How to Deter Crimes?*

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Abstract

Controlling crimes requires carefully targeted enforcement incentives to cope with moral hazard and, depending on incentive costs and the crime environment, optimal allocation of the enforcement budget. Incentive targets based on the crime level should be preferred for their *monotonicity in enforcement effort*. In some crime environments we identify, none of the detection-related data satisfy effort monotonicity.

For independent crimes, detection-based enforcement makes compliance beyond a certain level impossible due to moral hazard, whereas incentives based on a verifiable measure of crime enables the State to increase compliance at first-best cost. This means, if crime data is not available due to the unobservable nature of the crime or if survey-based measures are not reliable, resources may shift to prevention of crimes for which crime measures are available, such as burglaries and breaking of bank tills, from detection of unobservable crimes such as corruption.

For crimes that form an input-output chain the State should target to block 'ground zero', the origin on the feeder side of crimes, though not as an absolute priority, to deny their downstream operations (focus on foiling drug smuggling than catching drug peddlers; 'Say's Law' in enforcement). On two occasions this recommendation might be reversed: (i) downstream detections have a secondary trace-back effect in discouraging upstream crime, (ii) without the culmination of downstream crime the social harm of upstream crime is negligible. Thanks to the cross-detection effects of enforcement, these crimes can be deterred at first-best cost through crime- and detection-based targets.

Finally, for two crimes linked by causality, the root (cause) crime also gains priority in budget allocation over the effect crime. None of the detection measures is monotonic in effect crime's enforcement effort, which can further shift enforcement resources to the root crime when the State has to rely on detection-based incentives.

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Key Words: Decentralized crimes, interlinked crimes, crime chain, moral hazard, enforcement targets, crime data, detection data, budget allocation, limits of enforcement, Say's Law.

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1 Introduction

Every fiscal year federal and state governments branch out enormous sums to local and specialized law enforcement units, who in turn justify their budget demands by formulating verifiable performance targets. NYPD, for example, plans to spend in 2019 $1,596 million on patrol services, $569 million on its detective bureau, $287 million on its school safety division, $245 million on its transit bureau, $219 million on the transportation bureau and $149 million on the Citywide Operations. Budgets are smaller for special task units, like the additional $2.8 million spending in 27 new positions for the Co-Response Teams aimed at violence prevention and reaching individuals with mental health or substance abuse problems.[1] For the central management the question is, what apparent results should (or can) enforcement units deliver?

Moral hazard and accountability problems in law enforcement have led governments to develop verifiable correlates of performance, formulated as targets, which are becoming increasingly common and tied to agency budgets−a trend that is expected only to grow.[2] A large number of police departments issue their own activity and crime data, compiled at the state and national levels in annual reports assessing whether the targets are met. Some of these targets can be quite specific, like those in Garda Siochana's 2017 report stating that it failed to meet detection targets on burglary, robbery and assault, which fell respectively by 2.62%, 0.59% and 2.86%, whereas the 4.59% increase in detections of sexual offenses achieves the target. Besides indicators based on detection, apprehension and court conviction rates, one can find others based on crime levels measured by surveys or direct reports to the police.[3] We have limited knowledge about the power of these targets as incentives for law enforcement agencies operating in various, quite different, crime environments.

Crime environments are extremely diverse by their harm, nature, links and organization. Some crimes, like shoplifting and cybercrime, are unrelated, whereas large-scale drug smuggling and disparate small-scale drug peddling on street corners form an input-output chain. Selling drugs raises the benefit from other crimes like theft or other violent crimes due to the addiction and intoxicating effects of drugs. This diversity defies a unified approach to the budgeting and motivation of law enforcement activities. While it is unclear whether to set deterrence targets for independent crimes separately, curtailing one of two interrelated crimes should clearly affect the deterrence of the other. A relevant issue facing any government therefore is how to structure enforcement incentives for in-

[1]See the *Report of the Finance Division on the Fiscal 2019 Preliminary Budget* for the New York Police Department, The Council of the City of New York, March 12, 2018.

[2]Sherman (2013) traces the first major initiatives of statistical management in the UK to the Thatcher era. Police departments were ranked and assessed according to specific goals on the basis of key performance indicators. In the U.S., the Department of Justice Fiscal Years 2016-2017 priority goals include, besides specific national and cyber security targets (disruption of 400 terrorist groups or threats, dismantling of 1,000 cyber threat actors), five percent increases over Fiscal Year 2015 targets in the numbers of investigations concerning sexual exploitation of children and human trafficking by September 30, 2017; see U.S. Department of Justice (2016). In the fight against health care fraud, the Department formulates its success in monetary terms, by "$7.70 to $1 return on investment for law enforcement and detection efforts in Fiscal Year 2014."

[3]For example, the Metropolitan Police Department of DC (p12, 2004), Strategic Business Plan Key Results Measures, mentions ten measures: Percentage changes in the number of homicides, violent crime and property crime, besides clearance rates for homicides, forcible rape, robbery, aggravated assault, burglary, larceny-theft and motor vehicle theft.

dependent law enforcement agencies – through individualized incentives or a coordinated (or joint) incentives design? Does coordination imply the performance targets of one law enforcement division also depend on the performance of another division? That is, should the divisions be encouraged to compete or cooperate in controlling crimes? The literature we review in the sequel offers little guidance to the answers.

We consider a model with two independent law enforcement agencies each responsible for controlling one of two crimes. The output of each agency is an intensity of enforcement, i.e., probability of detection, produced by unobservable effort in combination with other resources. The objective of the State is to minimize total harm from crimes by allocating a budget of fixed size and designing incentives that include rewards to ensure that the agencies exert appropriate efforts and use their budgets most effectively. The analysis distinguishes between independent crimes and two cases of interlinked crimes. For the latter, in one variant commission of a root (cause) crime leads to an increase in the potential criminal population of another (effect) crime, and in a second variant the upstream crime supplies an indispensable input to a downstream crime. In this last environment, upstream criminals may be detected before matching with their downstream partners and hence before realizing their benefits, whereas downstream criminals commit the crime, and thus can be detected, after matching with an upstream partner. Deterrence of one of these crimes may create shortages on one side, reducing the matching prospect and the expected benefits for the other side. This creates positive enforcement externalities.

The need to motivate law enforcement brings in a second layer of distinction, according to the type of observable statistic available for that purpose, which depends on the crime. Some crimes, like open-air drug markets, are more visible than others like sexual crimes or corruption; in the case of the latter victims may not report the crime (e.g., in domestic abuse) for fear of retaliation. When a crime is not directly observable and its occurrence or non-occurrence can be ascertained by law enforcers only, the State may have to rely on detection or apprehension data to motivate the enforcement agency.[4] On the other hand, the combat against observable or predominantly reported crimes such as car theft admit both the crime data and the detection data.

Intuition may favor crime data over detection data in the provision of enforcement incentives for its apparent congruence with the harm-minimization objective of the State. Our analysis confirms the choice, if not the intuition: Crime-based incentive systems should be preferred because they satisfy a fundamental property for implementation of crime levels at first-best cost, namely, *monotonicity in enforcement effort.* We identify crime environments in which none of the detection-related data satisfy the effort monotonicity property. For the crime environments we consider in this paper, crime-based performance indicators weakly dominate those that are detection-based.

[4] Alternatively the State could use victimization surveys that indicate the extent of the unreported crime (in the United States, the FBI's Uniform Crime Reports and the National Crime Victimization Survey). It goes without saying, and we acknowledge, that the performance indicators could be subject to influence, even outright manipulation including flexibility in crime definitions, by the enforcement agencies themselves. However, safeguards against abuse seem to be operating with integrity, at least in the case of computerized statistics COMPSTAT (developed by William Bratton and Jack Maple in use in NYPD since 1996), as noted by Sherman (2013), in some instances leading to arrests of local commanders charged with manipulating crime statistics.

The potential limitation of detection-based incentives can be understood by visualizing the relation between total detected criminals and enforcement intensity (probability of detection), for an independent crime. The measure of detected criminals reaches a maximum in the probability of detection, falling to a minimal level in the limit if all criminals are sought to be detected with probability one. It follows that high enforcement efforts cannot be induced by setting a detection target because the agency can achieve the same detection target with a lower effort. Nor can the State prop up deterrence by relying on other operational supports subject to less stringent or no moral hazard constraints, for any such attempt will be continue to be upset by the agent further adjusting efforts downwards consistent with target detections (unless efforts and other operational supports are perfect complements). This highlights a feasibility problem in the combat against crimes for which the only data that can be produced are detections. In the case of an observable independent crime, if the State has a large budget with an ambitious crime reduction objective, performance targets of the agencies should be formulated in terms of crime levels, not detections. High deterrence levels are not compatible with detection-based incentives, due to moral hazard, though low deterrence levels can be induced just as effectively as under crime-based incentives.

Crime-based incentives continue to implement harm-minimizing crime levels for crimes linked by unidirectional causality or those forming a vertical input-output chain. These types of crime environments, however, also present a richer set of implementation possibilities for detection-based incentive systems. We allow for the possibility of tracing back criminals of root or upstream crimes from detection of downstream crimes, in addition to other cross-detection effects of enforcement that operate even without the possibility of backtracing. In the case of crimes that form input-output chain, we show that first-best enforcement incentives can be restored under detection-based systems provided upstream-crime detection data can be decomposed into its components as those purely owing to downstream enforcement and those due to upstream enforcement. In the case of causal links, i.e., when one root crime leads to an increase in the measure of potential criminals of another effect crime, root crime enforcement costs are first-best because, as we show, detections of the effect crime are monotonic in root crime enforcement. However, none of the detection measures are monotonic in effect-crime enforcement and therefore detection-based incentives are subject to the same limitation as in the case of independent crimes.[5]

The optimal budget allocation depends on enforcement costs, hence on the incentive systems and whether crime and/or detection data are available, as well as the relative harms from crimes and the criminal benefit distributions. In the reference case of symmetric independent crimes, it is optimal to allocate a larger enforcement budget to the crime that causes a larger harm, as expected, provided crime data is available for both crimes. In vertical crime chains we identify a structural mechanism which favors concentration of enforcement efforts on the upstream/root crime. *Undeterred* downstream criminals seek to match with *undetected* upstream criminals who

[5]We make the standard assumption that the detection probability affects, but is not affected by, the crime level−in order to focus on the structural factors determining the relative effectiveness of performance targets. If the crime level also affects the probability of detection, even crime measures may not be monotonic in enforcement effort (see Bar-Gill and Haren (2001), indicating the possibility of multiple crime equilibria.)

supply the instrument they need to complete the crime. Under conditions of symmetry, an equal distribution of the enforcement budget between the two crimes produces an excess demand for the instrument needed by *undeterred* downstream criminals. A fraction of these undeterred criminals will not be able to complete the crime. This *indirect deterrence* effect creates a tendency for the State to allocate a smaller budget to the downstream crime unless its social harm is substantially larger than the upstream crime. One possible exception to the above prescription is when the detection of downstream crime has an adversarial trace-back effect on the expected benefit from committing the upstream crime. There, spending a bigger proportion of the budget on downstream enforcement gains grounds.[6]

The tendency to favor the upstream root crime persists in the presence of a causal link between the crimes, qualified by the trace-back effect mentioned above. However, if crime data is not available, under detection-based incentives the State has an additional reason to allocate a larger budget to the root crime: Unless the enforcement budget and the target crime rate are small, detection-based incentives for the effect crime hit a roadblock feasibility problem.

■ **Related Literature.** The economics literature of prime relevance to our paper, formal analysis of incentives and budgeting in law enforcement, is small. The efficiency of crime-based incentives in coping with moral hazard was first pointed out by Harris and Raviv (1978) in a single crime context as an application of their agency model. Their analysis has not been extended any further than a few applications to specific enforcement contexts, notably Graetz et al. (1986) providing the first formal treatment of the incentive problem with an explicit objective for the agency enforcing tax compliance, and Besanko and Spulber (1989) who study the effort commitment problem in a game between the law enforcer and a representative criminal under the incentives designed by the state. A common ground of these models and ours is inclusion of the law enforcement agency as a separate decision maker. We follow their approach in taking criminal sanctions as exogenously given, to keep the focus on incentives and the allocation of enforcement resources.[7] The design of incentives when the enforcement agency can abuse delegated public authority for private gain has been studied by Polinsky and Shavell (2001), and notably by Echazu and Garoupa (2010) who consider two independent enforcement activities by one agency and show that corruption can distort the allocation of enforcement efforts in favor of the "lucrative" crime.

[6]The exact breakdown of the enforcement budget and how law enforcers' rewards should be designed will depend on whether overall detection of upstream crimes can be verifiably attributed to upstream and downstream initiatives or such a distinction is not feasible. In either environment, the externalities that exist between upstream and downstream deterrence can get aggravated or mitigated, if not completely internalized, based on how swiftly downstream detections may lead to its upstream root, from zero link to a complete capture. The details of the strategic interactions between the two layers' enforcement efforts are discussed in Section 4.

[7]Sanctions determined by the legislative branch of government can by and large be taken as exogenous for the executive branch and its law enforcement agency. Sanctions are bounded from above by fairness principles and constitutional requirements, but also by efficiency considerations such as minimizing the costs of errors in adjudication, coping with corruption and issues of marginal deterrence (see Shavell, 1992; Mookherjee and Png, 1994; Polinsky and Shavell, 2000), leaving the intensity of enforcement as the main policy instrument for the executive branch in the control of crimes. The optimal combination of sanctions and detection probabilities has been the subject of a large literature starting with Becker (1968), followed by Becker and Stigler (1974) and, to cite a few, Shavell (1991) and Mookherjee and Png (1992).

While research on the efficiency of public enforcement resources, predominantly by criminologists (e.g., Sherman, 2013, and the references therein), concentrated on allocations according to activity type such as imprisonment, policing and prevention, the impact of crime-based re-allocation of expenditures has been studied in one instance, to the best of our knowledge, the Comprehensive Crime Act of 1984 allowing police agencies to keep the proceeds of assets forfeited in drug enforcement activities. Benson et al. (1995) discuss the response of law enforcement to changes in incentives, arguing that the resulting resource shift from non-drug crime such as burglary to drug crimes has increased drug enforcement efforts, but with a pessimistic view of the results on crime. Their results are refined in Baicker and Jacobson (2007) who find that the policy has led to higher drug prices due to shortened supply. In our vertical crime-chain model a budget shift from downstream to upstream crime enforcement also leads to an increase in downstream crime, but the effect is less pronounced than if the crimes were independent. The resource shift reduces the probability that downstream criminals complete their crime, due to shortages of supply from upstream crime.

This last environment, input-output crime chains, can be a fertile ground for criminal organizations. In many crime chains organized and decentralized segments co-exist, for example, specialized gangs and individual burglars who sell the guns they stole to other potential criminals or to middlemen. The growing literature on organized crime has a branch that studies quasi-governmental models of gangs (Garoupa, 2000, Mansour et al., 2006; Chang et al., 2013) with emphasis on enforcement policy when a criminal organization responds to it by modifying its size and defensive strategy. The other branch models organized crime as networks, with implications on its members' detection probabilities (Ballester, Calvo-Armengol and Zenou, 2006; Baccara and Bar-Isaac, 2008; Goyal and Vigier, 2014).[8] Generally, if a criminal is caught then anyone connected to the criminal also risks being caught with a high probability in a follow-up investigation. In contrast to these integrated crime organizations, in our decentralized crime setup upstream criminals can be traced back with positive probability from detection of the downstream criminal with whom they interacted whereas interception of an upstream criminal (before any transaction) does not change the detection probability for downstream criminals. This asymmetry, we show, has implications on budget allocation as well as the choice of the incentive system.

The core model is presented in the next section. The case of independent crimes is analyzed in Section 3, and the interlinked crimes appear in Sections 4 and 5, followed by Conclusion.

[8] Ballester, Calvo-Armengol and Zenou (2006) define a *key player* based on a notion of intercentrality measure reflecting network payoff externalities, whose removal should weaken significantly the network's functionality. Baccara and Bar-Isaac (2008) study repeated interactions between an external authority and a criminal organization. The authority's objective is to restrain cooperation between members, to which the organization responds by modifying its internal information exchange structure. They show that the organization should arrange either isolate binary cells (allowing mutual exchanges) or unlinked agents or at most one hierarchy with a central agent, whereas the State may employ a symmetric or hierarchical scrutiny (i.e., some agents monitored more than others). In Goyal and Vigier (2014), an attack/defense contest is framed between a single Designer and a single Attacker, with the former considering what type of network to form and how to allocate resources between defending different parts of the network. The Attacker then decides which subset of nodes or links to attack. The main result is that in most situations the Designer should organize a 'star' network with all defence resources concentrated on the central node.

