector of the economy. In the multiplier analysis it is expected at every moment that resources are available to carry out any level of production, determined by demand. In that exercise it is also expected that technical coefficients remain, but there is no question on the allocation proportions. Each sector keeps its technology and sells its production as demanded. Intermediate demand coefficients are stable because there is no reason for either the technology or the structure of the system to change, but output in all the sectors will increase or decrease in a magnitude explained by the multipliers and the initial final demand modification. If coefficients changed, it would be impossible to estimate multipliers.

On the contrary, Oosterhaven (1988) concludes that an analogous analysis is not possible in the supply-driven model, because he apparently expects that both models are connected somehow. Nevertheless, when factoral income grows in one sector, according to equation (6) output in that sector should also increase, which implies that the factor's revenue should rise as well, by means of extended sales. The needed extra sales of the sector in question to all the rest is given by the multiplier matrix **H** in equation (6), together with the increased **v**. Clearly the latter producers need also to expand their revenue, in order to afford the extra amounts of inputs that they require to grow their own production; moreover, producing that extra amount may also require higher quantities of other inputs that may also include factors. Under the assumptions of the model, no sector should face difficulties to hire extra factors or to find consumers willing to demand the new production. There is no question about the profitability of the increased production: the model does not mention any of these. Perhaps these ideas need also the assumption that production in each sector is constrained by the supply of inputs, therefore, as soon as one input is available in bigger quantities, growth is a natural result. In fact this is the idea behind forward linkages (Bulmer-Thomas, 1982).

The IO model is static and maybe that is one major drawback, which has limited its development and application. It is also an equilibrium system and changing one coefficient may cause changes in the output of whole structure (Shintke and Stäglin, 1988); Leontief (1937) offers a detailed study of such possibility. The reason for that is that the main preoccupation of the model in the early days was the analysis of sectoral interdependence. When one or a few coefficients change, also changes the way sectors interrelate. Therefore, the only scheme admissible to consider the possibility of growth in the IO model is that of a balanced rate. When a sector expands faster or more slowly than the rest, the system faces disequilibrium and unbalances. Exercises of the kind considered in the previous two paragraphs are valid only as bounded simulations to measure impacts of exogenous moves in a system that eventually returns to equilibrium; otherwise the technical coefficients matrix is unattainable.

A numerical exercise

Looking for consistency between the two IO models, Leontief demand driven and Ghosh's supply-sided, quite a few authors have explored the numerical relationships between the two models and between matrices **A** and **E**. In an exercise on impacts analysis on the demand version of the model once final demand grows in any sector or group of sectors, it is possible to measure growth in the output of each sector and then to calculate a new exchange matrix. Technical coefficients will remain, as well as multipliers, but it is possible to calculate a new matrix **E**; an analogous process can be derived on the supply model. Chen and Rose (1986) explain that it is empirically interesting to investigate whether changes in the allocations matrix, keeping array **A** stable, or changes in the latter, keeping **E** fixed, are consistent with the simulated demand or value added increases or decreases, which has been defined as the problem of stability. They conclude that the models are jointly stable if the original growth (positive or negative) does not cause much difference in the matrix that changes. In any case, it is clear that the analysis is symmetrical for both models and when all sectors grow at a balanced rate would be the only case when both models comply with the joint stability condition.

Oosterhaven (1988) and Dietzenbacher (1997) perform similar analysis and conclude that joint stability can only be expected if sectoral growth is uniform. Oosterhaven derives two expressions for stability, which should be complied simultaneously:

$$\mathbf{A}_{t+1} = \hat{\mathbf{e}}\mathbf{A}_t\hat{\mathbf{e}}^{-1}$$

and

$$\mathbf{E}_{t+1} = \hat{\mathbf{e}}^{-1}\mathbf{E}_t\hat{\mathbf{e}}$$

where ê is the relative growth in total sectoral output and subindex *t+1* refers to the simulated matrix after final demand or value added grows. Those expressions mean that $\mathbf{A}_{t+1} = \mathbf{A}_t$ and $\mathbf{E}_{t+1} = \mathbf{E}_t$, since both arrays are premultiplied and postmultiplied by a diagonal matrix and its inverse[6].

A numerical example may be useful to understand the models and the stability problem. Let a three-sector economy be represented by the following:

Sector	1	2	3	Intermediate Demand	Final Demand	x
1	45	64	98	207	293	500
2	123	342	198	663	237	900
3	198	295	543	1036	64	1100
Intermediate Consumption	366	701	839			
Value Added	134	199	261			
x	500	900	1100			

Matrix **A** is:

0.09	0.07	0.09
0.25	0.38	0.18
0.40	0.33	0.49

and **E**:

[6] Given matrices **A** and **B**, $\mathbf{B}\,\mathbf{B}^{-1} = \mathbf{I}$, the identity matrix; so, $\mathbf{BAB}^{-1} = \mathbf{A}$

0.09	0.13	0.20
0.14	0.38	0.22
0.18	0.27	0.49

Let the new final demand vector if it grows at a uniform rate of 10% for all sectors:

322.3
260.7
70.4

Which gives rise to the new output vector $\mathbf{x}_1$, i.e., all sectors grow 10%:

550.0
990.0
1210.0

The new transactions table $\mathbf{Z}(\mathbf{A})_1$ is:

50.39	78.40	128.10
137.73	418.95	258.82
221.72	361.38	709.79

Matrix **A** remains and the new $\mathbf{E}_1$ array is:

0.09	0.14	0.23
0.14	0.42	0.26
0.18	0.30	0.59

Therefore, the model is unstable, according to the definition above.

If on the contrary, value added grows in 10% for each sector, the new vector **v'** is:

147.4
218.9
287.1

and the new output vector equals to the former vector x_1. The new exchange matriz $\mathbf{Z}(\mathbf{E})_1$ will be:

58.3	82.9	127.0
160.1	445.2	257.8
259.4	386.5	711.4

This gives rise to the new matrix **A1**, different from **A**, which means that balanced growth does not warrant that both coefficient matrices stay put after growth has been accepted on one side of the model and the stability problem remains:

0.11	0.08	0.10
0.29	0.45	0.21
0.47	0.39	0.59

Then, there are two possible cases to solve the stability problem, as defined above. It should be noted that this is just a mathematical problem (i.e. A and E are similar matrices) and not theoretical: demand and supply sided models are similar and effects of either demand or value added changes are analogous. One case would involve a symmetric matrix **Z**:

Sector	1	2	3	Intermediate Demand	Final Demand	x
1	323	189	298	810	288	1098
2	189	245	211	645	342	987
3	298	211	678	1187	133	1320
Intermediate Consumption	810	645	1187			
Value Added	288	342	133			
x	1098	987	1320			

Matrix **A** is:

0.29	0.19	0.23
0.17	0.25	0.16
0.27	0.21	0.51

and **E**:

0.29	0.17	0.27
0.19	0.25	0.21
0.23	0.16	0.51

A and **E** are transposed matrices because **Z** is symmetric. Let the new final demand vector and the new (transposed) value added array be as follows - if they grow at a uniform rate of 10% for all sectors:

316.8
376.2
146.3

Which gives rise to the new output vector $\mathbf{x}_1$ from both the demand and the supply driven model, i.e., all sectors grow at 10%

1207.8
1085.7
1452.0

The new transactions table $\mathbf{Z}(\mathbf{A})_1$ is:

355.3	207.9	327.8
207.9	269.5	232.1
327.8	232.1	745.8

Matrix **A** remains and the new array $\mathbf{E}_1$ is:

0.29	0.17	0.27
0.19	0.25	0.21
0.23	0.16	0.51

Then matrix **E** remains too: the model is stable when **Z** is symmetric.

A final example assumes that all sectors would involve a balanced economy that also grows at a balanced rate. The system is:

Sector	1	2	3	Intermediate Demand	Final Demand	x
1	45	64	98	207	993	1200
2	123	342	198	663	537	1200
3	198	295	543	1036	164	1200
Intermediate Consumption	366	701	839			
Value Added	834	499	361			
x	1200	1200	1200			

Matrix **A** is:

0.04	0.05	0.08
0.10	0.29	0.17
0.17	0.25	0.45

and **E**:

0.04	0.05	0.08
0.10	0.29	0.17
0.17	0.25	0.45

technical and distribution coefficients are equal because of the strict conditions imposed on vector **x** and matrix **Z**. Assuming that final demand grows 10% uniformly for all sectors:

1092.3
590.7
180.4

Similarly the expanded value added (**v'**) is

917.4
548.9
397.1

which give rise to the new output vector $\mathbf{x}_1$, i.e., all sectors grow at 10% in both linear models:

1320
1320
1320

The new transactions table $\mathbf{Z}(\mathbf{A})_1$ is:

49.5	70.4	107.8
135.3	376.2	217.8
217.8	324.5	597.3

which is equal to **Z**(**E**)$_{1.}$ Matrices **A** and **E** remain because in this example, the principle of proportionality between sectors is strictly observed.

In a word, unless the initial conditions of proportionality are strict, neither Leontief or Ghosh models remain stable after introducing changes in final demand or value added; the problem is that the principle of proportionately is not complied after the introduction of numerical changes in one of the models only.

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