2 Model and preliminaries

Consider two crimes, crime A and crime B. There are four actors, the State acting as the principal, two law enforcement units, and the citizens. The State, in its executive capacity, decides on the budgets, rules and incentives for law enforcement. Crime A and crime B are targeted by separate units. Each unit would be composed of police staffs managed by a chief law enforcement officer, whom we call '*the agent*'. Agent i is delegated the task of controlling crime $i = A, B$. The social harm from crime i is denoted by $h_i > 0$.

A fixed measure of population make up the potential criminals, with one-half prone to committing crime A only and the other half, crime B only. Each group size is normalized to one.

■ **Crimes**. Crimes will be classified according to two criteria.

(i) Observability/measurability. We say that a crime is (ex-post) *observable* if it leaves a physical mark of its occurrence behind, such as a victim, a witness or a property damage. Burglary, assault, robbery or hate speech are examples. A crime is (ex-post) non-observable otherwise; the occurrence or non-occurrence of this kind of crimes is almost unidentifiable unless detected by law enforcers. Bribery, smuggling, drug trafficking ensuring steady supply of drugs or taking drugs making up the demand side, are examples. Admittedly, crimes may differ in observability along a continuum (say, because victims are more likely to report some crimes than others); here we consider a binary classification for simplicity.

(ii) Interdependence. The second criterion relates to whether or not the two crimes form an activity chain. Crime A may be an indispensable input for crime B, as is transportation of illicit drugs from production sites, including smuggling through border controls, to city ghettos for sales to final users. Trade in unlicensed or illegal guns feeding other crimes in which they would be used, say, in assault or murder attempt, and human traffickers who supply labor to the informal black markets such as under-age labor, prostitution are other examples.

■ **Criminals**. Utility of not committing a crime is normalized to zero. Potential criminals of crime i derive a positive private benefit, b, from committing crime i only, distributed according to a continuous cdf $F_i(.)$ with support $[0, \bar{b}]$ and a continuous density function $f_i(.)$, strictly positive in this domain. We denote by s_i the sanction on crime i and assume it can be administered costlessly.

■ **The Agents**. Agent i, acting as the head of the enforcement unit specializing in crime i, determines an effort e to manage and organize his own unit. We assume that this effort is unobservable, hence not contractible. The cost of effort, $z(e)$, is increasing, differentiable and strictly convex in e, with $z(0) = z'(0) = 0$.

The agents' outside options are zero. An incentive system $r \in \{C, D\}$ in enforcement remunerates the agent according to a verifiable indicator, which in system $r = C$ is the measure of crime (or crime rate) and in system $r = D$ the level of detections (or, apprehensions). Denoting agent i's rewards under system r by w_i^r, his final utility is

$$w_i^r - z(e).$$

Although w_i^r cannot be conditioned on e, it will depend on the number of detections or crime data that reflects the agent's effort.

Data availability makes system D a more feasible option than system C, which can be used only for (ex-post) observable crimes if all incidences are reported and recorded; alternatively, the State may conduct periodic surveys to generate information about the level of crime and use it as a statistic in compensating its law enforcement agent.

■ **The State**. The State will allocate an exogenous budget R between the two enforcement agents, $R = R_A + R_B$. It also chooses an incentive system $r \in \{C, D\}$ under which each agent's expected reward payments Ew_i^r, together with the cost of other operational enforcement resources R_i^X, cannot exceed R_i. The State's objective is to minimize total expected harm from crimes.

■ **Enforcement technology**. Agent i's law enforcement efforts together with the budget R_i^X for all other enforcement resources, e.g., infrastructure, personnel, operational inputs produce an unverifiable probability of detection per criminal, $\mu_i = \mu(e, R_i^X)$, also referred as the "intensity of (crime i) enforcement".

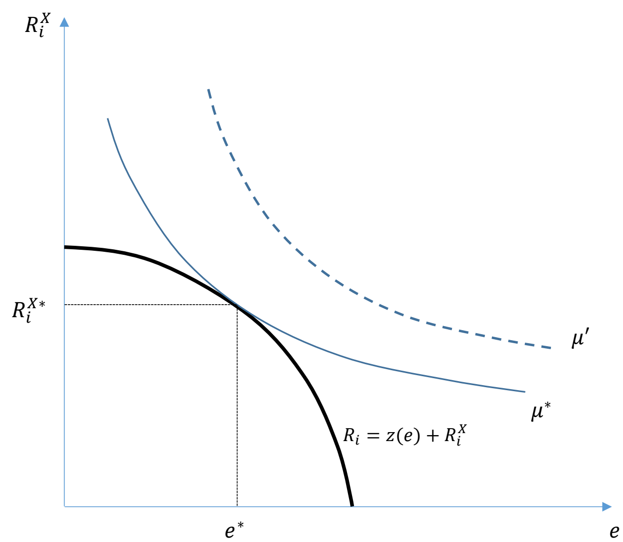

Figure 1. The optimal (R_i^{X*}, e) combination given budget R_i, inducing the maximal feasible detection probability μ^*.

Assumption 1. *The detection probability, $\mu_i(e, R_i^X)$, is increasing in each of its arguments and differentiable, with $\mu_i(0,0) = 0$, $\frac{\partial \mu_i(0, R_i^X)}{\partial e} = \infty$ and $\frac{\partial \mu_i(e,0)}{\partial R_i^X} = \infty$. Moreover, $\mu_i(e, R_i^X)$ is strictly concave, i.e., $\mu_i(\alpha * (e, R_i^X) + (1-\alpha) * (\tilde{e}, \tilde{R}_i^X)) > \alpha \cdot \mu_i(e, R_i^X) + (1-\alpha) \cdot \mu_i(\tilde{e}, \tilde{R}_i^X)$ for any $(e, R_i^X) \neq (\tilde{e}, \tilde{R}_i^X)$ and $\alpha \in (0,1)$.*

As illustrated in Fig. 1, the upper-contour sets defined by μ_i-levels are strictly convex. In the hypothetical scenario of efforts perfectly observable and contractible, for any given enforcement budget R_i for unit i, *feasible combinations* (e, R_i^X) are defined by the set $\mathcal{F}(R_i) = \{e \geq 0, R_i^X \geq$

$0|z(e) + R_i^X \leq R_i\}$. Because the effort cost function $z(.)$ is strictly convex, the boundary of this set defined by $z(e) + R_i^X = R_i$ is strictly concave.

The first-best input combination (e^*, R_i^{X*}) maximizing μ_i in the feasible set $\mathcal{F}(R_i)$ will be unique. Note also that by duality, R_i is the minimum budget required to induce the detection probability $\mu_i^* = \mu(e^*, R_i^{X*})$, so $R_i = c(\mu_i^*)$ $(= z(e^*) + R_i^{X*})$. Now using strict concavity of $\mu(e, R_i)$ (Assumption 1) and $z(0) = z'(0) = 0$, the following result is immediate (see, for example, Proposition 6.11, part (ix) of Manove (2005) Lecture Notes):

Lemma 1 (First-best enforcement)**.** *Under effort contractibility, first-best enforcement effort minimizing expected harm from crimes will be $e^* > 0$ for all $R_i > 0$. The enforcement cost function*

$$c(\mu_i) = z(e^*) + R_i^{X*} \tag{1}$$

is increasing and strictly convex in μ_i, with $c(0) = 0$ and $c'(0) = 0$.

3 Independent crimes

Committing crime i yields the benefit b if undetected, $b - s_i$ if detected and punished. Thus, a potential criminal commits the crime under incentive system $r \in \{C, D\}$ if

$$b > \mu_i s_i = b_i^r. \tag{2}$$

There are two ways to incentivize law enforcement. Under crime-based incentives, agent i's reward can be made contingent on the *level of (crime) deterrence*:

$$\sigma_i = F_i(\mu_i s_i). \tag{3}$$

Under detection-based incentives, the reward is contingent on the *level of (crime) detections*:

$$d_i = \mu_i(1 - F_i(\mu_i s_i)). \tag{4}$$

The difference between (3) and (4) is that in the former crimes can be controlled directly by linking higher rewards to higher μ_i whereas in the latter such direct linking to μ_i might not be possible due to the non-monotonicity of the $d_i(\mu_i)$ function. By Weistrass Theorem there will be a maximal detection $d_i^{\max}$ and a corresponding maximizer $\widehat{\mu}_i$. But the $d_i(\mu_i)$ curve can be of any shape depending on the form of the benefit distribution function $F_i(\cdot)$. To simplify the analysis we will focus on:

Assumption 2. *$d_i(\cdot)$ is single-peaked.*

Single-peakedness will be guaranteed for non-increasing benefit density functions such as the uniform and exponential;[9] see Fig. 2a. For any arbitrary level of detection $0 < d < d^{\max}$ there will be two μ_i's associated with it.

Definition 1. *For any $d \in [0, d_i^{\max}]$, let*

$$\mu_i(d) = \min\{\mu_i | d_i(\mu_i) = d\}$$

under the incentive system D. Further, for $d = d_i^{\max}$, denote

$$\widehat{\mu}_i = \mu_i(d_i^{\max}).$$

Finally, define

$$\widehat{b}_i = \widehat{\mu}_i s_i, \quad \text{and} \quad \sigma_i^{\max} = F_i(\widehat{b}_i).$$

Note that under detection-based incentives the State should never target implementing any μ_i in excess of $\widehat{\mu}_i$, for cost efficiency reasons. In fact, impossibility of implementing such μ_i's will be established in Lemma 3.

System C. Agent i is rewarded according to some crime deterrence target set. Let us return to Fig. 1. We know that for any μ_i the cost-minimizing solution under effort contractibility is given by the unique (e^*, R_i^{X*}) pair, resulting in cost $c(\mu_i)$ as in (1). The following simple *all-or-nothing* reward mechanism will induce the agent to exert e^* when effort is unobservable (using (3)) :

$$w_i^C(\sigma_i) = \begin{cases} z(e^*), & \text{if } \sigma_i \geq F_i(\mu_i s_i) \\ 0, & \text{otherwise.} \end{cases} \tag{5}$$

Failing to meet the target crime level and receiving zero reward can be interpreted as the agent being replaced or denied a promotion. We assume the agent will break the indifference in favor of effort e^* in accordance with the preference of the State.

Lemma 2. *Under crime-based incentives, $c^C(\mu_i) = c(\mu_i)$ for all μ_i as in (1).*

The State does not concede any moral hazard rent under crime-based incentives, inducing the first-best effort choice. This is a standard result in contract theory – no efficiency loss in the presence of a sufficient statistic capturing hidden agent effort (e.g., Harris and Raviv, 1978).

System D. Law enforcement officer is rewarded according to the achieved level of detections. First observe from Fig. 2a that for any $\mu_i \leq \widehat{\mu}_i$ there is a unique $d_i(\mu_i)$ that can be set as a target and the principal can implement μ_i at minimal cost by choosing (e^*, R_i^{X*}) at the tangency between the

[9] If F_i is uniform over $[0, \bar{b}]$, $d_i(\mu_i) = \mu_i \left(1 - \frac{\mu_i s_i}{\bar{b}}\right)$ is initially increasing and then decreasing with a global peak $d^{\max} = \frac{\bar{b}}{2s_i}$ at $\widehat{\mu}_i = \frac{\bar{b}}{2s_i}$.

For exponential distribution over $(0, \infty)$, i.e., $F_i = 1 - e^{-\lambda b_i}$, $d_i(\mu_i) = \mu_i e^{-\lambda(\mu_i s_i)}$ will have the slope $\frac{dd_i}{d\mu_i} = e^{-\lambda(\mu_i s_i)}\Big[1 - \lambda \mu_i s_i\Big]$, implying d_i achieves its global maximum at $\widehat{\mu}_i = \frac{1}{\lambda s_i}$ (with $\frac{d^2 d_i}{d\mu_i^2} = -\lambda s_i e^{-\lambda(\mu_i s_i)} < 0$).

See also the discussion in the Appendix on the property of the $d_i(\cdot)$ function.

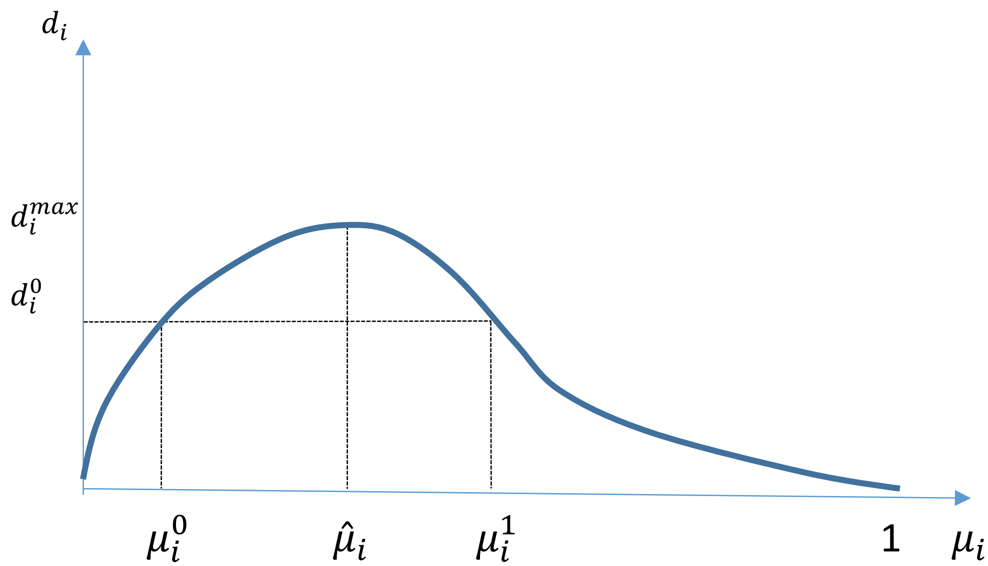

Figure 2a. Behavior of measure of detections as a function of the probability of detection. Same measure d_i^0 of detections is induced by the detection probabilities μ_i^0 and μ_i^1.

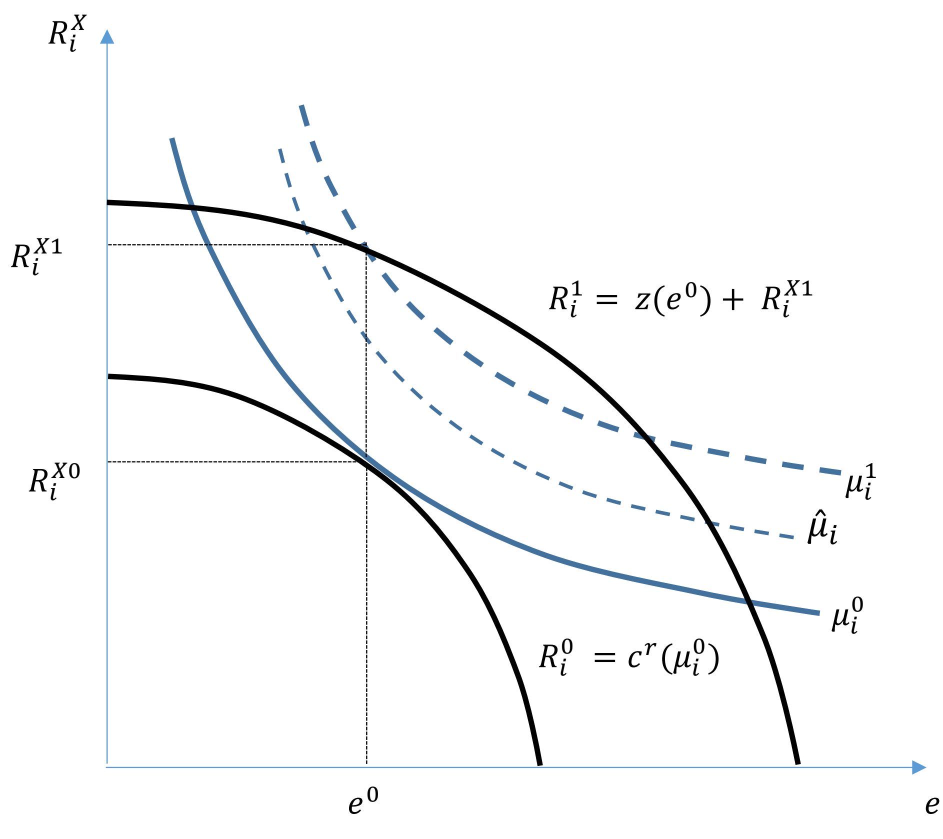

Figure 2b. The detection probability μ_i^1 can be induced under system D by the effort e^0 combined with other enforcement resources R_i^{X1}, at the cost $R_i^1 = z(e^0) + R_i^{X1}$, which is larger than the minimum cost $c(\mu_i^1)$.

R_i-curve and the μ_i-level curve as in Fig. 1. Inducement of agent i's effort e^* is done by choosing the following reward (using (4)):

$$w_i^D(d_i) = \begin{cases} z(e^*), & \text{if } d_i \geq \mu_i(1 - F_i(\mu_i s_i)) \\ 0, & \text{otherwise.} \end{cases} \tag{6}$$

The overall implementation cost is then given by

$$c^D(\mu_i) = c(\mu_i), \tag{7}$$

where the RHS is as in (1). The agent earns zero moral hazard rent.

However, for any $\mu_i > \widehat{\mu}_i$ (say μ_i^1 in Fig. 2a), there is a unique $d_i = \mu_i(1 - F_i(\mu_i s_i))$ associated with this μ_i (d_i^0 in the figure), which is also associated with at least one other μ_i below $\widehat{\mu}_i$ (μ_i^0 in the figure). The smaller μ_i^0 is incentive compatible with the unique d_i^0 whereas the higher μ_i^1 is not. In fact, the effort e^1 in any input combination (e^1, R_i^{X1}) that generates μ_i^1 cannot be induced by rewarding the agent for detecting at least d_i^0 criminals. There will always exist a lower effort, call it e^0 (Fig. 2b), which, combined with the resources R_i^{X1}, generates the same measure of detections d_i^0 but through a lower probability of detection, μ_i^0, as in Fig. 2a. It follows that the State cannot implement an enforcement input combination on the μ_i^1 schedule.

With the agent thus deviating to the minimal effort e^0 that meets the target $d_i = \mu_i(1-F_i(\mu_i s_i))$ associated with the specific $\mu_i > \widehat{\mu}_i$ and thus generating the lower $\mu_i^0 < \widehat{\mu}_i$ instead, it becomes impossible to implement any detection probability above $\widehat{\mu}_i$, hence any deterrence target above $\sigma_i^{\max}$ under system D. We obtain the following result:

Lemma 3. *Under detection-based incentives,*

$$c^D(\mu_i) = c(\mu_i) \quad \textit{for} \quad \mu_i \leq \widehat{\mu}_i, \tag{8}$$

whereas $\mu_i > \widehat{\mu}_i$ cannot be implemented.

We can now formally present the State's harm minimization problem $(\mathcal{P})$ as follows:

$$\min_{r \in \{C,D\}, R_A, R_B} \quad [1 - F_A(b_A^r)]h_A + [1 - F_B(b_B^r)]h_B, \tag{9}$$

$$\text{subject to} \quad b_i^r = \mu_i s_i, \tag{10}$$

$$R_i = c^r(\mu_i), \tag{11}$$

$$R_A + R_B \leq R, \tag{12}$$

where all μ_i's are admissible under system C whereas only the restricted range $\mu_i \leq \widehat{\mu}_i$ is admissible under system D.

Condition (12) is the budget feasibility requirement.[10] Condition (11) ensures that the budget R_i is efficiently divided between rewards and other operational support to generate the maximal

[10]Budget restrictions are in expectation with the understanding that sometimes the hard budget constraints may

μ_i possible, that in turn keeps deterrence b_i^r at a high level through (10). The cost functions $c^r(\mu_i)$ have already been solved in Lemmas 2 and 3, and (10) earlier appeared in (2).

We approach the problem ($\mathcal{P}$) by breaking it down into two parts: first whenever any incentive system $r \in \{C, D\}$ is an option translate the corresponding maximal achievable μ_i for any given R_i, then determine the optimal budget allocation together with a pair of incentive systems.

Definition 2. *Fix r and R_i, and recall $c^r(\mu_i)$ as constructed above. Using monotonicity of $c^r(.)$ in the admissible range of μ_i and (11), define the following:*

(i) *With slight abuse of notation, let $\mu^r(R_i) = c^{r^{-1}}(R_i)$ be the maximal detection probability that can be induced with budget R_i under the incentive system r.*

(ii) *Accordingly, using $\mu_i = \mu^r(R_i)$ in (10), let $\sigma^r(R_i) = F_i(\mu^r(R_i)s_i)$ be the level of deterrence and $d^r(R_i) = \mu^r(R_i)(1 - F_i(\mu^r(R_i)s_i))$ the corresponding measure of detections that can be induced with budget R_i.*

Observe that, in the admissible range of μ_i, $\mu^r(R_i)$ is increasing and strictly concave in R_i, the latter property following from Lemma 1–3, and bounded above by $\widehat{\mu}_i$ for $r = D$ (see Fig. 3). Thus, $\sigma^r(R_i)$ is also increasing in R_i, but bounded above by $\sigma_i^{\max}$ for $r = D$.

We can now translate our results in Lemmas 2 and 3 to a summary comparison between two incentive systems – crime deterrence vs. detections – for any specific budget allocation (R_A, R_B).

Rewrite the optimal incentives (5) and (6) from the previous section translating conditions in terms of μ_i to parallel conditions in terms of R_i, using monotonicity of $c^r(\mu_i)$, as:

$$w_i^r(\sigma_i) = \begin{cases} z(e^*(R_i)), & \text{if } \sigma_i \geq \sigma^r(R_i) \\ 0, & \text{otherwise,} \end{cases} \tag{13}$$

whereas incentives in (6) are transformed as:

$$w_i^D(d_i|\mu_i) = \begin{cases} z(e^*(R_i)), & \text{if } d_i \geq d^D(R_i) \text{ and } \mu_i \leq \widehat{\mu}_i \\ 0, & \text{otherwise.} \end{cases} \tag{14}$$

Note that detection probabilities $\mu_i > \widehat{\mu}_i$ cannot be induced via system D, thus excluded from (14). Also, the agent's effort is always unconstrained, i.e., corresponds to the point of tangency between the budget set and the implemented μ_i-level curve in Fig. 2b, in the determination of agent rewards in (13), and (14) for $\mu_i \leq \widehat{\mu}_i$. Proposition 1 summarizes the analysis of incentives under the two systems.

Proposition 1 (Limitation of detection-based system)**.** *Consider an independent crime. There exists a deterrence level $\sigma_i^{\max}$ and a corresponding detection probability $\widehat{\mu}_i$ such that:*

be violated. So long as on average total law enforcement costs do not exceed some limits by a big margin over a planning horizon, councilors and city chiefs can tide over modest shortfalls by raising local taxes.

(i) *Under system C all deterrence targets σ_i up to $\min\{1, F_i(s_i)\}$, and under system D deterrence targets $\sigma_i \leq \sigma_i^{\max}$, can be implemented at the first-best cost $c(\mu_i)$ as in (1).*

(ii) *Deterrence targets $\sigma_i > \sigma_i^{\max}$ cannot be implemented under system D.*

Overall, a simple rule implied by Proposition 1 is that crime-based incentives should be used to motivate law enforcement, whenever feasible. Part (i) identifies when systems C and D are interchangeable – for low enforcement budgets inducing high crime targets only. For large enforcement budgets and high deterrence targets as in part (ii), the corresponding large detection probabilities cannot be induced through detection-based incentives: giving it a hard try makes the actual criminal pool and, hence detections, thinner, but a lower effort also generates the same level of detections, so conversion into actual rewards hits a roadblock (Fig. 2b). Obviously, If the sanction s_i is so small that $F_i(s_i) < 1$, then deterrence targets $\sigma_i > F_i(s_i)$ cannot be implemented even if $\mu_i = 1$, under system C. To eliminate this analytically uninteresting case in the rest of the paper we shall assume $F_i(s_i) = 1$, $i = A, B$.

Observe that the critical deterrence measure $\sigma_i^{\max}$ is determined by enforcement costs $c(\mu_i)$ and benefit distribution $F_i(.)$. To widen the range of implementable deterrence of independent crimes under system D the State may improve the enforcement technology and/or implement social programs that "shift down" the criminal benefit distribution function.[11]

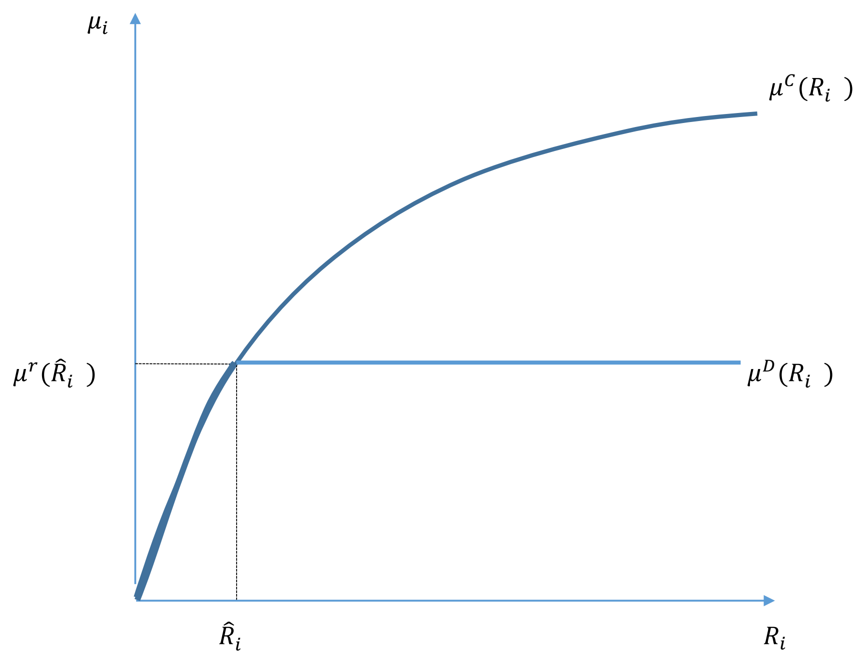

Figure 3. The maximal detection probability that can be induced with budget R_i, under systems C and D.

■ **Budget allocation**. We now turn to the State's objective in (9), the optimal allocation of the budget R between two law enforcement units. Our task is simplified, given Proposition 1: system

[11]Direct employment generation programs in the Keynesian mould or improving human capital through education are some examples, e.g., Brian and Lefgren (2003), Lochner and Moretti (2004).

C incentives will be used whenever crime data is available, system D will be used otherwise, by producing detection data.

Suppose, first, that both crime data are available. Under crime-based incentives (13) the optimal allocation (R_A^*, R_B^*) will equalize the marginal expected harms. First-best is achieved at two levels: allocating resources efficiently across crimes and within each crime, conceding zero moral hazard rent to the enforcement agents. The implemented crime levels are $1 - \sigma(R_i^*) = 1 - F_i(\mu^C(R_i^*)s_i)$, the lowest feasible crimes possible for budget R_i^* (by Proposition 1).

Formally, the State will choose R_A and R_B such that $R_A + R_B = R$, minimizing

$$[1 - F_A(\mu_A s_A)]h_A + [1 - F_B(\mu_B s_B)]h_B, \text{ where } \mu_i = \mu^C(R_i).$$

Thus (R_A^*, R_B^*) will satisfy the first-order condition,[12]

$$f_A(\mu^C(R_A^*)s_A)[\mu^{C\prime}(R_A^*)s_A]h_A = f_B(\mu^C(R_B^*)s_B)[\mu^{C\prime}(R_B^*)s_B]h_B. \tag{15}$$

Since $c^{r\prime}(0) = 0$ by Lemma 1, $\mu^r(R_i)$, the inverse of $c^r(\mu_i)$, will satisfy $\mu^{r\prime}(0) = \infty$, implying $R_i^* > 0$. Given a pair $(h_A, h_B) \gg 0$, social harms minimizing division of enforcement budget, (R_A^*, R_B^*), will be unique if the distribution of criminal benefits is concave, that is, if the density $f_i(.)$ is non-increasing (implying that each side of eq. (15) is increasing in R_i.) In a symmetric crime environment, i.e., if $F_A(.) = F_B(.)$, $s_A = s_B$, the crime causing a larger social harm receives a larger enforcement budget.

With a larger total enforcement budget R, the budget allotted to each agent's unit will be raised, along an "expansion path" which we denote by $r(R)$. The schedule $r(R)$ delivers a unique pair of optimal budgets (R_A^*, R_B^*) given R, such that $R_A^* + R_B^* = R$. Under crime -based incentives for both crimes, this expansion path is an increasing schedule in the (R_A, R_B)-plane.

When, say, crime A data is not available or the crime measure from surveys are not of admissible quality, enforcement can only be motivated by tying rewards to detections. Incentive system D will be used for crime A, incentive system C will be used for B. From Proposition 1 we know that it is impossible to raise the probability of detection above $\widehat{\mu}_A = \mu_A(\widehat{R}_A)$ by raising the budget above $\widehat{R}_A$. It follows that any budget $R_A > \widehat{R}_A$ has no impact on crime A deterrence. Because the budget components of the expansion path are each increasing in R, there exists a budget $\bar{R}$ such that $r(\bar{R}) = (\widehat{R}_A, R_B^*)$. Thus, if $R > \bar{R}$, the entire budget in excess of $\bar{R}$ should be allotted to crime B enforcement, whereas for $R \le \bar{R}$ the budget allocation is unconstrained, given by (R_A^*, R_B^*).

[12]Second-order condition,

$$\Big[- f_A'(\mu_A s_A)\big(\mu_A'(R_A)\big)^2 - f_A(\mu_A s_A)\mu_A''(R_A)\Big] s_A h_A$$
$$+ \Big[f_B'(\mu_B s_B)\big(\mu_B'(R - R_A)\big)^2 \cdot (-1) + f_B(\mu_B s_B)\mu_B''(R - R_A) \cdot (-1)\Big] s_B h_B > 0,$$

will be satisfied if $f_i'(b) \le 0$ (recall μ_i is strictly concave by Assumption 1). Cite Bagnoli and Bergstrom (2005) on log-concavity of the density function $f_i(b)$.

Finally, if the state has no choice but to rely on detection data for both crimes, it can only implement the feasible detection probability range $\mu_i \leq \widehat{\mu}_i$. The budget allocation for the feasible range will be optimal, satisfying eq. (15). Define $\bar{R}_m$ as the *lowest* total budget under which $R_i^* = \widehat{R}_i$, inducing $\widehat{\mu}_i$, for one of two crimes. Suppose it is crime A. With a total budget exceeding $\bar{R}_m = \widehat{R}_A + R_B^*$, the state will allocate the marginal budget to crime B enforcement, for any budget beyond $\widehat{R}_A$ has zero impact on crime A. It will continue to do so until $R_B = \widehat{R}_B$. It follows that under detection-based incentives, the total enforcement budget should not ever exceed an upper limit defined by $\bar{R}_M = \widehat{R}_A + \widehat{R}_B$. A budget $R > \bar{R}_M$ cannot have any deterrence effect; it should be used for other projects.

Proposition 2 (Second-best budget allocation)**.** *(i) Expected social harms are minimized by allocating resources uniquely, through eq. (15), (R_A^*, R_B^*), under*

- *all budgets R if system C can be used for both crimes;*
- *budgets $R \leq \bar{R}$ if system C can be used for one crime and system D is used for the other;*
- *budgets $R \leq \bar{R}_m$ if system D is used for both crimes, where $\bar{R}_m \leq \bar{R}$.*

(ii) When system C can be used for one crime and system D is used for the other, any excess budget $R - \bar{R}$ is allotted to the crime enforced under system C.

When system D is used for both crimes, budgets above $\bar{R}_m$ and up to $\bar{R}_M$ are fully allotted to the crime with remaining feasible deterrence range. Budgets above $\bar{R}_M$ have no deterrence impact.

One striking implication is that the State may allocate much more resources to the prevention of burglaries and breaking of bank tills, which only transfer values from rich to the poor, than for the detection of small-scale individual drug peddlers that may destroy many young lives. Burglaries and bank-tills theft are observable crimes; they are reported and thus recorded by the relevant affected parties, whereas a fraction of drug peddling will inevitably escape detection and thus never documented. The social inefficiency that results from crime non-observability can be quite significant in particular if the enforcement budget is large. For small enforcement budgets and modest deterrence targets, Proposition 2 implies detection-based incentive systems work just as effectively as crime-based systems.

4 Interlinked crimes: crime A an input into crime B

"Supply creates its own demand" – the famous quote that goes under the heading called 'Say's Law' (Baumol, 1999). For crimes that form an input-output chain, it is unclear where should enforcement start: at the top or bottom of the chain? An abundant supply of guns, one might argue, fosters the demand for guns and gun crimes. Illegal immigrants are forced into prostitution and slavery or working at black market wages in hazardous jobs without adequate training, or resort to street crimes.[13] Smuggling of substantial amount of banned drugs (heroin, cocaine etc.) eventually find their ways to underground drug users and fuel drug addiction. Illegal wildlife products such as ivory and rhino horn are sold in markets in south-east Asia, to feed which elephants and rhinos are routinely killed by poachers.[14]

When one crime supplies the instrument to potential criminals of another crime, enforcement policies become interdependent even if each crime deterrence strategy may be overseen by different law enforcement departments, for example, the border-control department and the city police anti-crime branch. Suppose crime A precedes crime B and, if undetected, provides the instrument for undeterred B-criminals to act. As such there is no guarantee that a successful implementation of crime A would culminate into a match with a B-criminal. A B-criminal must find criminal A to execute his plan. The probability of matching, as in any decentralized market, will depend primarily on the relative sizes of the two populations to be matched.

Apportioning social harms $h_A > 0$ and $h_B > 0$ separately to crime A and crime B is a way to reflect the seriousness with which the authorities might view the distinct parts of the crime production process. Alternatively, we could assign a single harm $h > 0$ on committing crime B (or A) and treat the other crime as an indispensable step to crime B (or A).[15] For cross-border crimes, law enforcement has to invest in monitoring at check points, intelligence and cooperating with the country where the supply originates, all of which we lump together under *enforcement* A. Then a separate law enforcement division monitoring inside the country will detect illegal residents and the related crimes such as prostitution, extortionary employment, street drug selling, sale of contraband goods (ivory, brand name cigarettes, wines) etc that are undocumented. This latter we label as *enforcement* B.

Before proceeding to the formal analysis we should emphasize that the approach here is very different from the crime networks literature (e.g., Ballester, Calvo-Armengol and Zenou, 2006; Baccara and Bar-Isaac, 2008; Goyal and Vigier, 2014). Ours is a decentralized matching mechanism between the perpetrators of crime, whereas the network approach is predominantly one of bilateral/far-sighted coordination among criminals. The question of optimal enforcement response in our formulation should thus be of independent interest.[16]

[13] See the reports on trafficking (illegal immigration) of women in the UK and women and children in the USA; https://www.theguardian.com/uk/2005/nov/02/immigration.ukcrime and https://www.oas.org/en/cim/docs/Trafficking-Paper%5BEN%5D.pdf (a study by *Inter-American Commission of Women*, an inter-governmental agency).

[14] BBC report, 26 August 2018, https://www.bbc.com/news/world-africa-45288429.

[15] For instance, transporting drugs from one location to another may not per se cause much harm, except facilitating the sales to final users, which is quite harmful. Or, if selling a rhino horn is not harmful, killing a rhino for its horn is quite harmful.

[16] The distinction between decentralized matching and bilateral/far-sighted coordination can be understood in the following example. In the latter, a player may connect to another player seeing the benefits of both the direct link with the player and the secondary benefits derived from links to other players the connected player generates; see, for example, Jackson and Wolinksy (1996). This way many players may be connected only to a star player, in a 'star' network. So enforcement by taking out the star criminal can weaken/destroy the entire criminal gang. In contrast, in our decentralized environment taking out a single (or small fraction of) A-criminal(s) or B-criminal(s) does not destabilize the matching. Instead, enforcements have to be at aggregate levels in different parts of the crime possibilities, as opposed to thinking about the interaction between enforcement directed at certain links of the criminal network

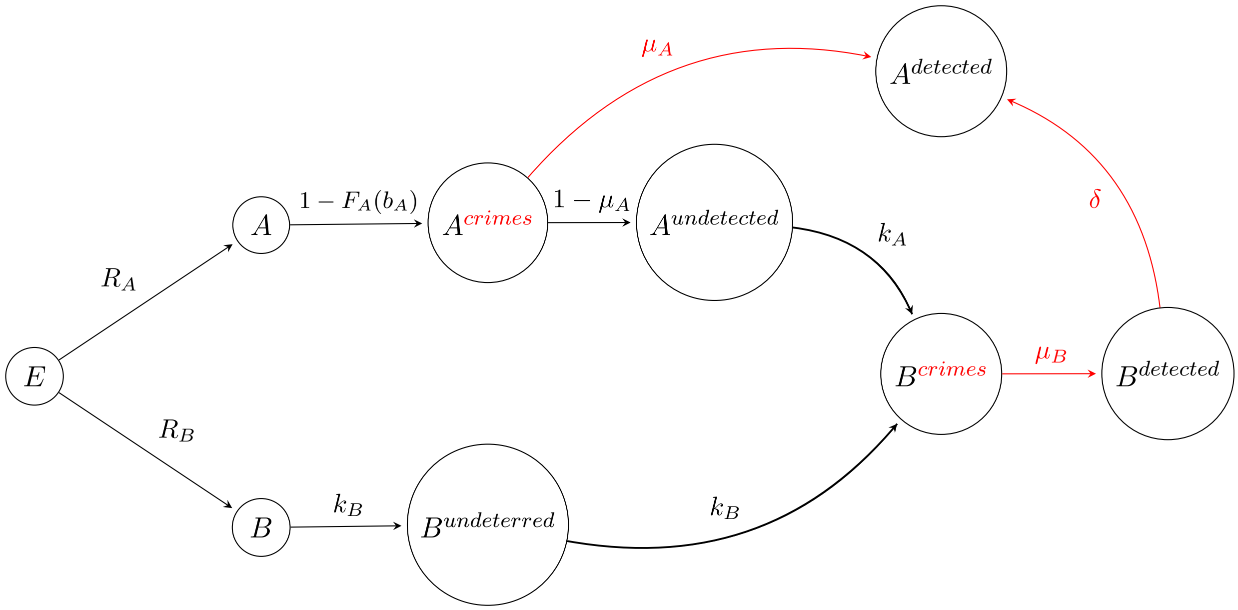

Figure 4: Crime chain; $k_A < k_B$ (A on short side), $k_A > k_B$ (B on short side) resulting in $\min\{k_A, k_B\}$-measure of the complete chain of (AB)-crimes. Given that it takes the two types of criminals to complete the final crime, B, either end may be considered as pivotal, although crime initiatives start with A-crime. This is particularly true in the decentralized environment with observable allocation of enforcement efforts, (R_A, R_B), as the only means of coordination. B's detection leads to A's detection with probability $0 \leq \delta \leq 1$.

■ **Sequence of events.** Following the State's decisions on budget allocation and enforcement incentives that are publicly announced, various parties choose their actions in the following order (see Fig. 4):

(i) Agent i determines effort e_i, hence μ_i, while potential criminals choose between compliance (deterred) and taking action to commit crime i (not deterred).

(ii) • In an early stage select criminals hatch a plan and pass on key information or instrument for execution of an eventual crime down the chain. Perpetrators of the early stage crime, called crime A, will be removed from the crime chain and sanctioned, if detected.

• Perpetrators of crime B down the chain may choose to stay away completely from making an attempt at execution. The rest are undeterred B-criminals.

(iii) Only undetected A-criminals of measure k_A and undeterred B-criminals of measure k_B search for each other to match.

(iv) Undetected A-criminals who find a B-partner realize their benefits.

(v) Undeterred B-criminals who find an A-partner commit the crime and realize their benefits. After their crime, they may be detected and sanctioned.

(vi) Detection of a B-criminal leads to a second shot at detection of the partner A-criminal with probability $0 \leq \delta \leq 1$.

and what equilibrium implications it might have for the emerging network. An analysis of enforcement capturing strategic interactions between the criminal network and multiple enforcement departments with an overall budget is certainly an interesting problem but also likely to be very challenging. The work of Ballester et al. (2006) offers a background structure that needs to be further extended to bring in multilateral enforcements as additional players.

Two remarks: First, the sequence of events excludes the possibility of detection at the moment of matching, when A-criminals and potential B-criminals meet to transfer the crime instrument (or, sometimes pass on valuable information). A-criminals are detected in one of two stages – (i) after committing the crime but prior to matching, (ii) after crime B has been completed – whereas B-criminals can be detected only after a match with A-criminals and thereupon successful execution of the crime. Second, as an alternative to the imperfect interim matching process, one could also posit an interim crime-instrument market where the price would reflect shortages of demand by B-criminals or of supply by A-criminals. Raising enforcement to combat crime B, for example, would deter B-criminals and reduce the demand for the instrument and its price, thereby, have a deterrent effect on A-criminals. This price mechanism should generate a similar qualitative relationship between enforcement efforts and their cross-deterrence effects, as the present approach.

■ **Deterrence and matching.** Denote by p_i the probability that an i-criminal finds a j-partner, $i \neq j$, $i, j = A, B$. These probabilities will later be determined in the overall equilibrium following enforcement decisions by the State.

The benefit from crime A will be realized with probability

$$(1 - \mu_A)p_A,$$

and thus a potential A-criminal with benefit b will commit the crime if

$$(1 - \mu_A)p_A b - [\mu_A + (1 - \mu_A)p_A \cdot (\mu_B \delta)]s_A > 0. \tag{16}$$

An A-criminal may be detected either in the preliminary crime-A stage or in a follow-up investigation on detection of the partner criminal B. In a crime environment with $\delta = 0$, A-criminals will escape untraced after transacting with B-criminals. On the other hand if $\delta > 0$, the detection mirrors part of the story of a crime chain in networks.[17] In the analysis to follow, we will consider δ unrestricted.

Given μ_A and μ_B, the measure of crime A is

$$1 - F_A(b_A), \quad \text{where} \quad b_A = \frac{[\mu_A + (1 - \mu_A)p_A \cdot (\mu_B \delta)]s_A}{(1 - \mu_A)p_A}. \tag{17}$$

On the other hand, undeterred B-criminals realize their benefits only if they find an A-partner, when they complete the crime, thus, with probability p_B. They will be detected and punished with probability $p_B \mu_B$, so, a potential B-criminal will commit the crime if

$$p_B b - \mu_B p_B s_B > 0. \tag{18}$$

[17] The only difference is that detection of an A-criminal in the early stage does not lead to apprehension of a B-criminal because the former hasn't yet met the latter.

Thus, the measure of crime B is[18]

$$[1 - F_B(b_B)]p_B, \quad \text{where} \quad b_B = \mu_B s_B. \tag{19}$$

We now turn to the determination of p_i, which we relate below to the endogenous variables k_A, the measure of initially undetected A-criminals, and k_B, the measure of undeterred B-criminals, where

$$k_A = (1 - \mu_A)(1 - F_A(b_A)) \quad \text{and} \quad k_B = 1 - F_B(b_B). \tag{20}$$

Assumption 3.

$$p_i = \rho(k_j/k_i) \begin{cases} = \pi & \textit{if } k_j/k_i \geq 1, \\ \in [0, \pi) & \textit{if } k_j/k_i < 1, \end{cases} \tag{21}$$

where $0 < \pi < 1$ *and* $\rho(k_j/k_i)$ *is strictly concave and increasing, with* $1 > \rho'(0) > 0$.

The matching technology stated in (21) involves frictions. The probability p_i is smaller than k_j/k_i; it is increasing in k_j/k_i at a decreasing rate until $k_j = k_i$, whereafter it remains constant at π. Thus, $p_i < 1 \Rightarrow p_j = \pi$, $i \neq j$: A criminal on the short side of the crime chain will find a partner with maximal probability, π. To illustrate, if $k_B > k_A$, undetected A-criminals match and get their private benefit with probability $p_A = \pi$, whereas undeterred B-criminals complete their crime with probability $p_B < \pi$. If smaller than π, p_B is increasing in k_A and decreasing in k_B.

Frictionless matching is obtained as a special case of Assumption 3 by setting $\pi = 1$ and $\rho(k_j/k_i) = k_j/k_i$ for $k_j/k_i < 1$. If the population of undetected A-criminals is 40 percent of the population of undeterred B-criminals, under frictionless matching each undeterred B-criminal expects to match with probability 0.4. With frictions, this probability is smaller than 0.4.

■ **Equilibrium analysis.** In the following lemma we define the crime equilibrium induced by a pair of enforcement intensities μ_A and μ_B.

Lemma 4. *Fix a pair of enforcement intensities* (μ_A, μ_B)*. An* <u>*induced crime equilibrium*</u> *consists of a pair of cutoff criminal types* $(\widetilde{b}_A, \widetilde{b}_B)$ *satisfying (17), (19) and (20), that is,*

$$\begin{aligned} b_A &= \left[\frac{\mu_A}{(1-\mu_A).\rho\left(\frac{1-F_B(b_B)}{(1-\mu_A)(1-F_A(b_A))}\right)} + (\mu_B \delta) \right] s_A, \\ \textit{and} \qquad b_B &= \mu_B s_B. \end{aligned} \tag{22}$$

A solution pair is easily guaranteed. The proof is integrated in the proof of the next proposition. The first equation in (22) is obtained by using (20) in the definition of p_A and (17). Note that the potential cross-deterrence effect of enforcement is unidirectional, whereas the cross-crime effects

[18] Notice the difference between (16) and (18). In (16), criminal A gets sanctioned for "killing the rhino" even if he fails to "deliver the horn" to (match with) criminal B. In contrast, in (18), criminal B is sanctioned only when executing his crime (marketing the horn) after *matching* with criminal A. The difference gets reflected in the cutoff benefits b_A and b_B, with the former dependent on the matching probability p_A whereas the latter is independent of p_B.

work in both directions. Deterrence of A-criminals does not marginally affect deterrence of crime B though it can affect the measure of undeterred B-criminals who complete the crime. On the other hand, enforcement effort by agent B will affect crime A deterrence via two potential channels − one through p_A, the probability that an A-criminal realizes his benefit, the other through δ, the likely detection of the A-partner following criminal B's detection.

Our first result on vertical crime chains is on the existence and uniqueness of induced crime outcomes and the related comparative statics.

Proposition 3 (Decentralized coordination). *(a) In the decentralized environment of linked crimes, a pair of enforcement intensities* (μ_A, μ_B) *induces a* unique *coordinated crime equilibrium* $(\widetilde{b}_A, \widetilde{b}_B)$.

(b) In any induced equilibrium, an increase in the enforcement intensity of crime i lowers crime i whereas an improvement in A's detection through the follow-up investigation, δ*, only lowers crime A without any cross-deterrence effect on crime B:*

(i) $\frac{d\widetilde{b}_A}{d\delta} > 0, \quad \frac{d\widetilde{b}_B}{d\delta} = 0,$

(ii) $\frac{d\widetilde{b}_A}{d\mu_A} > 0, \quad \frac{d\widetilde{b}_B}{d\mu_B} > 0, \quad \frac{d[p_B(1-F_B(\widetilde{b}_B))]}{d\mu_B} < 0.$

Uniqueness of equilibrium deterrence levels may be surprising, especially in our decentralized matching environment where greater crimes at either end enhance the attractiveness of crimes at the other end. The uniqueness is due to two reasons: (i) in downstream segment B, the crime can be committed and its benefits and potential costs be realized only after successful matching, so the matching probability p_B has no impact on the deterrence level $(\widetilde{b}_B)$ and the measure of undeterred B-criminals (k_B) is uniquely determined by μ_B; (ii) with k_B thus determined, moving up the chain only a unique pair of (p_A, b_A) can satisfy (17), (20) and (21), pinning down $\widetilde{b}_A$.

Let us now look at the comparative statics, beginning with downstream enforcement. Raising μ_B lowers downstream expected benefits and drives out the marginal criminal, increasing $\widetilde{b}_B$ and lowering $k_B = 1 - F_B(\widetilde{b}_B)$. Fewer attempts will be made towards completion of crime B, but the probability that these attempts will succeed is also affected by the rise in μ_B. There are two conflicting effects on p_B. The first is a deterrent effect on upstream crime A, increasing $\widetilde{b}_A$, due to the increased risk through detection of one's B-partner (the δ effect). This effect tends to decrease k_A and hence, p_B. Meanwhile on the downstream crime scene the fewer B-criminals who remain undeterred will see their chances of finding an upstream partner increased. This effect tends to increase p_B. Even if the latter effect dominates, the increase in p_B will be bounded by matching frictions (reflected in strict concavity of the matching probability function; see Assumption 3). So, when downstream enforcement intensifies the net result is a fall in the equilibrium measure of completed downstream crime, $p_B k_B$; there will be fewer active criminals, even if each attempt is completed with higher probability.

Similarly, raising μ_A leads to an increase in $\widetilde{b}_A$ at constant matching probability p_A. However, those who escape initial detection foresee an increase in their probability of completing the crime, which tends to decrease $\widetilde{b}_A$. Strict concavity of $\rho(.)$ guarantees that this second-order negative feedback on deterrence through the matching process does not offset the initial inductor effect. Overall, $\widetilde{b}_A$ increases and crime A falls. Also, since $\mu_i = 0 \Rightarrow \widetilde{b}_i = 0$, Proposition 3(b)-(ii) implies that $\widetilde{b}_i > 0$ for all $\mu_i > 0$. Finally, we note that crime B deterrence $\widetilde{b}_B$ is neutral to δ, but the measure of completed crime B, $p_B(1 - F_B(\widetilde{b}_B))$, will depend on δ through p_B.

4.1 Incentives

• **Crime-based incentives**. Suppose data is available to set separate verifiable crime A and crime B targets, $1 - F_A(b_A)$ and $(1 - F_B(b_B))p_B$. We know from Proposition 3(b) that an increase in the probability of detection μ_i raises $\widetilde{b}_i$ and lowers crime $i = A, B$. The monotonicity chain from enforcement inputs e_i and R_i^X to deterrence $\widetilde{b}_i$ through μ_i, coupled with verifiability of crime levels, implies that rewards conditional on crime targets as derived in Section 3 can successfully implement any crime level, budget permitting, at first-best cost.

Proposition 4. *Under crime-based incentives the budgets R_i uniquely induce a pair of deterrence levels $\{\widetilde{b}_A, \widetilde{b}_B\}$ as crime equilibrium through (22), at first-best cost, that is, $c_i^C(\mu_i) = c(\mu_i)$, $i = A, B$.*

However, the picture is quite different from independent crimes under detections-based incentives.

• **Detection-based incentives**. In a crime chain the agents' efforts produce cross-detection effects which the authorities can explore in designing incentives. Supplementary detection measures become available as indicators of each agent's enforcement effort. We identify these indicators below and check their monotonicity in enforcement efforts.

Detections of crime A and crime B are now given by

$$d_A = (\mu_A + (1 - \mu_A)p_A\mu_B\delta)(1 - F_A(\widetilde{b}_A)), \quad \text{and} \quad d_B = \mu_B p_B(1 - F_B(\widetilde{b}_B)). \tag{23}$$

Observe that in (23) upstream enforcement intensity μ_A affects downstream detections d_B through the probability p_B whereas downstream enforcement intensity μ_B affects upstream detections d_A through both p_A and δ. Thanks to this δ effect, two additional sets of detection data can be obtained by decomposing d_A according to its source, as $d_A = d_{AA} + d_{AB}$, where

$$d_{AA} = \mu_A(1 - F_A(\widetilde{b}_A)), \quad d_{AB} = (1 - \mu_A)p_A\mu_B\delta(1 - F_A(\widetilde{b}_A)). \tag{24}$$

The measure d_{AA} is owed to agent A's enforcement, d_{AB} to agent B's enforcement and follow-up investigations. Let us now focus on the relationship between these detection measures and enforcement intensities μ_i.

Lemma 5.

The signs of $\dfrac{d[d_A]}{d\mu_A}$, $\dfrac{d[d_B]}{d\mu_B}$, $\dfrac{d[d_{AA}]}{d\mu_A}$, $\dfrac{d[d_{AB}]}{d\mu_B}$ *and hence,* $\dfrac{d[d_A]}{d\mu_B}$, *are ambiguous.*

$$\textit{However, } \frac{d[d_{AB}]}{d\mu_A} < 0, \quad \frac{d[d_{AA}]}{d\mu_B} < 0, \textit{ and } \frac{d[d_B]}{d\mu_A} \begin{cases} > 0 & \textit{if } k_A/k_B < 1 \ (p_B < \pi) \\ = 0 & \textit{if } k_A/k_B \geq 1 \ (p_B = \pi). \end{cases}$$

As in the independent crimes case, d_i is not monotonic in μ_i, hence, not monotonic in own enforcement effort, e_i. We know that $\widetilde{b}_i > 0$ for all $\mu_i > 0$, given fixed μ_j. Holding thus μ_B constant in (23) it is easy to verify that d_A is positive and varies between the two limits,

$$lim_{\mu_A \to 0} d_A \equiv d_A^0 = p_A^0 \delta \mu_B (1 - F_A(\delta \mu_B s_A)) \quad \text{and} \quad lim_{\mu_A \to 1} d_A = 0, \tag{25}$$

whereas d_B converges to the same value, zero, as $\mu_B \to 0$ and $\mu_B \to 1$, taking strictly positive values in between.

Fig. 5 illustrates the shapes of d_{AA} and d_A which are not monotonic, and d_{AB} which is monotonic, in μ_A. Crime A detections must have a maximum, $d_A^{\max}$, at some enforcement intensity denoted $\widehat{\mu}_A = argmax[d_A(\mu_A)]$. Because enforcement effort is not contractible, the agent would exert the lowest effort and generate the lowest enforcement intensity $\mu_A(d_A) = \min\{\mu_A | d_A(\mu_A) = d_A\}$ in Definition 1 compatible with any detection target $d_A < d_A^{\max}$. It follows that the state cannot implement enforcement intensities $\mu_A > \widehat{\mu}_A$ through incentives based on d_A data alone.[19] However, in Lemma 5 we identify the cross-detection effect on d_B, downstream detections, which fall as upstream enforcement intensifies: $d[d_B]/d\mu_A < 0$, whenever $p_B < \pi$, i.e., whenever A-criminals are on the short side of the crime chain. The rise in μ_A deters crime A and reduces p_B, which in turn deters crime B and leads to a fall in detections of crime B, given μ_B. Thus, incentive targets based on d_B can be used to motivate enforcement effort by agent A.

When crime A detection data can be decomposed as in (24), two additional monotonic cross-detection effects become available. Crime A detections attributable to agent B's enforcement, d_{AB}, is unambiguously decreasing in μ_A. Basically a higher μ_A deters crime A and leaves a smaller measure of A-criminals for detection in follow-up investigations through their detected B-partners. This inverse relationship between d_{AB} and μ_A can be used to motivate agent A's enforcement effort.[20] The second monotonic relationship is between d_{AA} and μ_B. An increase in the intensity of enforcement by B reduces d_{AA} for two reasons. First, a higher μ_B means a higher expected sanction from crime A thanks to the increased risk of detection of B-partners. It thus deters crime A and lowers d_{AA}. Second, and reinforcing the first, when B-criminals are on the short side of the crime chain the rise in μ_B reduces p_A, hence the expected benefit from crime A. This effect further raises deterrence of crime A and lowers d_{AA}.

Proposition 5 (Costs and feasibility under system D)**.** *If crime A detection data is available in decomposed form, as d_{AA} and d_{AB}, any given pair of budgets (R_A, R_B) uniquely induces a pair of deterrence levels $\{\widetilde{b}_A, \widetilde{b}_B\}$ as a crime equilibrium via detection-based incentives, at first-best cost.*

[19] Exceptions could be the μ_A levels that are so high that the corresponding detections d_A are smaller than even d_A^0, the measure of detections owning solely to B's enforcement while agent A exerts zero effort.

[20] The impact of an increase in μ_B on d_{AB}, however, is ambiguous. Intuition may suggest that a larger measure of A-criminals should be traced through their B-partners (the δ effect) when agent B raises μ_B, given μ_A. The ambiguity arises because, first, the δ effect is dampened by rise in deterrence of crime A, and second, possibly also by the combined effect of these changes on the matching probability p_A.

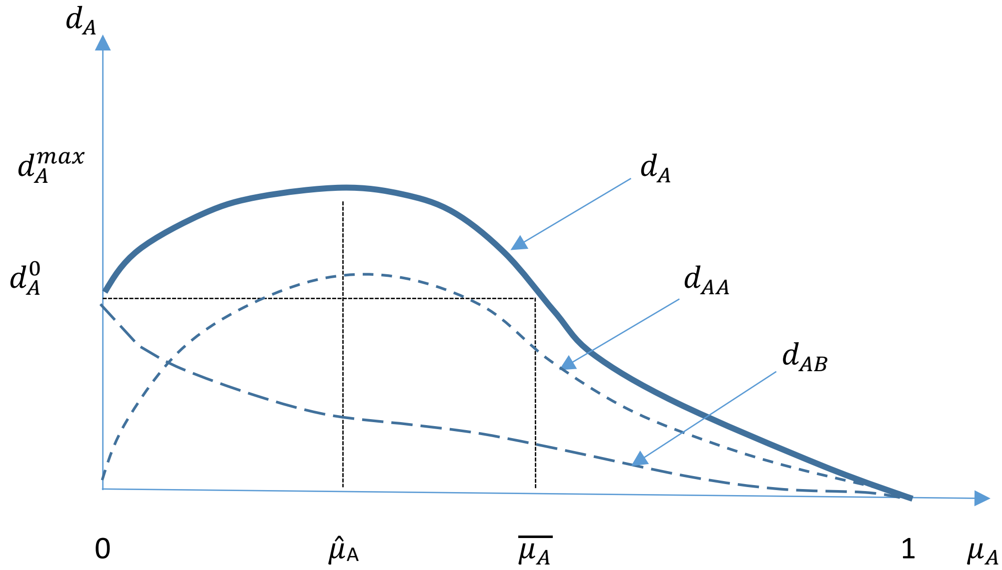

Figure 5. The measure of detections of crime A decomposed, $d_A = d_{AA} + d_{AB}$, as a function of the probability of detection, for $\delta > 0$. The same measure d_A^0 of detections is induced by the detection probabilities $\overline{\mu_A}$ and $\mu_A = 0$.

Given a budget allocation $\{R_A, R_B\}$, the optimal enforcement efforts $\widetilde{e}_i(R_i)$ generate first-best detection probabilities $\mu_i(R_i)$ which uniquely induce through (22) a deterrence pair $\{\widetilde{b}_A, \widetilde{b}_B\}$ as crime equilibrium, hence the detection measures d_{AA}, d_{AB} and d_B. Thus, if these detection data are available, the implementation problem that plagues the control of an independent crime via detection-based incentives disappears, thanks to the monotonicity properties in Lemma 5. The detection measures d_B and d_{AB} can be used as indicators of agent A's enforcement effort, and d_{AA} can be used for agent B's enforcement effort. To illustrate, incentives for the Border Force to deter illegal immigration and smuggling should be based on the number of detections within the borders by the police (d_B), alternatively on the detections by the Border Force as follow-up investigations upon the information provided by the police (d_{AB}). Rewards for the corresponding units of the police, on the other hand, should be based on the number of detections in the customs done independently by the Border Force (d_{AA}). If too large a fraction of crime A detections is owed to agent B, this is taken as signal of poor enforcement by agent A, and if agent A's own independent detections exceed the target this is taken as a signal that B is withholding enforcement effort. The State can tune the twin detection targets appropriately and implement *all* deterrence levels, budget permitting, at first-best cost.

It is important to note that these detection measures constitute the minimal tool set for incentive provision. Other detection measures which in general are not guaranteed to be monotonic in μ_i may be monotonic for specific parameters or in intervals of detection probabilities, thus, may also be used as such. Monotonicity of d_i in μ_i holds, for example, at sufficiently low levels of μ_i, hence high crime levels.

Combining Propositions 4 and 5, we conclude that any pair of deterrence levels $(\widetilde{b}_A, \widetilde{b}_B)$ for interlinked crimes can be implemented at the same, first-best, cost $c(\mu_i)$ through crime- and detection-based systems. To abstract from the sources of asymmetries other than the vertical structure of the crime chain, we analyze the budget allocation problem below assuming that each agent's incentives are first-best (crime or detection data in decomposed form are available so that Proposition 4 or 5 applies) and we consider a symmetric crime environment, defined in the sequel.

4.2 Budget allocation

The optimal budget allocation (R_A^*, R_B^*) minimizes the expected social harm

$$\begin{aligned} &\min_{\{R_A,R_B\}}\ SH = [1 - F_A(b_A)]h_A + [1 - F_B(b_B)]p_B h_B, \\ &\text{subject to} \quad \text{eqs. (12) and (22).} \end{aligned} \tag{26}$$

■ **Symmetric crime environments**. We say that two crimes form a *symmetric crime environment* if the sanctions and the criminal benefit distributions are identical, i.e., if $s_A = s_B$, and $F_A(.) = F_B(.)$. Denote by s the common sanction and by $F(.)$, the common distribution function.

For any budget allocation such that $R_A \geq \frac{R}{2} \geq R_B$, the induced crime equilibrium satisfies $\widetilde{b}_A \geq \widetilde{b}_B$, by (22):

$$\widetilde{b}_A = \left[\frac{\mu(R_A)}{(1-\mu(R_A)).p_A} + (\mu(R_B)\delta)\right] s > \mu(R_B)s = \widetilde{b}_B \text{ for any } p_A \leq \pi.$$

It follows that $1 - F(\widetilde{b}_A) < 1 - F(\widetilde{b}_B)$, hence, $k_A \equiv (1 - \mu(R_A))(1 - F(\widetilde{b}_A)) < 1 - F(\widetilde{b}_B) \equiv k_B$. We have a crime equilibrium with $p_A = \pi$ and $p_B < \pi$. An equal budget allocation in a symmetric crime environment generates stronger deterrence on crime A than crime B, $\widetilde{b}_A > \widetilde{b}_B$. This holds even if $\delta = 0$ because A criminals face the risk of apprehension before matching with B partners, thus, before realizing their criminal benefits, whereas B criminals are detected after matching and realizing their benefits.

We study below the impact of a small *balanced* budget adjustment, such that $dR_A = -dR_B$, on the expected social harm in (26) at an equal budget allocation. For a clear picture, in Proposition 6 we take δ sufficiently small. The only sources of asymmetry left are the harms and the chain structure of crimes.

Proposition 6 (Priority: upstream). *Consider a symmetric crime environment in which crime A is input to crime B, with δ sufficiently small. The budget for agent A should be larger than the budget for agent B, $R_A^* > R_B^*$, if*

$$h_A > h_B(1-\mu(\frac{R}{2}))^2 \left[\frac{f(\widetilde{b}_B)\pi}{f(\widetilde{b}_A)}(p_B - \rho'(.)\frac{k_A}{k_B}) - \frac{\rho'(.)}{1-\mu(\frac{R}{2})}(1 + \frac{\pi k_A.\mu(\frac{R}{2})}{sf(\widetilde{b}_A)\mu'(\frac{R}{2})})\right], \tag{27}$$

that is, unless crime B is sufficiently more harmful than crime A.

There are two main reasons for allotting a larger budget to upstream enforcement.

• *Stronger deterrence impact of resources upstream.* Starting from an equal budget allocation, a balanced-budget shift to upstream enforcement will raise deterrence upstream more than it dilutes downstream, unless $f(\widetilde{b}_A)$ is too small relative to $f(\widetilde{b}_B)$. This is so, because an increase in the probability of detection of crime A raises the expected sanction for A-criminals but it also reduces the expected benefit which they may realize later if matched with a B-partner, whereas reducing the probability of detection of B-criminals reduces their expected sanction but leaves their expected benefit unchanged. In addition, it is better to deter one A crime than one B crime because the harm from crime A is proportional to the measure k_A of undeterred A criminals (the harm occurs when undeterred poachers kill rhinos), whereas the harm from crime B is proportional to $p_B k_B$, the measure of undeterred B-criminals who complete their crime (by marketing the rhino horns). Thus, even if $f(\widetilde{b}_A) = \pi f(\widetilde{b}_B)$ so that the net increase in overall deterrence is zero, a rise in A-deterrence coupled with an equal fall in B-deterrence is not neutral for the objective of the State.

• *Reduction in the probability that downstream crime is completed.* The impact on $p_B = \rho(\frac{k_A}{k_B})$ operates through three variables: the deterrence levels $\widetilde{b}_A$ and $\widetilde{b}_B$ and the detection probability of crime A, $\mu_A = \mu(R_A)$. Induced changes in all three variables following a balanced budget shift contribute to the fall in p_B, as captured by the negative terms at the right-hand side of (27); the rise in R_A does it by increasing $\widetilde{b}_A$ and μ_A and hence by reducing k_A, and the fall in R_B, by raising $\widetilde{b}_B$ and hence k_B as well.

Thus, even if upstream and downstream crime environments and enforcement technologies are identical in every respect, the vertical crime chain structure favors a larger budget for upstream law enforcement unless the harm from downstream crime is sufficiently larger than the harm from upstream crime. Raising the budget to fight the latter will reduce expected harm from crimes.

It is worth noting that condition (27) prioritizing upstream crime can hold in vertical crime chains where h_A is very small relative to h_B, or even $h_A = 0$, if by deterring crime A the state can induce a large fall in p_B, the probability that B-criminals find a match and complete the crime. The larger is the cross-deterrence effects on p_B (represented by the negative terms in the coefficient of h_B in (27)), the more likely it is that the State will prioritize upstream enforcement even if the social harm from the upstream crime is very small or zero.

The symmetric opposite case where h_B is very small relative to h_A, or in the limit, $h_B = 0$, is also of interest. Whereas (27) suggests $R_B^* = 0$, the practice of enforcement in such crime chains does not involve a corner solution. Indeed, if δ is not small, it will be optimal to devote positive amount of resources to detect the almost harmless upstream actors for their potential help in tracing their harmful upstream partners.

■ **Large** δ. When δ is large, the positive externality from agent B's enforcement can lead to a large increase in detections of crime A. The expected harm from crimes will be smaller thanks also to the reduction in p_B.

In a symmetric crime environment with large δ and an equal budget allocation, the equilibrium conditions in (22) continue to imply $\widetilde{b}_A > \widetilde{b}_B$.[21] Now the condition $dSH < 0$ in (27) is modified as

$$h_A \left[\frac{1}{(1-\mu(\frac{R}{2}))^2 \pi} - \delta \right] > h_B \left[\frac{f(\widetilde{b}_B)}{f(\widetilde{b}_A)} (p_B - \rho'(.) \frac{k_A}{k_B}) - \rho'(.) \Big\{ \frac{\mu(\frac{R}{2})(1-F(\widetilde{b}_A))}{\mu'(\frac{R}{2}) s f(\widetilde{b}_A)} + (1-\mu(\frac{R}{2}))(\frac{1}{(1-\mu(\frac{R}{2}))^2 \pi} - \delta) \Big\} \right]. \quad (28)$$

The impact of $\delta > 0$ is apparent at the left and right hand sides of (28). Now a balanced-budget transfer to agent A dilutes deterrence of crime A by curbing the possibility of crime A traced back through detections of crime B. This is captured by the negative δ term in the coefficient of h_A, at the left-hand side of (28). Also, the now larger population of A-criminals seeking a B-partner will raise p_B and feed crime B. This second-order effect appears at the right hand side with the additional δ term. Both of effects work in the same direction, of increasing crime B and thus reducing the expected benefit from a balanced-budget transfer to agent A. If strong enough, these new effects can lead the State to prioritize the downstream enforcement.

5 Interlinked crimes: crime A causes crime B

A crime could fertilize the ground for another crime through a multitude of mechanisms. In this section we consider a unidirectional causality; we say that crime A causes crime B if it increases the population of *potential* B-criminals. Crime B does not cause, but is in part an effect of, the preceding crime A. Crime A could be child abuse and crime B juvenile crime. We know that children subject to parental abuse of any form are more likely to commit crimes than children that are not. Similarly there is ample evidence that illegal drug sales lead to an increase in the population of potential violent criminals.

The difference between Section 4 and the present section is not confined to a causal link replacing a matching process through which changes in the supply of one crime affects the supply of the other. In vertical crime chains neither the upstream crime nor the downstream crime can be identified as cause. Each crime needs the other: Without crime i there would be no crime j. In this section, undetected criminals don't need any partner to realize their benefits, so, each of the two crimes would still be committed if the other were eradicated. However, crime A increases the pool of potential B-criminals and this creates a positive externality from agent A's enforcement to crime B outcome. We ask if the cause-effect relation favors a larger budget for the root crime A and we address the incentive problem in enforcement.

Consider two groups of individuals, potential A-criminals and a B-population, each of measure one. Potential B-criminal population is partly endogenous, for perpetrators of crime A "affect" a fraction α of the B-population. The affected B-individuals see their probability of becoming

[21] It is easy to verify that $k_B > k_A$ holds even stronger under $\delta > 0$, hence, $p_A = \pi$, $p_B < \pi$.

potential B-criminals increase. Depending on the context, these B-individuals could be victims of, deal with, or happen to interact with, A-criminals. In the case of parental child abuse α could be taken equal to one (a parent for each child). If A-criminals are drug sellers and crime B is burglary or shoplifting, α would be smaller than one; some, but not all, shoplifters would be acting under drug influence.

Formally, a B-individual not affected by crime A becomes a potential B-criminal with probability p_L. These individuals are of measure $1 - \alpha(1 - F_A(b_A))$. For an affected B-individual this probability rises to p_H. These individuals are of measure $\alpha(1 - F_A(b_A))$. So, as the measure of crime A varies from zero to $1 - F_A(b_A)$, the size of *potential* B-criminals grows from p_L to $[(1-\alpha) + \alpha F_A(b_A)]p_L + (1 - F_A(b_A))\alpha p_H$. The two crimes become fully independent if $p_L = p_H$, with equal measures of potential A- and B-criminals if $p_L = p_H = 1$.

The sequence of events is a variant of Section 4. Given the budget allocation and enforcement incentives set by the State, agent i determines effort, hence, μ_i. Then:

- Undeterred potential A-criminals commit crime A, realize their benefits and, if detected, are sanctioned.
- The measure of potential B-criminals thus determined. Those who commit the crime realize their benefits and, if detected, are sanctioned.
- Detection of a B-criminal leads to detection of his "affector" A-criminal, if initially undetected, with probability $0 \leq \delta \leq 1$.[22]

■ **Crime equilibrium.** Fix a pair of enforcement intensities μ_A and μ_B and consider first a potential B-criminal. Committing the crime yields the expected utility $b - \mu_B s_B$ whereas the utility from compliance is zero. Define a critical benefit

$$b_B = \mu_B s_B \tag{29}$$

such that a potential B-criminal commits the crime if $b > b_B$ and complies otherwise.

Perpetrators of crime A are detected initially by agent A, with probability μ_A per criminal. Those who escape may be caught through investigations of the B-criminals whom they affected, later detected by agent B. Each A-criminal has a B-criminal affected with probability $\alpha p_H(1 - F(b_B))$. Thus, crime A is detected by agent B's enforcement with probability $\delta(1-\mu_A)\mu_B \alpha p_H(1 - F_B(b_B))$. Given this, a potential A-criminal will commit the crime if his benefit exceeds

$$b_A = [\mu_A + \delta(1 - \mu_A)\mu_B \alpha p_H(1 - F_B(b_B))]\, s_A. \tag{30}$$

A *crime equilibrium* can be defined as in Lemma 4, as a pair of deterrence levels $(\widetilde{b}_A, \widetilde{b}_B)$ that satisfy (29) and (30), given enforcement intensities (μ_A, μ_B). It is easy to verify that the crime equilibrium is unique: μ_B uniquely determines $\widetilde{b}_B$ through (29) and $\widetilde{b}_A$ is uniquely determined through (30)

[22] This possibility of backtracking originally undetected A-criminals should also be contextual. It may be difficult to prove parental abuse by detecting juvenile shoplifting but it is possible to trace the drug seller from a criminal acting under the influence of the drug.

given (μ_A, μ_B) and $\widetilde{b}_B$. Observe that if $\delta = 0$ the deterrence levels in (29) and (30) are identical to those for independent crimes, in (2).

The comparative statics results in Lemma 6 are verified easily. Own-deterrence effects of enforcement are positive. Whereas crime B deterrence level is independent of agent A's enforcement, agent B's enforcement affects deterrence of the cause crime A. The latter impact is not monotonic, however. Under Assumption 2 (the detection function $d_i(\cdot) = \mu_i(1 - F_i(\mu_i s_i))$ is single-peaked), the term $\delta(1 - \mu_A)\mu_B \alpha p_H(1 - F_B(\mu_B s_B))$, hence b_A in (30), are also single peaked at the same $\mu_B = \widehat{\mu}_B$, given μ_A.

Lemma 6. *$\frac{d\widetilde{b}_i}{d\mu_i} > 0$; $\frac{d\widetilde{b}_B}{d\mu_A} = 0$ and $\frac{d\widetilde{b}_A}{d\mu_B} \geq 0$ if and only if $\mu_B \leq \widehat{\mu}_B$.*

5.1 Incentives

■ **Crime-based incentives**. The two crime measures are given by:

$$\text{crime A: } 1 - F_A(\widetilde{b}_A); \quad \text{crime B: } \left[p_L + \alpha(1 - F_A(\widetilde{b}_A)).(p_H - p_L)\right](1 - F_B(\widetilde{b}_B)). \tag{31}$$

Since $\frac{d\widetilde{b}_i}{d\mu_i} > 0$, crime i is monotonically decreasing in μ_i. It follows that under crime-based incentives, given any budget allocation (R_A, R_B) the corresponding maximal detection probabilities $(\mu_A^C(R_A), \mu_B^C(R_B))$ (see Definition 2) will induce a crime equilibrium $(\widetilde{b}_A, \widetilde{b}_B)$ at first-best cost. Proposition 4 continues to hold in the case of causally linked crimes.

■ **Detection-based incentives**. The measures of crime detections are:

$$\begin{aligned} d_A &= \left[\mu_A + \delta(1 - \mu_A)\mu_B \alpha p_H(1 - F_B(\widetilde{b}_B))\right](1 - F_A(\widetilde{b}_A)), \\ d_B &= \mu_B\left[p_L + \alpha(1 - F_A(\widetilde{b}_A)).(p_H - p_L)\right](1 - F_B(\widetilde{b}_B)). \end{aligned} \tag{32}$$

Regarding own-detection effects of enforcement, an increase in μ_B raises d_B at constant $\widetilde{b}_B$ but reduces d_B by raising $\widetilde{b}_B$. The resulting change in d_B will then impact on $\widetilde{b}_A$ through δ (the backtracking effect) and the change in $\widetilde{b}_A$ will cause a change in the measure of potential B-criminals through the α effect, hence, feed back on crime B detections. The sign of $\frac{d[d_B]}{d\mu_B}$ is thus ambiguous.

When $\delta > 0$, detections of crime A can be decomposed into its components, $d_A = d_{AA} + d_{AB}$, as crime A detections by agent A's enforcement plus those detected through detection of affected B-criminals, where

$$d_{AA} = \mu_A(1 - F_A(\widetilde{b}_A)) \quad \text{and} \quad d_{AB} = \delta(1 - \mu_A)\mu_B \alpha p_H(1 - F_B(\widetilde{b}_B))(1 - F_A(\widetilde{b}_A)).$$

The expression of d_{AA} is identical to (4), which we know is not monotonic in μ_i. Thus, d_{AA} is not monotonic. On the other hand, d_{AB} is monotonically decreasing in agent A's own enforcement intensity. An increase in μ_A deters crime A, which shrinks the pool of A-criminals that can be detected through B's enforcement and reduces the cause effect on crime B, hence detections of crime B and d_{AB}. Thus, both effects lead to a fall in d_{AB}. Comparative statics results pertaining to detections are presented below.

Lemma 7. *(i) Assume $\delta > 0$. The detection measures d_B, d_A or its components d_{AA}, d_{AB}, are not monotonic in μ_B. Only d_{AB} and d_B are monotonic in μ_A, strictly decreasing in its full range, $[0, 1]$: The signs of the detection measures*

$$\frac{d[d_A]}{d\mu_A}, \frac{d[d_B]}{d\mu_B}, \frac{d[d_{AA}]}{d\mu_A}, \frac{d[d_A]}{d\mu_B}, \frac{d[d_{AB}]}{d\mu_B}, \frac{d[d_{AA}]}{d\mu_B} \text{ are ambiguous. However, } \frac{d[d_{AB}]}{d\mu_A} < 0, \frac{d[d_B]}{d\mu_A} < 0.$$

Under Assumption 2, d_{AA} has a single trough in μ_B, at $\hat{\mu}_B$: $\frac{d[d_{AA}]}{d\mu_B} < 0$ if $\mu_B \in [0, \hat{\mu}_B]$; otherwise $\frac{d[d_{AA}]}{d\mu_B} > 0$.

(ii) Assume $\delta = 0$. Then, d_B is monotonic in μ_A, strictly decreasing in its full range, $[0, 1]$. Moreover, under Assumption 2, d_A and d_B are single peaked, at $\hat{\mu}_A$ and $\hat{\mu}_B$, respectively.

Lemma 7 is the key to Proposition 7 on feasibility and implementation of deterrence levels for causally linked crimes, under detection-based incentives, which requires at least one measure of detections be monotonic in μ_i.

Part (i) implies that for $\delta > 0$ crime B detections d_B, or detections of crime A owing to agent B's enforcement, d_{AB}, can be used as targets to motivate enforcement effort by agent A. Fix a budget allocation and consider the maximal enforcement intensities $\mu_i(R_i)$ and the corresponding deterrence levels $\widetilde{b}_i$ induced through (29) and (30). Assuming that agent B exerts the effort that generates $\mu_B(R_B)$, an appropriate crime B detection target will motivate agent A to induce the enforcement intensity $\mu_A(R_A)$: Agent A is rewarded if actual crime B detections do not exceed the target. Otherwise agent A is not rewarded, for he must be withholding effort and generating a suboptimal enforcement intensity μ_A. Any deterrence target for the cause crime A can be implemented this way by modifying the budget allocation and the target d_B, at first-best cost, given μ_B.

However, none of the detection measures is monotonic in μ_B. The feasible range is then confined to an interval of low enforcement intensities, $\mu_B \in [0, \hat{\mu}_B]$, provided data on crime A detections is available in decomposed form, d_{AA} and d_{AB}. Only d_{AA} has a predictable regular shape (under Assumption 2), falling as μ_B increases to reach its minimum at $\hat{\mu}_B$ and rising thereafter. Thus, given a budget allocation with R_B not too large and fixed $\mu_A(R_A)$, detection targets d_{AA} can be set for agent B to induce $\mu_B(R_B) \leq \hat{\mu}_B$, implementing the corresponding pair of deterrence levels $\{\widetilde{b}_A, \widetilde{b}_B\}$ at first-best cost.

Whereas a positive δ has a beneficial deterrent effect, in the context of causal links it can obstruct incentive design for the enforcement unit B, in the "effect" crime. If $\delta = 0$ or very small, the picture on incentive provision as described in part (ii) of Lemma 7 is relatively clear with slightly improved possibilities. Cross detection data d_A, instead of its component d_{AA}, can now be used to motivate agent B, at least for enforcement intensities $\mu_B \in [0, \hat{\mu}_B]$.

Proposition 7 summarizes the analysis of enforcement costs under crime- and detection-based incentives.

Proposition 7. *Fix a budget allocation* $\{R_A, R_B\}$ *and let* $\{\widetilde{b}_A, \widetilde{b}_B\}$ *be the deterrence levels associated with the pair of enforcement intensities* $\{\mu_A(R_A), \mu_B(R_B)\}$*, satisfying (29) and (30).*

- Crime-based incentives*: The deterrence pair* $\{\widetilde{b}_A, \widetilde{b}_B\}$ *is induced at first-best cost.*
- Detection-based incentives*: If* $\widetilde{b}_B \leq \hat{\mu}_B s_B$*, the deterrence pair* $\{\widetilde{b}_A, \widetilde{b}_B\}$ *is feasible and it is then induced at first-best cost (requiring* d_{AA} *data if* $\delta > 0$*,* d_A *data if* $\delta = 0$*.)*

Deterrence levels $\{\widetilde{b}_A, \widetilde{b}_B\}$ such that $\widetilde{b}_B > \hat{\mu}_B s_B$ are not compatible with detection-based incentives. Such $\widetilde{b}_B$ can be induced only if μ_B is raised above $\hat{\mu}_B$ where $d_A > d_A^{min}$, but there is a lower effort and enforcement intensity that produces this same d_A to which agent B will switch, economize on his effort cost and upset the deterrence objectives. For the "effect" crime B, then, the deterrence implementation outcome and costs are qualitatively similar to the independent crimes case in that they hit the same roadblock: up to a threshold deterrence level only can be implemented, at first-best cost, via detection-based incentives.

5.2 Budget allocation

The optimal budget allocation (R_A, R_B) under incentive system $r = C, D$ minimizes

$$SH = (1 - F_A(\widetilde{b}_A)).h_A + \Big[p_L + \alpha(1 - F_A(\widetilde{b}_A)).(p_H - p_L)\Big]\,(1 - F_B(\widetilde{b}_B)).h_B \tag{33}$$

subject to (29) and (30), the budget constraint $R = R_A + R_B$ and the enforcement cost functions $R_i = c^r(\mu_i)$. These cost functions are first best and given by $c(\mu_i)$ defined in (1), as we assume below, if crime or detection data are available (subject to the feasibility limitations stated in Proposition 7).

■ $\delta = 0$. Proposition 8 compares the solution to this problem, (R_A^*, R_B^*), to the optimal budget allocation in the case of independent crimes, (R_A^I, R_B^I). To this end, we set the measures of potential criminals in each independent crime equal to those in the case of causally linked crimes, $k_i^I = k_i^*$. Thus, $k_B^I = k_B^* = p_L + \alpha(1 - F_A(\widetilde{b}_A^*)).(p_H - p_L)$ where $\widetilde{b}_A^*$ satisfies (30). We obtain a clear ranking of enforcement budgets in the case $\delta = 0$, under an intuitive condition, namely, diminishing marginal deterrence of enforcement, that is, $f_i(\widetilde{b}_i).\mu_i'(R_i)$, where $\widetilde{b}_i = s_i\mu_i(R_i)$, is decreasing in R_i. This translates to the benefit distribution functions as $F_i(b)$ not too convex at any b.[23]

Proposition 8 (Priority to the cause)**.** *Assume* $\delta = 0$ *and that* $f_i(\widetilde{b}_i).\mu_i'(R_i)$ *is decreasing in* R_i*, where* $\widetilde{b}_i = s_i\mu_i(R_i)$*. The optimal budgets when the crimes are independent and when crime A causes crime B compare as follows:*

(i) $R_A^I < R_A^*$*,* $R_B^I > R_B^*$*;*

(ii) in a symmetric crime environment, $R_A^* = R_B^* \Rightarrow \frac{h_A}{h_B} = p_L$ *and* $R_A^I = R_B^I \Rightarrow \frac{h_A}{h_B} = k_B^* > p_L$*.*

Part (i) states that introducing a causal link between two independent crimes leads to a budget transfer from the effect crime B to the cause crime A. Agent A's enforcement gains importance when

[23] More precisely, $\frac{F_i''(.)}{F_i'(.)} < -\frac{\mu_i''(.)}{s_i\mu_i'(.)^2}$.

crime A earns an additional harmful effect by causing crime B. Part (ii) carries the comparison to symmetric crime environments (same sanctions, identical benefit distributions). For an equal budget allocation to be optimal, the harm ratio h_A/h_B must be smaller in causally linked crimes than independent crimes. So, if in a symmetric environment the state allocates the budget equally when the crimes are independent, then it will allocate a larger budget to the cause crime relative to the effect crime, all else equal. These results pertain to zero or small δ contexts, for example, juvenile crime with parental child abuse as antecedent crime, where it is difficult to trace and convict a parent for child abuse through detection of a crime committed by the child.

■ $\delta > 0$. A large δ raises the productivity of agent B's enforcement, thereby the benefit from shifting some budget from crime A to crime B. This backtracking possibility can upset the budget ranking in Proposition 8. The enforcement budget for crimes committed under drug influence would rise to the detriment of the cause crime, illegal drug sales. Enforcement incentives can be adjusted according to the shift in the budgets if crime data is available. Otherwise, if the state has to rely on detection data, as we know from Proposition 7, high range of enforcement intensities are not compatible with detection-based incentives.

6 Conclusion

This paper addresses two issues in law enforcement, how to motivate enforcement units to exert effort under moral hazard, and the related issue as to the optimal allocation of an enforcement budget between the units. We study the moral hazard problem in various environments with multiple, related or independent, crimes, and we consider two indicators that correlate with enforcement efforts, crime levels and the number of detections/apprehensions.

The key property that determines whether an indicator can be used to implement crime deterrence targets at first-best cost is monotonicity of the indicator in the enforcement effort. The analysis reveals that crime-based incentives weakly dominate detection-based incentives. The crime level is monotonic in enforcement effort whereas detection measures in some environments and crimes are not. If available and reliable, crime-based incentives should be used, for they implement deterrence targets at first-best cost given any enforcement budget.

Whereas crime data may not always be available, as for unobservable crimes which can be known only if detected by law enforcers, or if measures of crime are not reasonably accurate, for many crimes the number of detected/apprehended suspects should, in principle, be available. However, detection-based incentive systems are not as effective in coping with moral hazard because many measures of detection are not in general monotonic in enforcement effort. Non-monotonicity is relevant particularly for independent crimes. For crimes interlinked through causality or an input-output chain, cross-detection data can be explored for incentive provision. Moreover, if upstream criminals can be traced back through detection of their downstream partners, two sets of upstream detection data become available, one owing purely to upstream enforcement, the other owing to the unit fighting the downstream crime. Including these cross-detection measures we get a rich set of detection measures, some of which are monotonic in downstream or upstream enforcement effort.

For each of the crimes in an input-output chain, effort-monotonic detection measures can be found to implement any level of deterrence at first-best cost. For these crimes, crime- and detection-based incentive systems are equally effective. This conclusion does not hold for the (effect) crime partially caused by another crime, that is, in the case of causally linked crimes. None of the detection measures is monotonic in the effort of the unit enforcing the effect crime. This offers an additional reason for favoring the unit enforcing the root crime in budget allocation, if crime data cannot be used for incentive provision.

We identify other structural factors that favor larger budget allocations to the upstream (input or cause) crime. For interlinked crimes forming an input-output chain, raising deterrence of the upstream crime reduces the benefit from downstream crime. In addition, because undeterred upstream criminals complete their crimes with probability one whereas for downstream criminals this probability is less than one, the upstream crime has some priority in the budget allocation. This is possible even if the upstream crime generates little or no social harm. The backtracking effect, if present and powerful enough, can shift the balance in the opposite direction, to the downstream (output or effect) crime.

An interesting question about enforcement incentives we did not address is whether crime- and detection-based systems can be used in conjunction, complementing each other, rather than separately as implied in our set-up where the only concern is effort unobservability. Concerns about measurement errors, relative manipulability and lags in availability of crime and detections data, if added on top of moral hazard, we conjecture, could lead the State to use both crime and detections data in designing incentives for its law enforcement units. Crime and detection observations can also be used together by the state in order to assess changes in the level of deterrence, an unobservable statistic.

A Appendix

■ **Property of d_i function.** Note that $\frac{\partial d_i}{\partial \mu_i} = 1 - F_i(.) + \mu_i\{-\frac{\partial F_i}{\partial \mu_i}\}$, which is positive at low μ_i values and negative at high μ_i values. So $\widehat{\mu}_i$ is bounded away from 1. Further, $\frac{\partial^2 d_i}{\partial \mu_i^2} = -2\frac{\partial F_i}{\partial \mu_i} - \mu_i \frac{\partial^2 F_i}{\partial \mu_i^2}$, where $\frac{\partial^2 F_i}{\partial \mu_i^2} = f_i'(.)\frac{s_i^2}{(1-\mu_i)^4} + f_i(.)\frac{-2s_i}{(1-\mu_i)^3}$. So long as $\frac{\partial^2 F_i}{\partial \mu_i^2} \geq 0$,[24] $d_i(.)$ will be strictly concave in μ_i, because $\frac{\partial F_i}{\partial \mu_i} > 0$ (of course it is possible that $\frac{\partial^2 F_i}{\partial \mu_i^2} < 0$ and yet $d_i(.)$ is strictly concave). This would guarantee a unique global maximum $d_i^{\max} = \widehat{\mu}_i\left(1 - F_i(\frac{\widehat{\mu}_i s_i}{1-\widehat{\mu}_i})\right)$ at a positive $\widehat{\mu}_i$. Marginal detections decline and become negative at μ_i levels above $\widehat{\mu}_i$. ||

Proof of Proposition 3.

(a) Given R_B, $\widetilde{b}_B$ is uniquely determined by the second equation in (22). Consider the first equation in (22), given R_B, $\widetilde{b}_B$ and R_A. The right-hand side is continuous and monotonically

[24] This will be guaranteed by (weak) log-convexity of the density $f_i(b)$.

decreasing in b_A for $\rho(.) < \pi$, constant for $\rho(.) = \pi$, with limits (under the assumption that the upperbound $\bar{b}$ of benefits is sufficiently large):

$$\left[\frac{\mu_A}{(1-\mu_A)\rho(\frac{1-F_B(b_B)}{1-\mu_A})} + \mu_B\delta\right] s_A > 0 \text{ as } b_A \to 0; \qquad \left[\frac{\mu_A}{1-\mu_A} + \mu_B\delta\right] s_A < \bar{b} \text{ as } b_A \to \bar{b},$$

where $\mu_A = \mu(R_A)$ is bounded away from 1. The limit at $b_A \to 0$ is larger than the limit at $b_A \to \bar{b}$. Applying the intermediate value theorem to the difference $LHS - RHS$ of this equation and using monotonicity, a unique fixed-point solution, $\widetilde{b}_A$, is guaranteed.

(b) The following signs are immediate from (22): $\frac{d\widetilde{b}_B}{d\mu_B} = s_B > 0$ and $\frac{d\widetilde{b}_B}{d\delta} = 0$. In the rest of the proof we consider two cases.

- $p_B < \pi$, thus, $p_A = \pi$. First, observe that

$$\frac{d\widetilde{b}_A}{d\mu_A} = \frac{s_A}{(1-\mu_A)^2\pi} > 0, \quad \frac{d\widetilde{b}_A}{d\delta} = \mu_B s_A > 0.$$

Now consider

$$\frac{d[p_B(1-F_B(\widetilde{b}_B))]}{d\mu_B} = (1-F_B(\widetilde{b}_B))\frac{dp_B}{d\mu_B} - p_B f_B(\widetilde{b}_B)\frac{d\widetilde{b}_B}{d\mu_B} \quad \text{where}$$

$$\frac{dp_B}{d\mu_B} = \rho'(.)\frac{k_A}{k_B^2} f_B(\widetilde{b}_B)\frac{d\widetilde{b}_B}{d\mu_B} - \rho'(.)\frac{(1-\mu_A)}{k_B} f_A(\widetilde{b}_A)\frac{d\widetilde{b}_A}{d\mu_B}.$$

Since μ_A is constant, $\frac{d\widetilde{b}_A}{d\mu_B} = \delta s_A$. Using these facts, we can write

$$\frac{d[p_B(1-F_B(\widetilde{b}_B))]}{d\mu_B} = [\rho'(.)\frac{k_A}{k_B^2}(1-F_B(\widetilde{b}_B))f_B(\widetilde{b}_B) - p_B f_B(\widetilde{b}_B)]\frac{d\widetilde{b}_B}{d\mu_B} - \rho'(.)(1-\mu_A)f_A(\widetilde{b}_A)\delta s_A.$$

This expression is negative if $\rho'(.)\frac{k_A}{k_B^2}(1-F_B(\widetilde{b}_B)) - p_B < 0$ or, using $1-F_B(\widetilde{b}_B) = k_B$, if $\rho'(.)\frac{k_A}{k_B} < p_B$, which is equivalent to strict concavity of $p_B = \rho(.)$ for $p_B < \pi$, stated in Assumption 3.

- $p_A < \pi$ and $p_B = \pi$. Since $dp_B = 0$, we have $\frac{d[p_B(1-F_B(\widetilde{b}_B))]}{d\mu_B} = -p_B f_B(\widetilde{b}_B)\frac{d\widetilde{b}_B}{d\mu_B} < 0$, unambiguously.

We differentiate the first equilibrium condition in (22) combined with (21):

$$d\widetilde{b}_A = \frac{s_A}{(1-\mu_A)^2 p_A} d\mu_A + \delta s_A d\mu_B - \frac{\mu_A s_A}{(1-\mu_A)p_A^2} dp_A; \tag{34}$$

$$dp_A = \rho'(\frac{k_B}{k_A})\left[\frac{(1-F_A(\widetilde{b}_A))k_B}{k_A^2} d\mu_A + \frac{f_A(\widetilde{b}_A))(1-\mu_A)k_B}{k_A^2} d\widetilde{b}_A - \frac{f_B(\widetilde{b}_B)}{k_A} d\widetilde{b}_B\right]. \tag{35}$$

Setting $d\mu_A = 0$ above and rearranging terms yields $\frac{d\widetilde{b}_A}{d\delta} > 0$, unambiguously.

Setting $d\delta = 0$ and using $k_A = (1-F_A(\widetilde{b}_A))(1-\mu_A)$, we get $\frac{d\widetilde{b}_A}{d\mu_A} > 0$ if $p_A - \mu_A\rho'(.)\frac{k_B}{k_A} > 0$, which holds by strict concavity of $\rho(.)$, i.e., $p_A > \rho'(.)\frac{k_B}{k_A}$, and the fact that $\mu_A \leq 1$. **Q.E.D.**

Proof of Lemma 5. We begin by differentiating the second equilibrium condition in (22) and the expression for p_B in (20) (thus completing the set, coupled with (34) and (35)):

$$d\widetilde{b}_B = s_B d\mu_B, \tag{36}$$

$$dp_B = \rho'(\frac{k_A}{k_B})\left[-\frac{(1-F_A(\widetilde{b}_A))}{k_B}d\mu_A - \frac{f_A(\widetilde{b}_A))(1-\mu_A)}{k_B}d\widetilde{b}_A + \frac{k_A}{k_B^2}f_B(\widetilde{b}_B)d\widetilde{b}_B\right]. \tag{37}$$

Now consider the expressions in (23) and (24), beginning with d_B.

Set $d\mu_B = 0$, hence by (36), $d\widetilde{b}_B = 0$. Clearly, if $p_B = \pi$, then $\frac{d[d_B]}{d\mu_A} = 0$. Suppose $p_B < \pi$ and thus $p_A = \pi$, hence $dp_A = 0$. Using $d\widetilde{b}_A = \frac{s_A}{(1-\mu_A)^2 p_A}d\mu_A > 0$ from (34) in the expression for dp_B in (37), we get

$$\frac{d[d_B]}{d\mu_A} = \mu_B(1-F_B(\widetilde{b}_B))\frac{dp_B}{d\mu_A} < 0 \quad \text{because} \quad \frac{dp_B}{d\mu_A} < 0.$$

Set $d\mu_A = 0$ and consider

$$\frac{d[d_B]}{d\mu_B} = \mu_B(1-F_B(\widetilde{b}_B))\frac{dp_B}{d\mu_B} + p_B(1-F_B(\widetilde{b}_B)) - p_B\mu_B f_B(\widetilde{b}_B)\frac{d\widetilde{b}_B}{d\mu_B}.$$

The second term is positive but the third is negative, because $\frac{d\widetilde{b}_B}{d\mu_B} > 0$. Therefore, the sign of $\frac{d[d_B]}{d\mu_B}$ is ambiguous regardless the sign of the first term (which is equal to zero if $p_B = \pi$, non-zero if $p_B < \pi$).

Now consider the components of d_A, beginning with

$$\frac{d[d_{AA}]}{d\mu_A} = 1 - F_A(\widetilde{b}_A) - \mu_A f_A(\widetilde{b}_A)\frac{d\widetilde{b}_A}{d\mu_A}.$$

By Proposition 3(b), $\frac{d\widetilde{b}_A}{d\mu_A} > 0$, but the sign of the expression above is ambiguous because it depends on the magnitude of $\frac{d\widetilde{b}_A}{d\mu_A}$. Therefore, the sign of $\frac{d[d_A]}{d\mu_A}$ is also ambiguous.

Set $d\mu_A = 0$ and consider the expression

$$\frac{d[d_{AA}]}{d\mu_B} = -\mu_A f_A(\widetilde{b}_A)\frac{d\widetilde{b}_A}{d\mu_B}.$$

In the case $p_A = \pi$ we have $dp_A = 0$ and thus from (34) we get $\frac{d\widetilde{b}_A}{d\mu_B} = \delta s_A > 0$. If $p_A < \pi$ and so $dp_A \neq 0$, using (35) in (34) it is easy to verify that $\frac{d\widetilde{b}_A}{d\mu_B} > 0$. Thus, $\frac{d[d_{AA}]}{d\mu_B} < 0$, unambiguously.

The last detection measure is d_{AB}. Holding μ_B constant and differentiating the corresponding expression in (24) yields

$$\frac{d[d_{AB}]}{d\mu_A} = -\mu_B\delta\left[p_A(1-F_A(\widetilde{b}_A)) - (1-\mu_A)(1-F_A(\widetilde{b}_A))\frac{dp_A}{d\mu_A} + (1-\mu_A)p_A f_A(\widetilde{b}_A)\frac{d\widetilde{b}_A}{d\mu_A}\right]. \tag{38}$$

If $p_A = \pi$, the second term vanishes and thus, given $\frac{\partial\widetilde{b}_A}{\partial\mu_A} > 0$, the expression of $\frac{d[d_{AB}]}{d\mu_A}$ in (38) is negative. Suppose $p_A < \pi$ and thus $dp_B = 0$. Using (34) in (35) we can express the second term in the squared brackets in (38) as

$$-k_A \frac{d\widetilde{p}_A}{d\mu_A} = -\frac{k_B}{k_A}\rho'(\frac{k_B}{k_A})\Big[(1 - F_A(\widetilde{b}_A)) + f_A(\widetilde{b}_A)\frac{d\widetilde{b}_A}{d\mu_A})\Big].$$

By substitution, the term in the squared brackets in (38) can be written as

$$p_A(1 - F_A(\widetilde{b}_A)) - \frac{k_B}{k_A}\rho'(\frac{k_B}{k_A})\Big[(1 - F_A(\widetilde{b}_A)) + (1-\mu_A)f_A(\widetilde{b}_A)\frac{d\widetilde{b}_A}{d\mu_A})\Big] + (1-\mu_A)p_A f_A(\widetilde{b}_A)\frac{d\widetilde{b}_A}{d\mu_A},$$

or, grouping the terms, as

$$(1 - F_A(\widetilde{b}_A))[p_A - \frac{k_B}{k_A}\rho'(\frac{k_B}{k_A})] + f_A(\widetilde{b}_A)(1-\mu_A)\frac{d\widetilde{b}_A}{d\mu_A}[p_A - \frac{k_B}{k_A}\rho'(\frac{k_B}{k_A})],$$

which is positive because $p_A > \frac{k_B}{k_A}\rho'(\frac{k_B}{k_A})$ by strict concavity of $\rho(.)$ and $\frac{d\widetilde{b}_A}{d\mu_A} > 0$ by Proposition 3(b). Hence, $\frac{d[d_{AB}]}{d\mu_A} < 0$.

Finally, consider

$$\frac{d[d_{AB}]}{d\mu_B} = (1-\mu_A)\delta\left[p_A(1 - F_A(\widetilde{b}_A)) + \mu_B(1 - F_A(\widetilde{b}_A))\frac{dp_A}{d\mu_B} - \mu_B p_A f_A(\widetilde{b}_A)\frac{d\widetilde{b}_A}{d\mu_B}\right]. \tag{39}$$

If $p_A = \pi$ and thus $\frac{dp_A}{d\mu_B} = 0$, $\frac{d\widetilde{b}_A}{d\mu_B} = \delta s_A > 0$, implying $\frac{d[d_{AB}]}{d\mu_B} = (1-\mu_A)\delta\pi[(1 - F_A(\widetilde{b}_A)) - \mu_B f_A(\widetilde{b}_A)\delta s_A]$, whose sign is ambiguous. If $p_A < \pi$ and thus $\frac{dp_A}{d\mu_B} \neq 0$, again no clear statement can be made about the sign of $\frac{d[d_{AB}]}{d\mu_B}$ because (see (34) and (35)) the signs of $\frac{d\widetilde{b}_A}{d\mu_B}$ when p_A can adjust and the sign of $\frac{dp_A}{d\mu_B}$ when $\widetilde{b}_A$ can adjust are ambiguous. **Q.E.D.**

Proof of Proposition 5. Fix a budget allocation (R_A, R_B) such that $R_i > 0$, $i = A, B$. Denote by $\widetilde{e}(R_i)$ the first-best effort under budget R_i, generating the maximal detection probability $\widetilde{\mu}_i = \mu_i(R_i)$ (see Definition 2). By duality, $R_i = c(\widetilde{\mu}_i)$ is the lowest cost for $\widetilde{\mu}_i$.

We know from Proposition 3-(a) that the pair $\{\widetilde{\mu}_A, \widetilde{\mu}_B\}$ uniquely induces a pair of deterrence levels $\{\widetilde{b}_A, \widetilde{b}_B\}$ through (22). Denote the resulting d_{AB} detections, expressed in (24), by $\widetilde{d}^D_{AB}(R_A|\widetilde{\mu}_B)$, which satisfies $\frac{d[\widetilde{d}^D_{AB}]}{d\mu_A} < 0$ by Lemma 5, thus, $\frac{d[\widetilde{d}^D_{AB}]}{dR_A} < 0$ given $\widetilde{\mu}_B$, hence given R_B. By part (b) of Proposition 3, $\frac{d\widetilde{b}_i}{d\widetilde{\mu}_i} > 0$. Thus a higher R_A corresponds to a higher $\widetilde{e}(R_A)$, hence a higher $\widetilde{\mu}_A = \mu_A(R_A)$, which induces a higher $\widetilde{b}_A$ and a lower $\widetilde{d}^D_{AB}(R_A|\widetilde{\mu}_B)$.

Fix agent B's effort $\widetilde{e}(R_B)$, hence, $\widetilde{\mu}_B$. It is easy to verify that the following incentive system induces agent A to exert effort $\widetilde{e}(R_A)$.

$$w^D_A(d_{AB}|\mu_B) = \begin{cases} z(\widetilde{e}(R_A)), & \text{if } \; d_{AB} \leq \widetilde{d}^D_{AB}(R_A|\widetilde{\mu}_B) \\ 0, & \text{otherwise.} \end{cases} \tag{40}$$

Agent A's payoff from choosing effort $\widetilde{e}(R_A)$ is zero. Deviating to a higher effort e^x leads to

$d^x_{AB} < d_{AB}$ and yields the payoff $z(\widetilde{e}(R_A)) - z(e^x) < 0$, whereas deviating to a lower effort e^x leads to $d^x_{AB} > d_{AB}$ and yields the payoff $-z(e^x) < 0$.

Given R_A, hence $\widetilde{e}(R_A)$, Agent B's incentives will be based on d_{AA}, as stated in (40) and based on d_{AB} for agent A. The arguments are similar. **Q.E.D.**

Proof of Proposition 6. We know that under an equal budget allocation in a symmetric crime equilibrium, $p_A = \pi$, $p_B < \pi$, and $\widetilde{b}_A > \widetilde{b}_B$.

Total differentiation of the state's objective function at the induced crime equilibrium yields

$$dSH = -h_A f_A(\widetilde{b}_A)[d\widetilde{b}_A] - p_B h_B f_B(\widetilde{b}_B)[d\widetilde{b}_B] + h_B(1 - F_B(\widetilde{b}_B))[dp_B]. \quad (41)$$

Since $p_A = \pi$ and hence $dp_A = 0$, equations (34)-(37) become:

$$\begin{aligned} d\widetilde{b}_A &= \frac{\mu'(R_A)s_A}{(1-\mu(R_A))^2\pi}dR_A + \mu'(R_B)\delta s_A dR_B, \qquad d\widetilde{b}_B = \mu'(R_B)s_B dR_B, \\ dp_B &= \rho'(.)\left[-\frac{\mu'(R_A)(1-F_A(\widetilde{b}_A))}{(1-F_B(\widetilde{b}_B))}dR_A - \frac{f_A(\widetilde{b}_A))(1-\mu(R_A))}{(1-F_B(\widetilde{b}_B))}d\widetilde{b}_A + \frac{(1-\mu(R_A))(1-F_A(\widetilde{b}_A))}{(1-F_B(\widetilde{b}_B))^2}f_B(\widetilde{b}_B)d\widetilde{b}_B\right]. \end{aligned}$$

Substituting the expressions for $d\widetilde{b}_A$, $d\widetilde{b}_B$ and dp_B in (41) we get

$$\begin{aligned} dSH &= -h_A f_A(\widetilde{b}_A)[\frac{\mu'(R_A)s_A}{(1-\mu(R_A))^2\pi}dR_A + \mu'(R_B)\delta s_A dR_B] - p_B h_B f_B(\widetilde{b}_B)\mu'(R_B)s_B dR_B \\ &\quad + h_B\rho'(.)\Big[-\mu'(R_A)(1-F_A(\widetilde{b}_A))dR_A + \frac{k_A}{k_B}f_B(\widetilde{b}_B)\mu'(R_B)s_B dR_B \\ &\quad - f_A(\widetilde{b}_A)(1-\mu(R_A))\left(\frac{\mu'(R_A)s_A}{(1-\mu(R_A))^2\pi}dR_A + \mu'(R_B)\delta s_A dR_B\right)\Big]. \end{aligned} \quad (42)$$

Set $R_A = R_B$, $dR_A = -dR_B$ and let $F_i(.) = F(.)$, $s_i = s$, $i = A, B$ (symmetric crime environment) in (42). Rearranging the terms and simplifying (42), as $\delta \to 0$ we have $dSH < 0$ if

$$h_A > h_B(1-\mu(\frac{R}{2}))^2\left[\frac{f(\widetilde{b}_B)\pi}{f(\widetilde{b}_A)}(p_B - \rho'(.)\frac{k_A}{k_B}) - \frac{\rho'(.)}{1-\mu(\frac{R}{2})}(1 + \frac{\pi k_A.\mu(\frac{R}{2})}{sf(\widetilde{b}_A)\mu'(\frac{R}{2})})\right],$$

which reproduces (27). If $f(\widetilde{b}_B)\pi \le f(\widetilde{b}_A)$, the coefficient of h_B in the squared brackets is definitely smaller than one because $\widetilde{p}_B < 1$ and $\rho'(x) > 0$ for $x < 1$. Then, (27) holds and the adjustment $dR_A = -dR_B > 0$ at $R_i = R/2$ reduces SH unless h_B is sufficiently larger than h_A. **Q.E.D.**

Proof of Lemma 6. Differentiating the equilibrium conditions (29) and (30) yields $\frac{d\widetilde{b}_B}{d\mu_B} = s_B > 0$ and $\frac{d\widetilde{b}_A}{d\mu_A} = [1 - \delta\mu_B\alpha p_H(1 - F_B(\widetilde{b}_B))]s_A > 0$ as own-deterrence effects. The cross-deterrence effects are $\frac{d\widetilde{b}_B}{d\mu_A} = 0$ and $\frac{d\widetilde{b}_A}{d\mu_B} = \delta(1-\mu_A)\alpha p_H s_A \frac{d[\mu_B(1-F_B(\mu_B s_B))]}{d\mu_B}$. By Assumption 2, the function $\mu_B(1 - F_B(\mu_B s_B))$ is single peaked, at the maximand $\hat{\mu}_B$. Since $\delta(1-\mu_A)\alpha p_H s_A > 0$, $\frac{d\widetilde{b}_A}{d\mu_B}$ is also single peaked, with maximand denoted by $\hat{\mu}^x_B$ in Definition , increasing at $\mu_B < \hat{\mu}^x_B$ and decreasing at $\mu_B < \hat{\mu}^x_B$. **Q.E.D.**

$\widehat{\mu}_B^x = argmax_{\mu_B}\left[\delta(1-\mu_A)\mu_B\alpha p_H(1-F_B(\mu_B s_B))\right]$.

The term $\delta(1-\mu_A)\mu_B\alpha p_H(1-F_B(\mu_B s_B))$, hence $\widetilde{b}_A$, is increasing in μ_B for $\mu_B < \widehat{\mu}_B$ and decreasing otherwise.

Proof of Lemma 7. Consider first the impact of μ_A on the detection measures for crime A. Because $\frac{d\widetilde{b}_A}{d\mu_A} > 0$,

$$\begin{aligned}
\frac{d[d_{AA}]}{d\mu_A} &= (1-F_A(\widetilde{b}_A)) - \mu_A f_A(\widetilde{b}_A)\frac{d\widetilde{b}_A}{d\mu_A}, \quad \text{sign ambiguous,}\\
\frac{d[d_{AB}]}{d\mu_A} &= -\delta\mu_B\alpha p_H(1-F_B(\widetilde{b}_B))\left[(1-F_A(\widetilde{b}_A)) + (1-\mu_A)f_A(\widetilde{b}_A)\frac{d\widetilde{b}_A}{d\mu_A}\right] < 0.
\end{aligned}$$

On the other hand, The sign of $\frac{d[d_{AA}]}{d\mu_B}$ is ambiguous because $\frac{d\widetilde{b}_A}{d\mu_B}$ has an ambiguous sign.

$$\begin{aligned}
\frac{d[d_{AA}]}{d\mu_B} &= -\mu_A f_A(\widetilde{b}_A)\frac{d\widetilde{b}_A}{d\mu_B};\\
\frac{d[d_{AB}]}{d\mu_B} &= (1-\mu_A)\delta p_H\alpha\left[(1-F_A(\widetilde{b}_A))\left[1-F_B(\widetilde{b}_B) - \mu_B f_B(\widetilde{b}_B)\frac{d\widetilde{b}_B}{d\mu_B}\right] - \mu_B(1-F_B(\widetilde{b}_B))f_A(\widetilde{b}_A)\frac{d\widetilde{b}_A}{d\mu_B}\right].
\end{aligned}$$

Using $\frac{d\widetilde{b}_B}{d\mu_B} = s_B$ and $\frac{d\widetilde{b}_A}{d\mu_B} = (1-\mu_A)\delta\alpha p_H[1-F_B(\widetilde{b}_B) - \mu_B s_B f_B(\widetilde{b}_B)]s_A$ in the expression above reveals that the sign of $\frac{d[d_{AB}]}{d\mu_B}$ depends on the sign of $1-F_A(\widetilde{b}_A) - \mu_B f_B(\widetilde{b}_B)f_A(\widetilde{b}_A)(1-\mu_A)\delta p_H s_A$, which is ambiguous.

The sign of $\frac{d[d_B]}{d\mu_B}$ is also ambiguous; d_B is in the form $\mu_B X_B(1-F_B(\widetilde{b}_B))$ where X_B is the measure of potential B-criminals and $\widetilde{b}_B$ is increasing in μ_B. However, $\frac{d[d_B]}{d\mu_A} = -\alpha\mu_A f_A(\widetilde{b}_A)\frac{d\widetilde{b}_A}{d\mu_A} < 0$.

For the proof of the claim that d_{AA}, d_{AB} and d_B are monotonic in μ_B in a range of small μ_B levels, it suffices to verify that the signs of the expressions of $\frac{d[d_{AA}]}{d\mu_B}$, $\frac{d[d_{AB}]}{d\mu_B}$ and $\frac{d[d_B]}{d\mu_B}$ become definite as $\mu_B \to 0$. **Q.E.D.**

Proof of Proposition 8. (i) In the independent crimes case with $k_A = 1$ and $k_B = k_B^*$, the objective of the state is modified as $SH_I = (1-F_A(\widetilde{b}_A)).h_A + k_B^*(1-F_B(\widetilde{b}_B)).h_B$. The optimal budget allocation (R_A^I, R_B^I) then satisfies the analogue of the first-order condition (15), adjusted for the measures of potential criminals:

$$\frac{h_A}{h_B} = \left[p_L + \alpha(p_H - p_L)(1-F_A(\widetilde{b}_A^*))\right]\frac{s_B f_B(\widetilde{b}_B^I)\mu'(R_B^I)}{s_A f_A(\widetilde{b}_A^I)\mu'(R_A^I)}. \tag{43}$$

Consider the case where crime A causes crime B, with $\delta = 0$. Differentiation of (33) with respect to the endogenous variables yields

$$dSH = -f_A(\widetilde{b}_A)\left[h_A + \alpha(p_H - p_L)(1-F_B(\widetilde{b}_B))h_B\right]d\widetilde{b}_A - \left[p_L + \alpha(1-F_A(\widetilde{b}_A)).(p_H - p_L)\right]f_B(\widetilde{b}_B))h_B d\widetilde{b}_B. \tag{44}$$

From the equilibrium conditions (29) and (30), $d\widetilde{b}_i = s_i\mu_i'(R_i)dR_i$. Using this and the fact that at an optimal allocation the impact of a balanced budget shift $dR_A = -dR_B$ on dSC is zero, the condition (44) can be arranged as follows:

$$\frac{h_A}{h_B} = \left[p_L + \alpha(p_H - p_L)(1 - F_A(\widetilde{b}_A^*))\right] \frac{s_B f_B(\widetilde{b}_B^*)\mu'(R_B^*)}{s_A f_A(\widetilde{b}_A^*)\mu'(R_A^*)} - \alpha(p_H - p_L)(1 - F_B(\widetilde{b}_B^*)). \qquad (45)$$

The last term, $\alpha(p_H - p_L)(1 - F_B(\widetilde{b}_B^*))$, represents the impact of agent A's enforcement on the measure of potential B criminals. Absent this external effect, the two optimality conditions (43) and (45), hence their solutions, would be identical.

(ii) In a symmetric crime environment and under an equal budget allocation $R_A = R_B$, $\mu_A = \mu_B$ and thus, $\widetilde{b}_A = \widetilde{b}_B$, $f_A(\widetilde{b}_A) = f_B(\widetilde{b}_B)$ and $F_A(\widetilde{b}_A) = F_B(\widetilde{b}_B)$. Using these facts in (44) with a negative sign for $d\mu_B$ and arranging the terms yields, $dSH < 0$ if

$$f(\widetilde{b}_A)s\frac{h_A}{h_B} > \left[p_L + \alpha(1 - F(\widetilde{b}_A)).(p_H - p_L)\right] f(\widetilde{b}_B))s - f(\widetilde{b}_A)\alpha(1 - F(\widetilde{b}_B)).(p_H - p_L)s.$$

Thus, $dSH < 0$ if $h_A > p_L.h_B$.

Q.E.D.

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